

1 **Estimating changes in temperature distributions in a large ensemble of**
2 **climate simulations using quantile regression**

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ABSTRACT

9 Understanding future changes in extreme temperature events in a transient
10 climate is inherently challenging. A single model simulation is generally in-
11 sufficient to empirically characterize the statistical properties of the underly-
12 ing physical processes governing the climate. Ensembles of repeated sim-
13 ulations (with different initial conditions providing independence) not only
14 alleviate this lack of data but also allow additional new approaches for charac-
15 terizing changes in extremes. We present here one such new approach, using
16 ensembles that allow characterizing changes in temperature distributions us-
17 ing a continuous representation of seasonality rather than breaking the dataset
18 into seasonal blocks. That is, we assume that temperature distributions evolve
19 smoothly both day-to-day over an annual cycle and year-to-year over longer
20 secular trends. To demonstrate our method’s utility, we analyze an ensem-
21 ble of 50 simulations of the Community Earth System Model (CESM) under
22 a scenario of increasing radiative forcing to 2100, focusing on North Amer-
23 ica. The results both confirm aspects of climate system behavior known from
24 previous studies and also elucidate new features. Confirming results include
25 that daily temperature bulk variability generally decreases in wintertime in the
26 continental mid- and high-latitudes ($> 40^\circ$). One new result is that these same
27 wintertime distributions, the low tails “stick”, i.e. experience a lesser reduc-
28 tion in variability, producing a more negative skew. Although the examples
29 above concern temperature only, the technique is sufficiently general that it
30 can be used to generate precise estimates of distribution changes in a broad
31 range of climate variables by exploiting the power of ensembles.

32 1. Introduction

33 Climate timeseries have generally been assumed to be separable into two components: random-
34 ness inherent in the underlying physical processes, which we call natural variability, and forced
35 secular trends that follow from increasing concentrations of greenhouse gases. Recently, the de-
36 gree to which natural variability may itself be changing has received significant scientific interest
37 (e.g. Trenberth 2011; Donat and Alexander 2012; Deser et al. 2012a; Thompson et al. 2015; Kay
38 et al. 2015). Potential changes in climate extremes, because of their heightened societal impacts,
39 are of special concern (e.g. Davison and Smith 1990; Stott et al. 2004; Chavez-Demoulin and Davi-
40 son 2005; Eastoe and Tawn 2009; Otto et al. 2012; Swain et al. 2014; Singh et al. 2014; Trenberth
41 et al. 2015; Diffenbaugh et al. 2015; Huang et al. 2015a; Jalbert et al. 2017). However, detecting
42 and understanding such changes on timescales relevant to human activities is inherently challeng-
43 ing and generally data-limited. The long equilibration time of the climate system means that on
44 the timescales of interest to human society, the climate state will be evolving (or ‘transient’), so
45 that its statistical properties are not stationary. Fully characterizing this evolving natural variability
46 of rare events is intrinsically challenging due to the limited amount of available observations or
47 simulation data. Studies of future climate extremes often employ statistical extreme value theory
48 to make inferences about rare events with modest amounts of data.

49 In this work, we study the entire distribution of temperatures in a transient climate, including
50 rare events, by employing quantile regression on an ensemble of simulations of an identical forcing
51 scenario from a single climate model. Given sufficient care in choosing different initial conditions,
52 such a large ensemble will reflect the natural variability of the system, since each simulation will be
53 statistically independent in terms of its natural variability. The increased data provided by multiple
54 simulations then admits more confident statements about changes in the statistical behavior of the

55 system than can be made with a single simulation. While the use of ensembles is a relatively recent
56 development, it is growing rapidly (e.g. Deser et al. 2012b,a, 2014; Fischer and Knutti 2014; Kay
57 et al. 2015; Srivastava et al. 2015; Rodgers et al. 2015; Hagos et al. 2016). Deser et al. (2012b),
58 Deser et al. (2012a) and Fischer and Knutti (2014) in particular discuss how ensembles help in
59 distinguishing internal climate variability from anthropogenic effects and allow more accurate
60 estimates for the forced model response.

61 Ensembles of multiple simulations offer at least three advantages that are currently under-
62 exploited. The most obvious advantage is that the increased data volume allows examining the
63 entire distribution of a climate variable. Studies of climate variability to date are generally di-
64 vided between those that address the center of the distribution (e.g. Semenov and Bengtsson 2002;
65 Räisänen 2002; Kitoh and Mukano 2009; Screen 2014; Schneider et al. 2015), and those that ad-
66 dress its tails (e.g. Katz and Brown 1992; Meehl et al. 2009; Northrop and Jonathan 2011; Davison
67 et al. 2012; Huser and Davison 2014; Trenberth et al. 2015; Huang et al. 2015b; Jalbert et al. 2017),
68 generally via extreme value theory. A more limited body of studies address overall distributional
69 changes in climate variables, but these generally focus on observations or observation-based data
70 products, which are necessarily limited in terms of data amount and therefore require spatial or
71 temporal aggregation (Donat and Alexander 2012; Stainforth et al. 2013; Chapman et al. 2013;
72 Huybers et al. 2014; McKinnon et al. 2016; Rhines et al. 2017). When studying model projec-
73 tions using ensembles, the large amount of data at each location allows us to accurately estimate
74 changes in the distribution of climate variables (e.g. temperature) without spatial aggregation.

75 A second potential advantage provided by data-rich ensembles is that trends in both means
76 and variability need not be modeled as linear in time (Franzke 2015; Gao and Franzke 2017).
77 Typically, analyses assume linear trends, but in realistic scenarios, forcings are not linear over
78 centennial timescales, and a linear approximation can be misleading (see for example Poppick

79 et al. 2017). The increased data provided by ensembles means that we can consider more flexible
80 statistical models to better represent complex climate responses. As we will show, distributions
81 of daily temperature evolve nonlinearly, and follow different trajectories even as a function of
82 quantiles (i.e. different parts of the distribution). Analysis methods should therefore be able to
83 take into account nonlinearities both in time and across quantiles.

84 Finally, a third advantage of ensembles is that they allow a more natural treatment of seasonal
85 variation in climate variables. In situations of limited data, it is standard practice to treat seasons
86 separately, assuming that each season has a temporally constant average and stationary statistical
87 properties discontinuous from neighboring seasons. With ensembles of simulations, we can allow
88 for a smooth change in the underlying trend from day to day, using a parsimonious set of param-
89 eters. By modeling the entire year on a continuum, we can explore how each season transitions
90 to the next and how seasonal patterns change over time, features that may be highly dependent on
91 both geographic location and quantile.

92 We describe here a methodology for exploiting ensembles to study changing climate variabil-
93 ity that captures these advantages: we model the complete distribution of daily temperatures as
94 a continuous function of both seasonality and secular climate change over time. Although the
95 methodology is applied to temperature here, it is general and can be applied to other climate vari-
96 ables of interest. We also show how such an ensemble-based approach is well-positioned for the
97 purposes of uncertainty quantification. Because each simulation is treated as an independent sam-
98 ple drawn from the ensemble of simulations, we circumvent the issue of dependency within each
99 simulation. We can therefore obtain uncertainty quantifications for all estimates by resampling
100 complete simulations from the ensemble.

101 In the sections that follow, we describe estimated changes in both bulk and tail variability as
102 differences in two quantiles; a large quantile difference implies more variability in a given region

of the distribution. When those quantiles lie in the high or low tails, the quantile difference is a measure of the spread or thickness of the tail. Figure 1 gives a pictorial explanation of how quantile differences reflect bulk and tail variability. Although the estimated model is seasonally continuous, we also present results assuming seasonally constant conditions, and show that the seasonal effect on temperature can indeed be explained with a reasonably smooth function. When applied to model runs of a realistic future climate scenario, results reproduce some well-understood changes (e.g. strong reduction in wintertime variability at continental mid-latitudes) and produce some new insights (e.g. strong changes in skewness driven by low tail behavior).

2. Data

We apply our algorithm to an ensemble of 50 historical/future simulations of the Community Earth System Model (CESM) (Sriver et al. 2015). The atmospheric component is the low-resolution Community Atmosphere Model version 4, with T31 spectral resolution ($\sim 3.75^\circ \times 3.75^\circ$) and 26 vertical levels. The model ocean component is the low-resolution version of the Parallel Ocean Program version 2 (Smith et al. 2010) with a nominal horizontal grid resolution of 3° , augmented to approximately 1° at the equator. The ocean model contains 60 vertical levels, down to a maximum depth of 5,500 m.

The ensemble is especially appropriate for the purpose of studying variability because it is based on a $\sim 10,000$ year pre-industrial control simulation. After a ~ 4000 year spin-up using constant preindustrial conditions, we initialize 50 historical hindcasts (1850-2005) from snapshots of the coupled model state taken every 100 years, so that the last hindcast is initialized after approximately 9000 years of the control simulation. We then extend each hindcast to 2100 using the Representative Concentration Pathway (RCP) 8.5 scenario. The 100-year gap between each new initialization ensures nearly independent ensemble members that fully capture internal variability

126 within the coupled system. RCP8.5 corresponds to anthropogenic radiative forcing of roughly 8.5
127 W m^{-2} by 2100 (Moss et al. 2010). More information about the model and ensemble design can
128 be found in Sriver et al. (2015).

129 CESM does show some known biases that affect primarily temperature means (and possibly
130 trends in means), but also to some extent the higher-order moments of the temperature distribution,
131 e.g. variance and skewness. Known model biases include reduced ocean heat transport, low north
132 Atlantic sea surface temperature, and excessive northern hemisphere sea ice (Shields et al. 2012).
133 The model generally underestimates both temperature and precipitation extremes compared with
134 observations, i.e. the mean of the extreme value distributions is biased, but the scale and shape are
135 consistent with observations for the continental United States (Sriver et al. 2015).

136 To evaluate whether the CESM simulations provide sufficiently realistic temperature distribu-
137 tions for the purpose of this analysis, we compare CESM temperatures with those from the ERA-
138 Interim (European Reanalysis) data product (Dee et al. 2011). Figure 2 shows the model/reanalysis
139 comparison for winter; for summer see Supplementary Online Material Figure S1. The model un-
140 derestimates variability in some places, and produces excessively cold winter temperatures in the
141 Arctic. The resulting exaggerated temperature gradients contribute to excess variability and ex-
142 cessively negative skew in the northern mid-latitudes. (Skewness is proportional to the cube of
143 temperature after subtracting off the average seasonal temperature; see Appendix A1.) Through-
144 out this work, we will show in-depth analysis from three locations with distinct temperature dis-
145 tributions to highlight our proposed method (**a**, **b**, and **c** shown in Figure 2). See Supplementary
146 Online Material Figure S2 for comparison of model and reanalysis temperature distributions in
147 both summer and winter for these locations.

3. Methods

In the methodology presented here, we model temperature at each location as a function of both seasonality and long-term change of the annual mean climate. We use two independent variables, with seasonality represented by a variable d , the day of the year (spanning values 1 to 365), and change in annual mean temperature represented by a variable t , years elapsed since 1850 (spanning 0 to 250 for these scenarios). We thus assume that each temperature quantile can be described by two sets of basis functions that represent the two variables' independent relationships with temperature (called here $\{f_i(d)\}$ and $\{g_j(t)\}$), and interaction terms $h_i(d)s_j(t)$, where f_i, g_j, h_i , and s_j are all smooth functions of the appropriate variable. The interaction terms are required to capture effects in which long-term temperature evolution differs between seasons, e.g. the robust projection that winter temperatures warm more than summer temperatures. To impose our smoothness condition, we assume that f_i, g_j, h_i , and s_j are piecewise cubic polynomials with a continuous second derivative, also called splines. (For a review of cubic polynomial basis functions, see Hastie et al. 2009, Chapter 5.) Because the seasonality variable d is periodic, its basis functions are also assumed periodic. For more details, see Appendix A2a.

We choose the number of basis functions by evaluating a metric representing model sufficiency. Our model sufficiency criterion is aimed at capturing the long term underlying signal. We do not require estimated quantile functions to capture transient events during the historical period like volcanic eruptions. Details on how we select the number of basis functions is given in Appendix A2b. In our climate simulation output, the intra-seasonal effect requires more detailed modeling than the inter-seasonal effect. In the results shown here, we fit the model with 15 terms (that is, basis functions) for the main seasonal effect $\{f_i\}$, but the interaction terms require less seasonal complexity, so we use only 3 terms for $\{h_i\}$. We use 4 terms for both the temporal change $\{g_j\}$

171 and the interaction terms $\{s_j\}$. That is, modeling long-term change generally requires fewer terms
 172 than modeling seasonality. In summary, we use 32 basis functions in total including an intercept
 173 term. We then fit each q^{th} quantile of temperature

$$T_q(d, t) = \alpha + \sum_i a_i f_i(d) + \sum_j b_j g_j(t) + \sum_{i,j} c_{i,j} h_i(d) s_j(t), \quad (1)$$

174 where all of the coefficients depend on q but we suppress the dependence for convenience. This
 175 fit determines coefficients $a_i, b_j, c_{i,j}$ for each quantile at each location.

176 To simplify notation, we construct a matrix X where each column contains a basis function and
 177 each row refers to a unique value of d and t . Using this matrix, X , we construct our temperature
 178 model in vectorized form,

$$T_q = X\beta_q, \quad (2)$$

179 where β_q contains the basis coefficients $a_i, b_j, c_{i,j}$. If the total number of basis functions is p and
 180 the total number of observations is n , then the predictor matrix X will have dimensions $n \times p$ and
 181 β_q will be a p -length vector. To get a confidence interval around T_q , we re-estimate the coefficients,
 182 β_q , using a resampled data set. Because we have 50 simulations we resample the data by drawing
 183 whole simulations from our ensemble of 50 simulations. Each simulation is treated as a contiguous
 184 block of data, and the dependency structure within these blocks is maintained when resampling
 185 whole simulations at a time. By repeating this resampling and re-estimation procedure 100 times
 186 we obtain pointwise confidence bands around each estimated T_q . Appendix A2c provides further
 187 details about uncertainty quantification.

188 As an example of a typical model fit, we show in Figure 3 the seasonal cycle in CESM daily
 189 temperatures for three locations, along with estimates of low, median and high quantiles. We show
 190 here data from 1850 to demonstrate the seasonal fit rather than that of the long-term trend. All
 191 locations show strong seasonal differences in variance that are well-represented by our smooth

estimates. Relevant features that are captured include an asymmetrical seasonal cycle in all locations; a clear left skewness in wintertime in all three locations (although most pronounced in the higher-latitude **a** and **b**); and a distinct springtime shoulder in the higher-latitude locations. These characteristics show the benefit of explicitly modeling seasonal variations as smoothly varying functions as opposed to a set of four constant functions changing value with season. Nuances like the decrease in winter temperature spread (variability) from early to late winter would not be captured by a piecewise constant model.

4. Results

To facilitate comparison to previous studies, we first perform a preliminary analysis where we replicate more standard methods. That is, we examine changes in the aggregate distribution of temperatures over multi-week and multi-month intervals, before we show results from our new approach that calculates responses for individual days. Even the standard analysis readily shows that temperature distributions in the CESM ensemble change over the RCP 8.5 scenario (Figures 4, 5, and 6, which compare the initial and final time windows 1850-1864 and 2086-2100). Means uniformly shift warmer, but the shapes of the distributions also change in terms of variance and skewness. Figure 4 shows initial and final distributions in our example locations for aggregated 15-day periods in winter and summer. In at least two of the three depicted locations, it is clear that the distributions are becoming narrower, although quantifying exactly how the tails are changing requires a quantification of the tail size and shape.

Regarding the spatial characteristics of temperature distributions, we see the expected strong decrease in variance in winter over land, especially in the northern mid-latitudes (Figures 5 and 6). By contrast, summer variance changes are much smaller and differ in sign in different locations. Temperature skewness, i.e. the asymmetry of the distribution, shows strong changes in winter

215 over land in a dipole pattern. Winter temperature distributions are in all time periods negatively
216 skewed throughout most of the domain, but in the north (including locations **a** and **b**), they become
217 more negatively skewed in the future, while in the south (including location **c**), they become more
218 symmetric. Summer skewness changes are again smaller and with less spatial coherence, other
219 than the strong transitions in the Southern Great Plains and in Mexico/Central America, where
220 skewness in temperature distributions actually changes sign.

221 Our methodology for quantile estimation provides additional information that helps to quantify
222 how temperature distributions are changing and to estimate the uncertainty associated with each
223 change. We can evaluate not only bulk variability – the interquartile range (IQR), the difference
224 between the 0.25th and 0.75th quantiles – but differences between any two quantiles. We there-
225 fore evaluate the difference between two low or high quantiles, denoted Δq_{low} and Δq_{high} , which
226 measure tail variability in the same way that interquartile range measures the variability of the
227 bulk distribution. If the skewness of a distribution changes over time, then future distributions
228 are not simply scaled versions of present distributions. That is, their tail variabilities must change
229 differently than does the IQR. In the case of the northern mid-latitudes winter temperatures shown
230 in Figure 5, where distributions become more negatively skewed as bulk variability decreases in
231 the future, the effect could result from either/both a low tail contracting less than the bulk (or ac-
232 tually increasing), or a high tail contracting more than the bulk. Our methodology allows readily
233 differentiating these cases.

234 To assess whether the high tail and/or the low tail is driving changes in skewness, we consider
235 the fractional changes in low, high, and bulk variability. If we denote the initial and final quantile
236 difference as $\Delta q_{p,i}$ and $\Delta q_{p,f}$ at the p^{th} percentile, the temporal change in quantile differences

relative to the initial year is then

$$\rho = \frac{\Delta q_{p,f} - \Delta q_{p,i}}{\Delta q_{p,i}}. \quad (3)$$

Because we model the complete temperature distribution for each day of the year for all years, we choose a representative day to understand winter and summer changes (Jan 1 and July 5, respectively), and consider the difference between the beginning and end of the scenarios, the years 1850 and 2000. For these representative days, we show in Figure 7 the fractional variability changes of ρ for low and high tails as well as the IQR.

Results show that tail changes can indeed differ strongly from changes in the bulk of the distribution. In wintertime (Figure 7, top row), in much of the northern mid-latitudes (including locations **a** and **b**), low tails change in a way that contributes to more a more negative skew. Low tail variability contracts less than does the IQR, while high tail variability contracts more strongly. (High tails would contribute to more negative winter skew predominantly in the Hudson Bay region, where the model shows distinct bias.) In summertime (Figure 7, bottom row), the high tail dominates the transition to positive skew in the Southern Great Plains region (including location **c**).

To clarify the relative contributions of high and low tails to skewness changes, we also examine evolving temperature variability in the bulk and tails as a function of seasonality as well as long term change. Figure 8 shows absolute variability changes for the three example locations **a**, **b**, and **c** estimated using our quantile model, and for fractional changes see Supplementary Online Material Figure S6. The uncertainty around our estimates is quantified by resampling the original simulations (with replacement) and recomputing the estimates using this new set of simulations (see Appendix A2c for details). In all locations, wintertime skewness changes are driven by the relative changes in IQR and low tails. In the higher-latitude locations **a** and **b**, more negative winter skew results because the IQR contracts even more strongly than does the low tail variability. In

259 other words, the low tails “stick”. In the lower-latitude location **c**, more positive winter skew
260 results because the IQR barely changes while the low tail variability contracts strongly.

261 The complexity of the relationships in Figure 8 also shows how misleading it may be to use
262 a three-month block to represent a season. While all three locations show IQR in winter than
263 summer, the transition from winter to summer happens more quickly in some locations than at
264 others, more abruptly in the northernmost location **a** and more gradually in **c**. Low-tail variability
265 seasonal transitions are even sharper than those of IQR in **a** and **b**, but more gradual in **c**. In
266 contrast, high-tail variability is more seasonally constant overall than low-tail variability. Through
267 these examples, we see how our method offers detailed information about changes in variability
268 across seasons and annual change, usually unavailable when analyzing each season separately.

269 While we show only three locations in the text here, an online interactive application allows
270 similar in-depth examination of changes in model temperature distributions at all locations within
271 North America, available at <https://matzhaugen.com/links.html>. The application allows the user to
272 browse through any desired location to see how the variability changes as a function of season,
273 year and quantile difference. We include temperature histograms of the first and last simulation
274 year for the designated location, as well as maps that show the variability change spatially.

275 **5. Conclusions**

276 We present a method to quantify changes in tail variability of temperature with high precision in
277 a transient climate model. Using data from the whole year and the whole span from 1850-2100 we
278 estimate temperature as a function of seasonality and long term change. Analyzing the whole year
279 simultaneously as opposed to analyzing each season separately allows for more flexible modeling
280 of seasonality. The large ensemble makes it possible to fit such models stably.

281 By resampling entire simulations from the ensemble of climate simulations and recalculating
282 the quantiles, we obtain confidence bands that do not require any assumptions of independence
283 within any one simulation. We show that the smooth quantile estimates are accurate even across
284 small intervals of the domain of the predictors. The fidelity of these intervals serves as a criterion
285 to determine the required complexity in the statistical model.

286 The techniques presented in this study are validated in part by the fact that they replicate several
287 prior conclusions made in the literature, e.g. the well-known projected decrease in winter variabil-
288 ity in the northern mid-latitudes (e.g. Schneider et al. 2015) most likely due to amplified warming
289 in the arctic (Screen 2014). Our approach furthermore allows us to quantify tail variability and
290 give corresponding confidence intervals around our estimates. In the case study of CESM runs
291 analyzed here, we relate the changes in tail variability to changes in skewness of the tempera-
292 ture distributions, and find that in most of the domain analyzed, wintertime skewness changes are
293 driven largely by the relative behavior of IQR and low tails. For example, in much of the conti-
294 nental northern U.S. and Canada, the low tail of temperature contracts substantially less than does
295 the overall temperature variability.

296 These results may inform physical explanations for the projection that skewness in winter tem-
297 perature changes in a dipole pattern across North America. It is possible that the skewness change
298 is a result of a change in the mean location and variability of the mid-latitude jet stream (e.g.
299 Barnes and Polvani 2013); this possibility may warrant further study.

300 The abundance of data available in ensemble simulations relative to single simulations allows
301 using quantile regression to accurately estimate high quantiles, avoiding some of the limitations
302 of extreme value theory. Unlike quantile regression, methods using extreme value theory require
303 making assumptions about the shape of the tail of the distribution. By parameterizing the season-
304 ally time-varying distribution of temperature through smooth functions using the whole year as

our domain, we also reveal previously unavailable details about seasonal transitions. For example, we show here that springtime variability decreases occur later in the year at lower latitudes, and that seasonal transitions in tail variability differ from those in IQR. While we analyze only temperature here, our method is intended to be general enough to be applied to other climate variables such as precipitation or humidity. These detailed insights into climate variable distributions may be valuable for risk assessment studies that emphasize extreme events.

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A1. Model and reanalysis comparisons

Following the discussion on the paper, we define sample mean, variance and skewness as

$$\begin{aligned}\bar{x} &= \frac{1}{n} \sum_{i=1}^n x_i \\ s^2 &= \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \\ \gamma &= \frac{1}{n} \sum \left(\frac{x_i - \bar{x}}{s} \right)^3.\end{aligned}\tag{A1}$$

These definitions are used in Figures 2, 5, and 6 in the main text and in Supplementary Online Material Figures S1 and S2. We plot the standard deviation s rather than the variance s^2 .

A2. Model Details

In the following, we first give details regarding the regression of temperature quantiles on a fixed set of basis functions. We then discuss how to select the number of basis functions, through a “sufficiency criterion”. Lastly, we describe how we quantify uncertainty in the quantile estimates.

a. Model estimation

Given the number of basis functions in our model, represented by the columns in a matrix X with number of rows equal to the number of observations in the data set, we construct our temperature quantile estimate, \hat{T}_q , and corresponding coefficients, $\hat{\beta}_q$, viz.

$$\hat{T}_q = X\hat{\beta}_q\tag{A2}$$

such that the q^{th} fraction of residuals between the observations T at a particular location and their estimates, $T - \hat{T}_q$, are greater than zero and a fraction $1 - q$ are less than zero. With the

335 temperature model in Equation 2, our coefficient vector estimate, $\hat{\beta}$, contains the estimates of
 336 $a_i, b_j, c_{i,j}$. Note that the seasonal interaction terms corresponding to the coefficients $c_{i,j}$ are not
 337 necessarily the same as the main seasonal terms corresponding to a_i . In fact, we find that fewer
 338 seasonal interaction terms are needed to describe the interaction behavior.

339 Computationally, obtaining the above quantile is equivalent to solving the following optimiza-
 340 tion problem (Koenker and Bassett Jr 1978),

$$\min_{\beta} \left\{ \sum_{d,t: T(d,t) \geq X(d,t)\beta} q|T(d,t) - X^T(d,t)\beta| + \sum_{d,t: T(d,t) < X(d,t)\beta} (1-q)|T(d,t) - X^T(d,t)\beta| \right\}, \quad (\text{A3})$$

341 and can be implemented in either R or MATLAB using existing libraries¹. Because we have
 342 access to 50 simulations, each location provides us with $365 \times 250 \times 50$ or approximately 4.5
 343 million observations. Consequently, even fairly high quantiles can be accurately estimated without
 344 borrowing data from neighboring locations through a spatial model as done by e.g. Reich et al.
 345 (2011). However, making inferences about more extreme quantiles, such as the quantiles .001 or
 346 .999, cannot be guaranteed to work as well with our methods.

347 We do not experience issues with quantile estimates crossing in our study area even though the
 348 optimization framework above does not explicitly enforce monotonicity with increasing percentile
 349 estimates. The absence of crossing quantiles is likely also due to the large sample size. For strict
 350 enforcement of monotonicity in the quantile curves see e.g. Bondell et al. (2010).

¹We use the R library `rq` and the function `rq.fit.pfn`, developed by Portnoy and Koenker (1997). Basis functions are created using `pbs` for periodic spline basis functions and `ns` for non-periodic splines. The non-periodic splines are constrained to be linear beyond the domain, 1850-2100, and are called *natural splines*.

351 *b. Model selection*

352 We describe our approach to selecting a modest set of basis functions that can accurately rep-
 353 resent the temperature data. If the model chosen has too many basis functions we run the risk of
 354 overfitting out-of-sample observations. To make sure this does not happen we need a metric to
 355 quantify the goodness-of-fit of the model.

356 Any reasonable temperature model we fit to the data will by definition contain the desired
 357 amount of positive and negative residuals *globally* according to the desired quantile q . A more
 358 stringent requirement would be that the smooth temperature estimate contains approximately an
 359 appropriate fraction of positive and negative residuals on a *daily* basis: for each d and t ,

$$S(d, t) = \frac{1}{n} \sum_{i=1}^n I [\hat{T}_i(d, t) - T_i(d, t) > 0] \approx q, \quad (\text{A4})$$

360 where I is the indicator function and n is the total number of samples (i.e. 50 for our CESM
 361 ensemble data set). If $S(d, t)$ is close to the value q for each d and t , the model would accurately
 362 describe the data and the number of basis functions is sufficient. In reality, we are looking basis
 363 functions that obey A4 with d averaged over blocks of days to increase the sample size, e.g. 10
 364 days blocks. It is also not the goal to capture the quantile at too short a timescale as events like
 365 volcanic eruptions would interfere with the estimate.

366 In order to estimate the appropriate number of basis functions, we hold out 5 simulations from
 367 the fitting process and use these to calculate our exceedences, which we call $S_{test}(d, t)$. We repeat
 368 this 10 times so that all the simulations are eventually held out, giving 10 samples of $S_{test}(d, t)$. As
 369 we increase model complexity through degrees of freedom in the basis functions, the variability
 370 of S_{test} should reach a minimum when the necessary number of basis functions is reached and
 371 the quantile estimate is the same for each time point. If the number of basis functions is increased
 372 beyond this point, we start to overfit the data and the out-of-sample variability of S_{test} will increase.

373 To estimate S_{test} , we block the variables in two ways, one for each variable. First, we divide
 374 each year in 10-day bins and calculate the average exceedence estimate, \hat{S}_{test} , in each bin. We
 375 sum over the whole domain of long term change, t , and a subset of the seasonality variable, d .
 376 Specifically, let A be a set of non-overlapping contiguous blocks of days that together cover the
 377 whole year, where a_j , $j = 1, \dots, m$ are the elements of the set. Also let T be the index set for
 378 long term change, $T = [1850, 2100]$, measured in years. Then, for all $a_j \in A$,

$$\hat{S}_{test}(a_j) = \frac{1}{n} \sum_{i=[1,n], d \in a_j, t \in T} I[\hat{T}_i(d, t) - T_i(d, t) > 0]. \quad (A5)$$

379 To get an equal number of days in each bin we use the first 360 days of the year only.

380 Second, we divide the long term change variable, t , in bins and repeat the process by flipping the
 381 role of the variables in Equation A5 to get a set of $\hat{S}_{test}(b_j)$ with $b_j \in B$, a set of non-overlapping
 382 contiguous blocks of long-term change indices in T . An example of the blocked exceedence
 383 estimate is shown in Figure 9. Note that the pointwise quantile estimate is contained between
 384 the error bars, suggesting that the model is sufficiently complex. The standard deviation of these
 385 estimates of \hat{S}_{test} is our measure of exceedence variability.

386 We seek the simplest model that gives good calibration of the quantile estimates (so close to
 387 0.05 in Figure 9). At the same time we have to watch out to not overfit the data so we also want to
 388 minimize out-of-sample variability. We find that a model with 15 seasonal, 3 seasonal-interaction
 389 and 4 temporal degrees of freedom minimizes the variability of exceedences \hat{S}_{test} , shown in Figure
 390 10, where seasonality has been binned. The out-of-sample fit when binning long-term change is
 391 shown in Figure S7 in the supplement. Here, models 4-6 have approximately equal test error, so
 392 since binning seasonality suggests the complexity of model 6, we chose model 6 as the overall
 393 model. Including the possible interaction terms, the full model has 32 free parameters to be fitted,

394 or $\hat{\beta} \in \mathbb{R}^{32}$. All model candidates are shown in Table 1. We reach the same conclusion when
395 blocking the long term change, t , and when analyzing different spatial locations (see Figure S7).

396 *c. Uncertainty Estimation*

397 With a reasonable model chosen through cross-validation, we present a way to quantify its un-
398 certainty. Because we are using multiple simulations that are assumed independent, we resample
399 entire simulations from the set of 50 simulations. Resampling 50 new simulations with replace-
400 ment from the original set of simulations yields a new dataset. From the new data set we obtain
401 another temperature estimate with the same model basis functions but different coefficients, β^* .
402 After repeating this resampling and re-estimating procedure 100 times we generate pointwise con-
403 fidence intervals for temperature quantiles. For example, in Figure 8 we show the 90% confidence
404 interval by selecting the pointwise 5^{th} and 95^{th} percentiles of temperature variability estimates.
405 Because the confidence intervals are quite tight we deem the 100 new estimates (or bootstraps)
406 sufficient to indicate that the results we describe in section 4 are not due to random variation.
407 Larger number of bootstrap replicates might give slightly more accurate intervals but would not
408 change our conclusions. One might also consider fewer simulations as a compromise between
409 computation time and quality of the estimates. Assuming normally distributed confidence inter-
410 vals, we would expect the standard error to scale as $1/\sqrt{n}$. Thus, if one is willing to widen the
411 confidence intervals by a factor of 2 (approximately) only 10 simulations would suffice. However,
412 one could compensate for this greater variability by using fewer basis functions at a cost, of course,
413 of obtaining less resolved estimates of seasonal patterns and long-term trends in the quantiles.

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535 LIST OF TABLES

536	Table 1.	Degrees of freedom in the spline basis for each independent variable, with the	
537		interaction terms including the reduced set of seasonal polynomials with de-	
538		grees of freedom listed in the middle column. The temporal polynomials are	
539		the same in both the main and interaction terms.	29

	Seasonal	Seasonal-Int.	Temporal
1	5	3	3
2	7	3	3
3	10	3	3
4	10	3	4
5	12	3	4
6	15	3	4
7	15	3	5
8	15	5	5
9	18	5	5

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LIST OF FIGURES

- 544 **Fig. 1.** An illustration of concepts and values related to distributions used in this paper. The cartoon shows a
 545 positively skewed (or “right-skewed”) probability distribution and the three quantile differences discussed
 546 in this paper, in the low tail, high tail, and middle of the distribution. The p^{th} quantile in a distribution is the
 547 value such that the probability of being below it has probability p . Here Δq_{low} is the difference between the
 548 0.025^{th} and 0.05^{th} quantiles; the IQR or Interquartile Range that between the 0.25^{th} and 0.75^{th} quantiles;
 549 and Δq_{high} that between the 0.95^{th} and 0.975^{th} quantiles. The values Δq_{low} and Δq_{high} quantify variability
 550 in extreme values while IQR quantifies variability in the bulk of the distribution. 32
- 551 **Fig. 2.** Comparison of daily temperature distribution properties (mean, standard deviation, and skewness) between
 552 the CESM ensemble and ERA-Interim, for winter (DJF; aggregating all daily temperatures without deseasonalizing). We compare the years 1979-1994, the first available 15 years of the ERA-Interim dataset, and
 553 upscale the reanalysis from 0.75° to 3.75° resolution to match CESM. Units on top two rows are degrees
 554 Celsius; bottom row showing skewness is dimensionless. Winter skewness over the continental U.S. is neg-
 555 ative in both model and reanalysis, implying a thicker lower tail; see Figure 1 and Appendix A1 for example
 556 and definitions. Overall, large-scale geospatial patterns are similar in both data sets, though some discrep-
 557 ancies are present (e.g. abnormally cold model Arctic winters). Letters **a-c** mark locations that will be used
 558 in examples throughout the paper; these are ordered from north to south, with latitudes and longitudes **a**
 559 $(50.1, -101)$, **b** $(42.7, -82)$, **c** $(35.3, -98)$ 33
- 561 **Fig. 3.** Illustration of results of our quantile estimation procedure using the 50-member CESM ensemble. The
 562 figure shows ensemble daily mean temperatures for the year 1850 for the three representative locations
 563 **a**, **b**, and **c** defined in Figure 2. The ensemble provides 50 points per day but for clarity we show only
 564 10% of the data. Solid lines show the median daily temperature and dashed lines the 2.5^{th} and 97.5^{th}
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 566 the points exceeding the smooth median estimate are approximately uniform across time (notwithstanding
 567 the amplitudes of residuals), suggesting that the quantile estimate is accurate for each day. XX put in
 568 Michael’s text here At all sites, the estimated quantile curves capture the seasonally changing patterns in
 569 the distributions reasonably well. 34
- 570 **Fig. 4.** Evolving distributions of daily mean temperature in the CESM ensemble RCP8.5 model runs at the loca-
 571 tions **a**, **b**, **c** defined in Figure 2. Each distribution includes temperatures from a 15-day period over 15
 572 model years for a total of 11,250 observations ($15 \text{ days} \times 15 \text{ years} \times 50 \text{ ensemble members}$). Winter distri-
 573 butions are taken from Jan 1-15 and summer July 5-19; “initial” distributions include years 1850-1864 and
 574 “final” years 2096-2100. Changes in distributions are readily apparent, especially in winter at higher lati-
 575 tudes (locations **a** and **b**), but detailed quantification, especially of tail changes, requires more sophisticated
 576 techniques. 35
- 577 **Fig. 5.** Initial temperature distribution properties (left) and their changes over time (right) in the CESM ensemble
 578 RCP8.5 model runs, for aggregate wintertime (DJF) daily temperature. Initial (“pre-industrial”) and final
 579 periods are defined as in Figure 4, as 15-year periods 1850–1864 and 2086–2100. Distributional moments
 580 (mean, standard deviation, and skewness) are defined as in Figure 2. Units on the top two rows are degrees
 581 Celsius, while the bottom row showing skewness is dimensionless. Gray crosses mark locations where the
 582 changes are not significant at the 0.05 level, obtained by resampling the set of 50 simulations (with replace-
 583 ment) and recalculating the sample moments. *Top right:* Mean temperature universally increases; note that
 584 color scale begins at XX degrees. Extreme warming in the Hudson’s Bay region occurs where the model is
 585 biased low in present-day simulations. *Middle right:* As expected, standard deviation decreases strongly at
 586 higher latitudes. *Bottom right:* Changes in winter skewness show a dipole pattern, which enhances negative
 587 skew above $\sim 40^\circ$ but reduces it at lower latitudes. 36
- 588 **Fig. 6.** As in Figure 5 but for aggregate summer (JJA) temperatures, and note that scales differ from those in Figure
 589 5. Except in the desert Southwest and Mexico, changes in standard deviation (*middle right*) and skewness
 590 (*bottom right*) are generally smaller in summer than in winter and often not significant at the 0.05 level. 37
- 591 **Fig. 7.** Changes in daily temperature variability (quantile differences) over time in CESM ensemble RCP8.5 runs
 592 estimated using our statistical approach. Because our approach removes the need to aggregate over time

when presenting changes, we show here differences in distributions for a single day and year: Jan 1 for winter (*top*) and July 5 for summer (*bottom*), with differences evaluated between the years 1850 and 2100. Changes are expressed as fractions of initial variability, so that the value 0 indicates no change with respect to the initial year. *Left, middle, and right* columns show, respectively, changes in low tail variability, IQR, and high tail variability, as previously defined. Gray crosses mark grid points where the change is less than 3 standard deviations from the original estimate. As expected, estimated changes in IQR (*middle*) are similar to changes in standard deviation seen in Figures 5 and 6. Changes in tail variability are clearly different from those in IQR, meaning that future distributions are not simply a rescaling of the present-day distributions. 38

Fig. 8. Evolving daily temperature variability (quantile differences) over time in CESM ensemble RCP8.5 runs estimated using our statistical approach, for locations **a**, **b**, and **c**. Using the analysis described in Figure 7, we show absolute IQR and tail variability as a function of seasonality, with different years (at 40 year intervals) shown as different colored lines, from 1850 (dark blue) to 2090 (dark red). Dashed lines represent pointwise 90% confidence intervals. Note the complexity of seasonal cycles in variability at different locations. These results show that the dipole pattern of changes in wintertime skewness changes seen in Figure 5 is driven by low rather than high tail behavior. In wintertime, in the more northern locations **a** and **b**, IQR reduces more strongly than does low tail variability, making skew more negative. In the more southern location **c**, IQR change is negligible while low tail variability reduces strongly, making skew more positive. In all locations, absolute changes in wintertime low tail variability are larger than changes in high tails. For fractional changes, see Supplementary Online Material Figure S6. 39

Fig. 9. Exceedence probability of temperature events above the 95th quantile estimate. The density is obtained by making 10-day bins and counting the number of observations that are above the quantile estimate and normalizing by the total number of exceedences aggregated across all model runs. Each bin is represented by the bin start day, i.e. an x-axis value of 0 includes the interval (0, 10]. We hold out 10 different sets of simulations to obtain 10 different estimates for each block of time, from which we calculate their mean shown as points and standard deviation shown as error bars around \hat{S}_{test} 40

Fig. 10. Training and test exceedence standard deviation as a function of model number, where increasing model number signifies increasing degrees of freedom in the spline basis functions. The data were extracted from the gridbox located at (lat, lon) = (31.5, -93.8). The exceedence is calculated by binning seasonality in 10-day blocks and summing over the long term change. 41

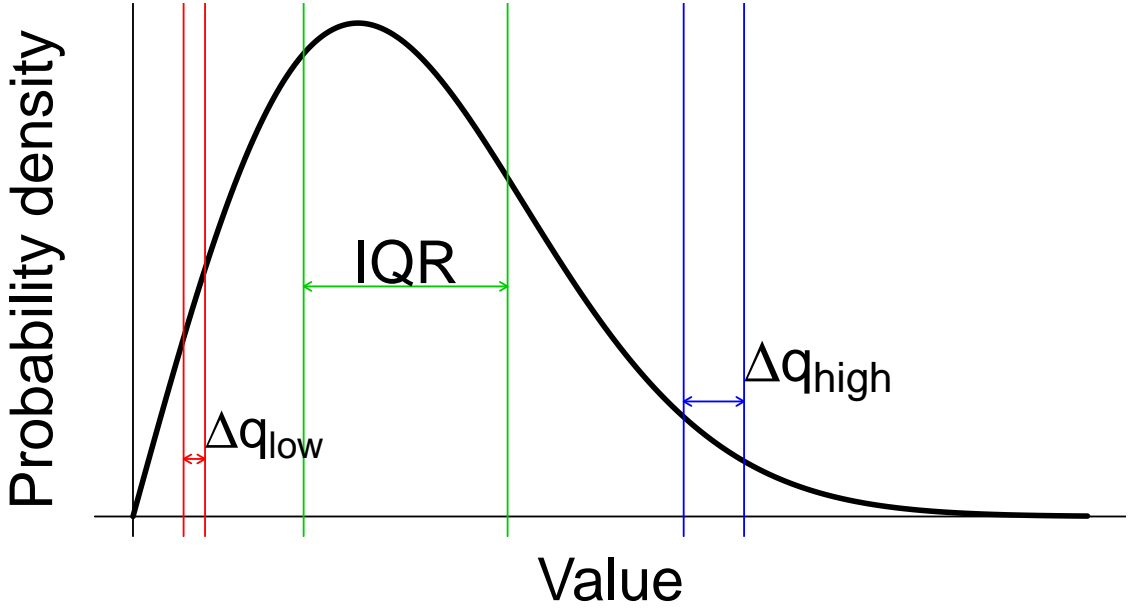


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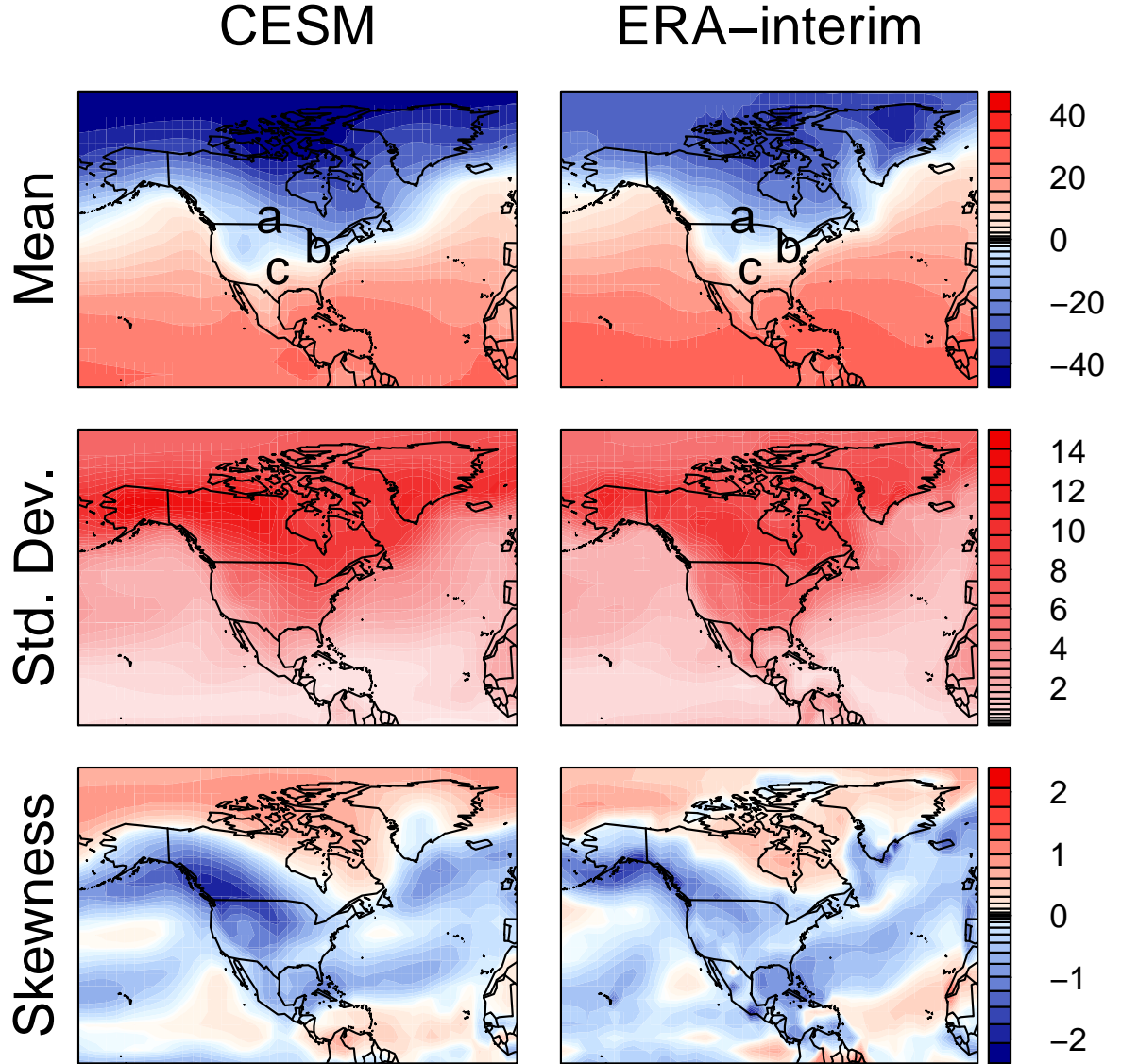
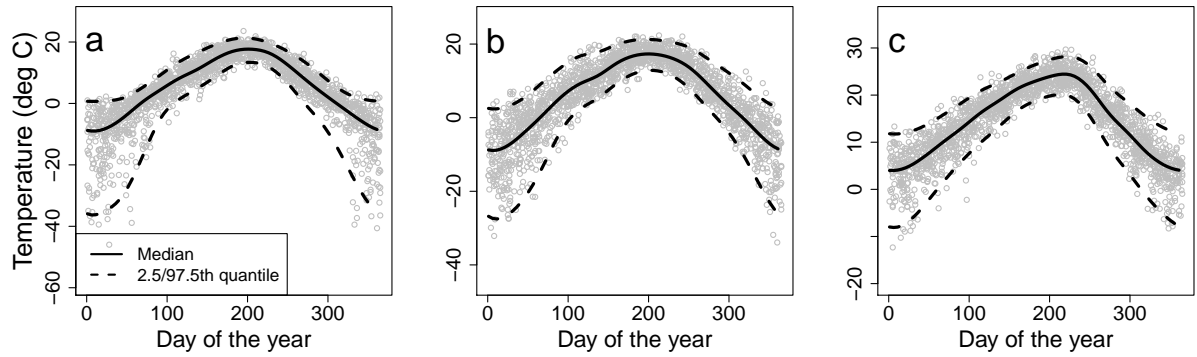


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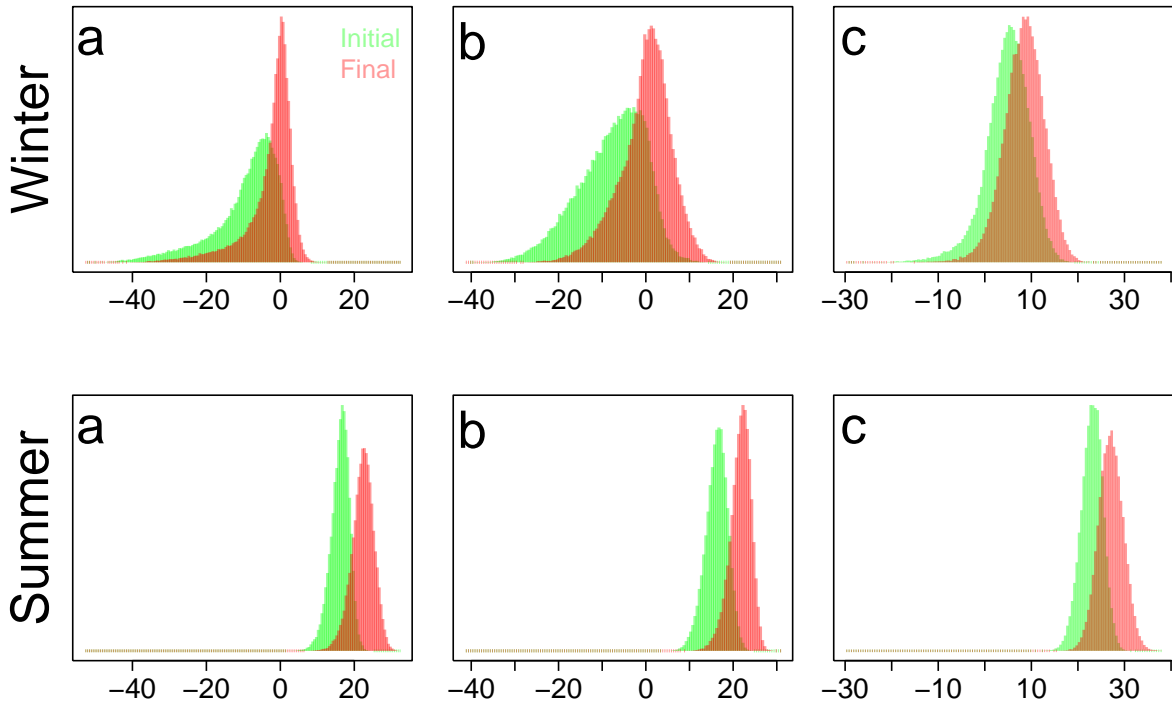


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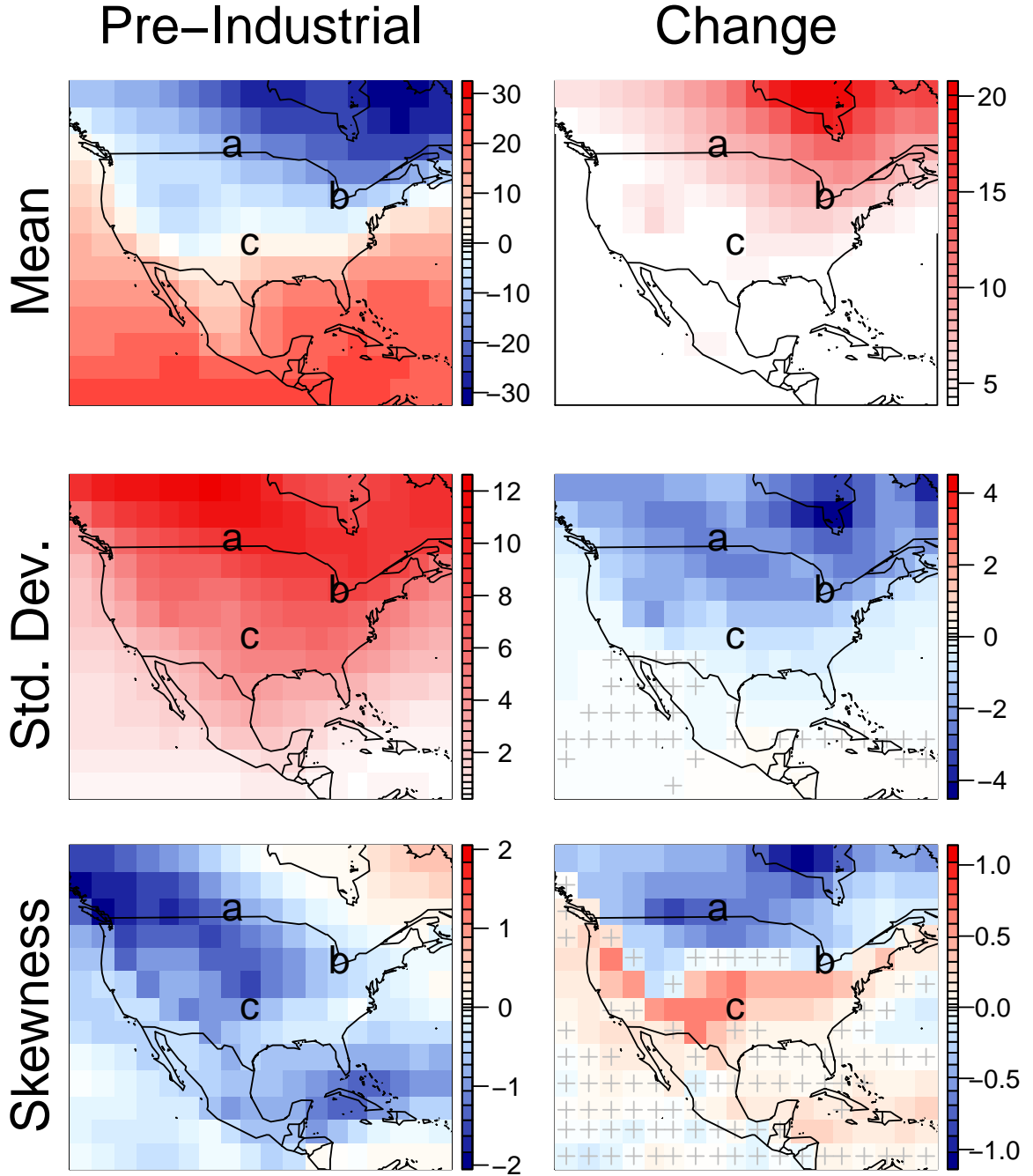


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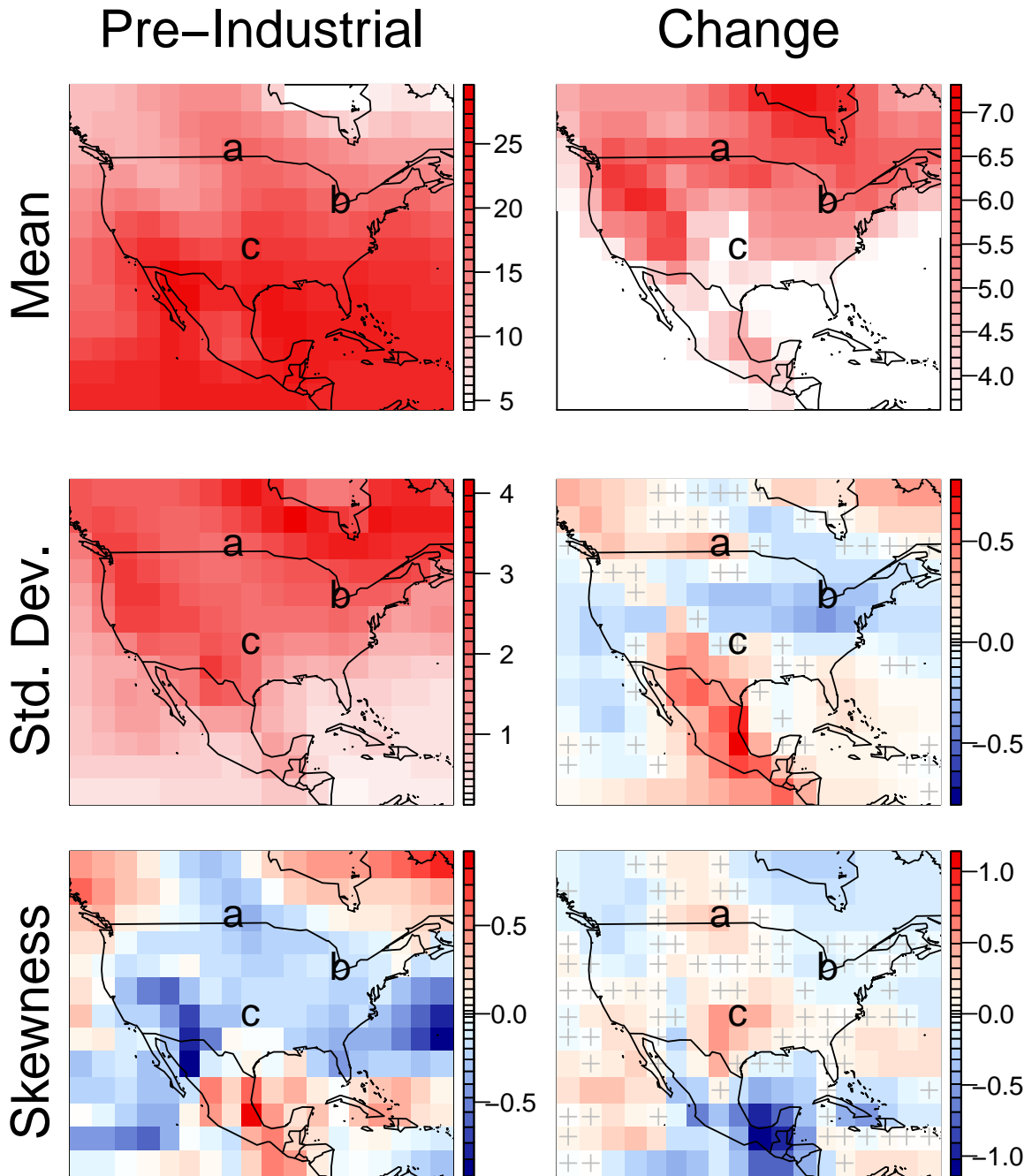


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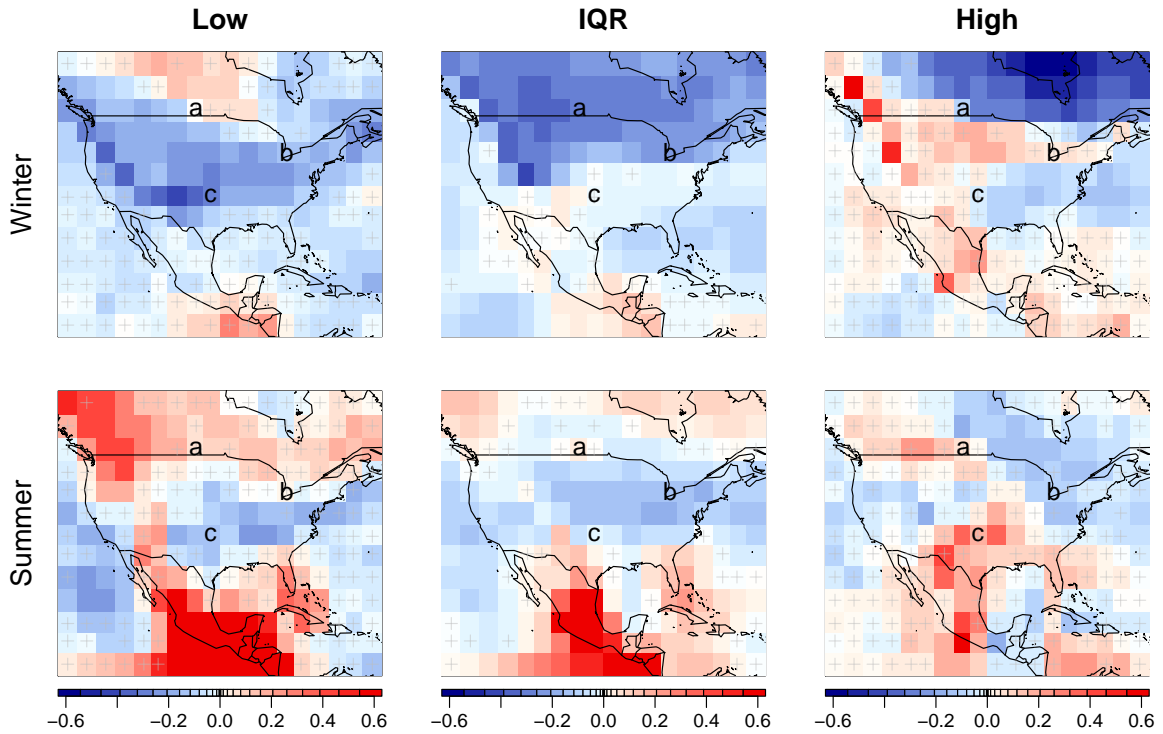


FIG. 7. Changes in daily temperature variability (quantile differences) over time in CESM ensemble RCP8.5 runs estimated using our statistical approach. Because our approach removes the need to aggregate over time when presenting changes, we show here differences in distributions for a single day and year: Jan 1 for winter (*top*) and July 5 for summer (*bottom*), with differences evaluated between the years 1850 and 2100. Changes are expressed as fractions of initial variability, so that the value 0 indicates no change with respect to the initial year. *Left, middle, and right* columns show, respectively, changes in low tail variability, IQR, and high tail variability, as previously defined. Gray crosses mark grid points where the change is less than 3 standard deviations from the original estimate. As expected, estimated changes in IQR (*middle*) are similar to changes in standard deviation seen in Figures 5 and 6. Changes in tail variability are clearly different from those in IQR, meaning that future distributions are not simply a rescaling of the present-day distributions.

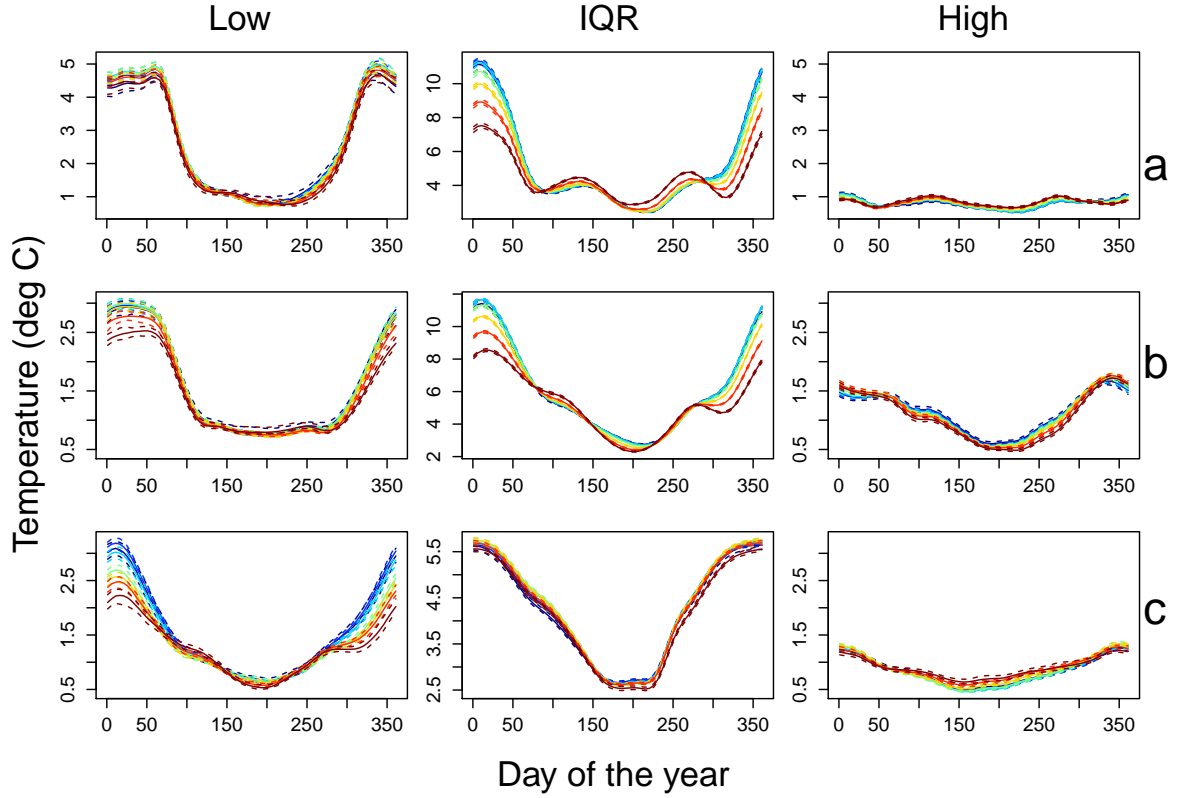


FIG. 8. Evolving daily temperature variability (quantile differences) over time in CESM ensemble RCP8.5 runs estimated using our statistical approach, for locations **a**, **b**, and **c**. Using the analysis described in Figure 7, we show absolute IQR and tail variability as a function of seasonality, with different years (at 40 year intervals) shown as different colored lines, from 1850 (dark blue) to 2090 (dark red). Dashed lines represent pointwise 90% confidence intervals. Note the complexity of seasonal cycles in variability at different locations. These results show that the dipole pattern of changes in wintertime skewness changes seen in Figure 5 is driven by low rather than high tail behavior. In wintertime, in the more northern locations **a** and **b**, IQR reduces more strongly than does low tail variability, making skew more negative. In the more southern location **c**, IQR change is negligible while low tail variability reduces strongly, making skew more positive. In all locations, absolute changes in wintertime low tail variability are larger than changes in high tails. For fractional changes, see Supplementary Online Material Figure S6.

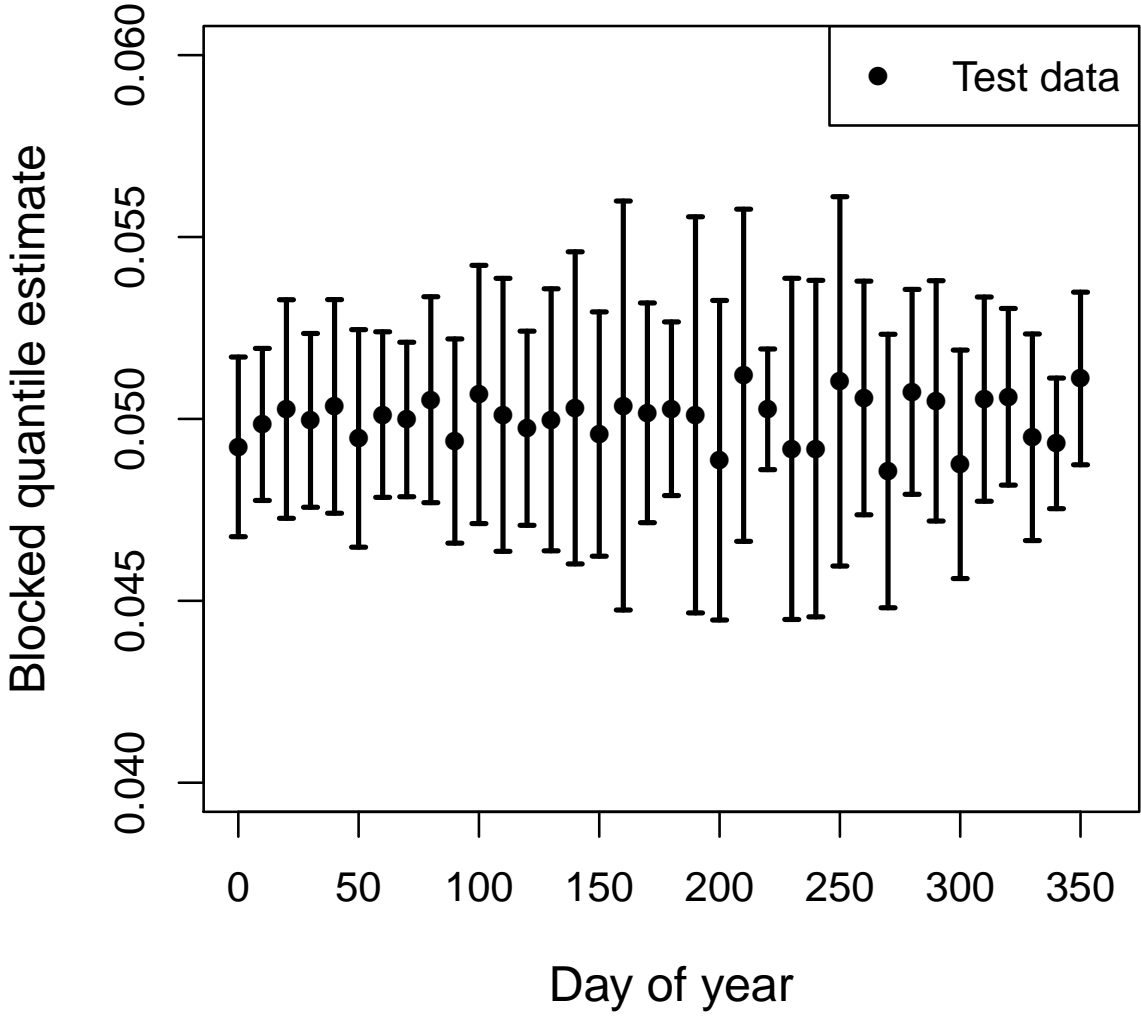


FIG. 9. Exceedence probability of temperature events above the 95th quantile estimate. The density is obtained by making 10-day bins and counting the number of observations that are above the quantile estimate and normalizing by the total number of exceedences aggregated across all model runs. Each bin is represented by the bin start day, i.e. an x-axis value of 0 includes the interval (0, 10]. We hold out 10 different sets of simulations to obtain 10 different estimates for each block of time, from which we calculate their mean shown as points and standard deviation shown as error bars around \hat{S}_{test} .

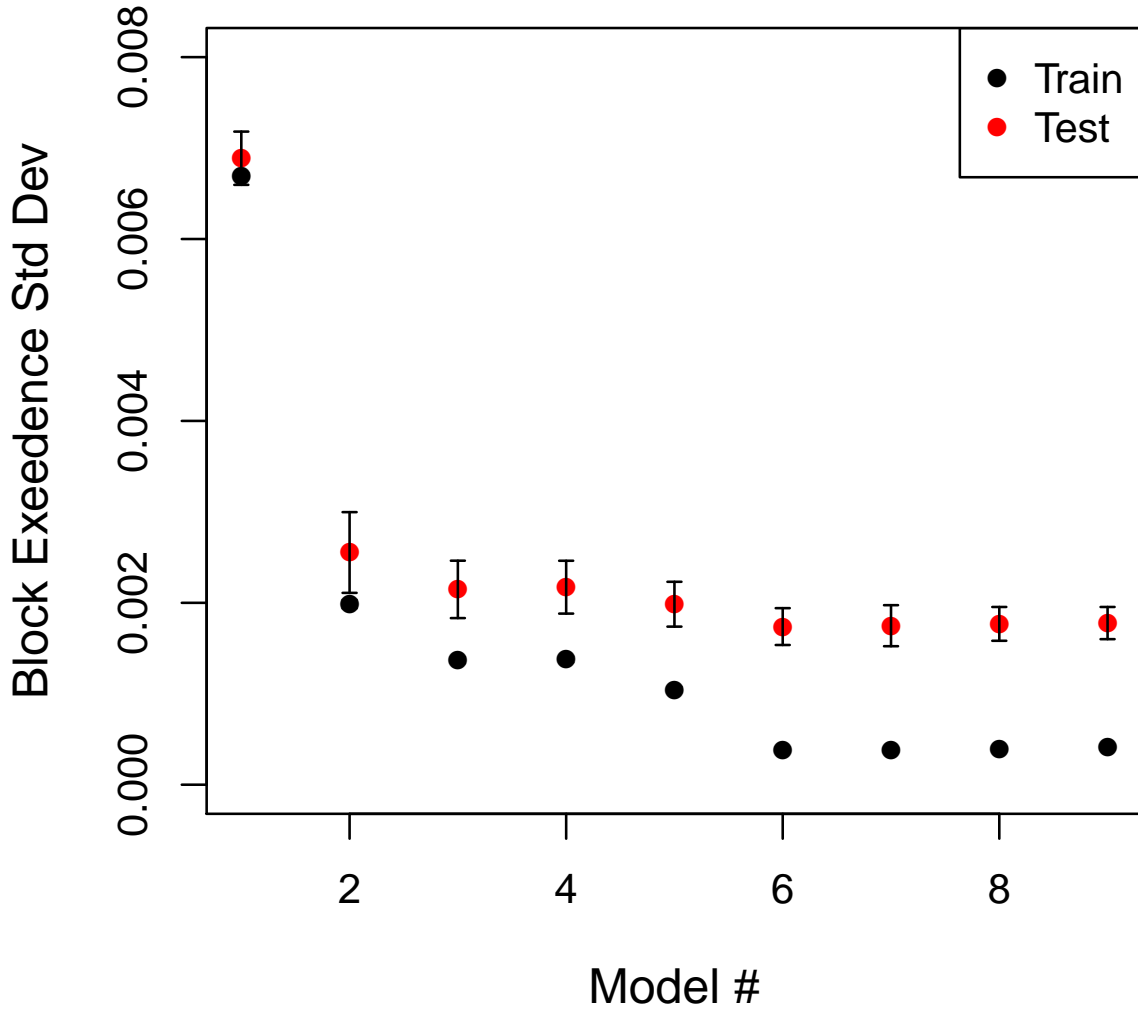


FIG. 10. Training and test exceedence standard deviation as a function of model number, where increasing model number signifies increasing degrees of freedom in the spline basis functions. The data were extracted from the gridbox located at (lat, lon) = (31.5, -93.8). The exceedence is calculated by binning seasonality in 10-day blocks and summing over the long term change.