My Solution

I chose to implement the Euler–Rodrigues formula where rotation about the origin is represented by four real numbers, a, b, c, d such that

$$a^2 + b^2 + c^2 + d^2 = 1 \tag{1}$$

When the rotation is applied, a point at position $\vec{x}$ rotates to its new position

$$\vec{x}' = \begin{pmatrix} a^2+b^2-c^2-d^2 & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & a^2+c^2-b^2-d^2 & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & a^2+d^2-b^2-c^2 \end{pmatrix} \vec{x} \tag{2}$$

The composition of two rotations is itself a rotation. Let (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2) be the Euler parameters of two rotations. The parameters for the compound rotation (rotation 2 after rotation 1) are as follows:

$$a = a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 \tag{3}$$

$$b = a_1b_2 + b_1a_2 - c_1d_2 + d_1c_2 \tag{4}$$

$$c = a_1c_2 + c_1a_2 - d_1b_2 + b_1d_2 \tag{5}$$

$$d = a_1d_2 + d_1a_2 - b_1c_2 + c_1b_2 \tag{6}$$

Any central rotation in three dimensions is uniquely determined by its axis of rotation (represented by a unit vector $\vec{k} = (k_x, k_y, k_z)$)and the rotation angle ϕ. The Euler parameters for this rotation are calculated as follows:

$$a = cos\frac{\varphi}{2} \tag{7}$$

$$b = k_x \sin\frac{\varphi}{2} \tag{8}$$

$$c = k_y \sin\frac{\varphi}{2} \tag{9}$$

$$d = k_z \sin\frac{\varphi}{2} \tag{10}$$

Note that if ϕ is increased by a full rotation of 360 degrees, the arguments of sine and cosine only increase by 180 degrees. The resulting parameters are the opposite of the original values, (a, b, c, d) they represent the same rotation.

In particular, the identity transformation (null rotation, ϕ = 0) corresponds to parameter values $(a, b, c, d) = (1, 0, 0, 0)$. Rotations of 180 degrees about any axis result in $a = 0$.

$$\begin{bmatrix} a^2+b^2-c^2-d^2 & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & a^2+c^2-b^2-d^2 & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & a^2+d^2-b^2-c^2 \end{bmatrix}$$