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RESEARCH ARTICLE

Capital flow freezes

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Abstract
The period following the 2008 financial crisis focused attention on "twin-crises," where banking crises precipitate sovereign crises due to increased bank support. We show that when private sector debt is renegotiated centrally, and bargaining power is low, it results in suboptimally low levels of debt and default rates (haircuts). If, instead, the bargaining power is sufficiently high, the supply of debt exceeds its demand and capital inflows "freeze". These inefficiencies arise because the decentralized borrowers fail to consider how their bond supply impacts debt renegotiation outcomes, affecting both bond prices and the asset span. These issues can be addressed through macroprudential policies in the form of taxing capital inflows.

Keywords Open economy · Capital flows · Capital flow freezes · Debt · Default · Renegotiation

JEL classification F34 · G15 · G18

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1 Introduction

The financial crisis of 2008 brought to the forefront the link between government debt and the state of the banking-financial sector. Consequently, the literature turned attention to "twin-crises" of the banking system and the sovereign.[1] Such crises may originate from either sector: in Iceland they began in the banking sector whereas in Greece they originated in the sovereign. In both cases, debt that was accrued by the private sector was ultimately borne by the sovereign through bail-outs and subsequently renegotiated. Importantly, the quantity of additional debt undertaken by the sovereign was determined by the private sector.

In this paper we focus on crises originating from private-sector generated external debt that is collectively renegotiated by the sovereign. In this setting there is an important pecuniary externality: debtor households do not internalize how the quantity of debt they issue would affect the anticipated default rate and the bond price. As a result, the equilibrium debt level is generally inefficient—although equilibrium outcomes vary with the bargaining power of the sovereign. For low and moderate levels of bargaining power, the quantity of debt issued is positive but suboptimally low. When bargaining power is sufficiently high, international lending may cease or "freeze". We show that both these scenarios can be improved upon through macroprudential policy.

We develop a small open economy model with incomplete markets that incorporates 1) private sector cross border flows (modeled as decentralized borrowing) and 2) renegotiation of private debt handled by the government (modeled as Nash Bargaining). The sovereign negotiates with foreign lenders over the outstanding national stock of external debt. This process results in a "haircut", or partial default on what is to be repaid, that is then relayed to private sector borrowers. This haircut is higher when there is a substantial level of debt to be repaid—a relationship the private borrowers do not internalize. This is a crucial distinction between our model and those featuring a centralized borrower (such as e.g. Yue 2010). In models featuring a borrowing government, the government internalizes how its borrowing decisions might affect haircuts and bond prices. In our model, private borrowers take equilibrium haircuts and bond prices as given. This gives rise to externalities that models with centralized borrowing do not have.

In our model, the nexus between decentralized borrowing and centralized Nash bargaining results in two types of externalities. First, when the bargaining power of the sovereign is low, there is underborrowing in equilibrium: private borrowers do not recognize that by borrowing more they could achieve higher haircuts in "bad times" which would allow for better consumption smoothing across states. Second, when the bargaining power of borrowers is *too high*, capital markets would cease to function. This is because the desired revenue from issuing bonds always exceeds the maximum amount lenders would be willing to provide at compatible anticipated default rates and bond prices. This result can be understood through an off-equilibrium argument. On

[1] See CGFS (2021). Balteanu and Erce (2014) documents systematic differences between 'single' and 'twin' crises and examine the feedback loop between fiscal and financial distress. Reinhart and Rogoff (2013) show that banking crises cause severe contractions in fiscal revenues, raising government debt by about 86% in the years following the crisis. Correa et al. (2014) shows how pronounced and general the relationship between sovereign risk and financial sector support is.

the one hand, with high borrower bargaining power lenders would rationally expect Nash bargaining to result in high haircuts—and market bond prices would be low. On the other hand, at those bond prices, borrowers would want to issue more debt in order to raise their desired revenue; higher debt, in turn, increases anticipated default rates and brings bond prices further down. This loop would continue with ever increasing quantities of debt issued, higher anticipated default rates and lower bond prices.

We show that both types of externalities can be corrected with a macroprudential capital-flow tax. When bargaining power is low, the government could incentivise the households to borrow more by imposing a negative tax (i.e. subsidy) on debt issued—this would yield higher haircuts in "bad times", and in effect allow for better consumption smoothing. When bargaining power is high, the government could impose a positive capital-flow tax to constrain household borrowing and sustain capital flows when they would otherwise freeze.

Our model yields endogenous partial defaults that occur in "bad times" following unfavorable output shocks. In this context default can be thought of as an insurance against economic slowdowns.[2] We argue, however, that the decentralized nature of the economy and the bargaining process introduce inefficiencies that impair consumption smoothing.

We develop the ideas of Jeske (2006), Wright (2006) and Kim and Zhang (2012) who highlight the importance of and channels through which decentralized debt affects capital flows and default. Kim and Zhang (2012) study an environment similar to ours where there is decentralized borrowing and a centralized default decision. However, in their setup the decision to default is binary: the sovereign evaluates whether welfare is higher under repayment or full default given the associated default cost. In our setup there is partial default, and the haircut is decided through Nash bargaining. The externalities we point out arise because of this bargaining protocol. Hatchondo et al. (2009) study an Eaton-Gersowitz economy where there are sequential governments that take the stock of debt of the previous government as given. This is closely related to our setting where previously accumulated debt (albeit by the private sector) is negotiated by a government. Whereas their focus is on the relationship between political stability and default decisions, we focus on the effect of pecuniary externalities on debt accumulation and capital flows.

Our framework shares some key elements with Yue (2010) who introduces Generalized Nash Bargaining as a means of determining default decisions and recovery rates.[3] Although we follow Yue (2010) in the way we model bargaining between risk-neutral lenders and the government, in our model borrowing decisions are made by atomistic households that do not internalize the effect their current choices will have on future bargaining outcome. We show that in this setup lower expected equilibrium default

[2] Our model shares this feature with the prominent studies of the field targeting developing economies, such as Arellano (2008), Aguiar and Gopinath (2006), Yue (2010).

[3] Other examples of models with defaults that feature Nash Bargaining protocol are Asonuma and Trebesch (2016), where the borrower may also choose to initiate a preemptive restructuring, and Arellano and Bai (2014), in which two borrowing countries simultaneously renegotiate with a common lender.

rates hinder potential for consumption smoothing across states.[4] Close to our framework is Araujo et al. (2017) who uses Rubinstein bargaining and explores haircuts, but on government debt rather than private debt bailed out by the government.

In practice governments may bail out banking-financial institutions by transferring their indebtedness onto the sovereign balance sheet. In the period following 2008 governments of advanced economies issued guarantees of bank debt, initiated renegotiations with bank creditors, undertook bank bailouts and nationalizations and sought assistance from big international lenders (e.g. the 'troika') to finance bank recapitalizations. The unraveling of the Irish crisis is one striking example of this dynamic (see Appendix A).

An important lesson we can draw from these episodes: when faced with looming banking crises, governments attempt to mitigate losses of the financial system by taking charge over the fate of bank debt (see, for example, Brandao-Marques et al. (2020)). For this reason sovereign spreads are affected by risks in the banking system. Acharya et al. (2014) examine the period of 2007–2009 associated with bank bailouts in Europe and find that the bailouts of banks by European governments triggered a hike in sovereign bond spreads. Reinhart and Rogoff (2011) document that banking crises usually either precede or coincide with sovereign debt crises, and find that banking crises help explain sovereign defaults. Arellano and Kocherlakota (2014) arrive at a similar conclusion, and stipulate that domestic financial crises are typically associated with large transfers from the sovereigns to the private sector.

Our results are related to the literature following Giavazzi and Pagano (1989) and Cole and Kehoe (1996), that has focused on the expectations driven equilibria in which short-term debt cannot be rolled over. In these settings it is the strategic interaction between the bond price function and government debt management decisions which allows for multiple equilibrium solutions. In our model, rollover risk caused by short-term debt also drives the result. However, the result itself does not arise from the presence of multiple equilibria, but because an equilibria with trade cannot be supported.

Our approach to modeling the "twin-crises" is in contrast to much of the literature that has focused on the "diabolical loop" between governments decisions to bail out banks and banks incentives to hold government debt (see, for example, Gennaioli et al. (2014), Farhi and Tirole (2017), Leonello (2018), Cooper and Nikolov (2018), and Sosa-Padilla (2018)). We abstract away from the moral hazard considerations and narrow our focus on the interaction between the private debt and government renegotiation or bailout.

We show that when the threat of autarky is sufficient to guarantee debt, then the surplus from trade vs autarky can be represented as if it were a non-pecuniary penalty or sanction (see Appendix B). In this way we contribute to literature that models default costs as non-pecuniary losses developed by Shubik and Wilson (1977) and Dubey et al. (2005) and applied in Tsomocos (2003), Goodhart et al. (2005), Goodhart et al. (2006), Walque et al. (2010) and Goodhart et al. (2018). We show that the allocation obtained in a model with bargaining can be replicated in a model with properly specified non-

[4] Another important difference between our setup and that of Yue (2010) is that in our model the economy does not experience output losses or market exclusion in an equilibrium with orderly renegotiation. We believe, however, that these features would not affect our qualitative results.

pecuniary loss. Within this literature our model shares many features with Peiris and Tsomocos (2015), who set up a two period large open international economy model with incomplete markets and default, and Walsh (2015a) and Walsh (2015b), that examine a small open dynamic incomplete markets economy.

Our study contributes to the literature on enforcement of debt contracts and shows the link between enforcement of contracts through an "adjudicator" (i.e. Nash Bargaining) and enforcement through penalties such as in Krasa and Villamil (2000), and especially Krasa et al. (2008). Analogous to the bargaining power parameter that we utilize, the latter paper shows the link between the default *rate* and a parameter that governs debtor vs creditor protection.

Recent studies have examined the complexities and impacts of bankruptcy and default penalties within competitive general equilibrium models. For instance, da Rocha et al. (2023) analyze how bankruptcy risk and financial default penalties affect the existence of competitive equilibria in complete markets. They build on Araujo and Sandroni (1999), showing that stringent default penalties are necessary to prevent agents from making unfeasible promises. In the context of sovereign debt, Alonso-Ortiz et al. (2017) and Conesa and Kehoe (2017) investigate total factor productivity losses as penalties, exploring the dynamics of sovereign debt crises and the strategic interactions between governments and creditors. They highlight the role of expectations and strategic default in debt sustainability. Elkamhi et al. (2023) and D'Amato et al. (2024) consider covenants, which relates to the implicit financial covenants we consider and their impact on debt renegotiation. Birkner et al. (2023) and Mitra (2024) study the link between aggregate policy and inequality while Desierto and Koyama (2024) analyze coalition formation in feudal economies, paralleling our focus on decentralized debt decisions made by a centralized authority. We build on the strategic bargaining frameworks of Araujo et al. (2017) and da Rocha et al. (2017), incorporating the effects of financial externalities and bargaining dynamics in debt renegotiation (Pitchford and Wright 2017). Our research offers a new perspective on mitigating the inefficiencies arising from decentralized debt decisions through policy interventions.

Finally, in our model there is a close link between the insurance opportunities that debt (and partial default) provides when markets are incomplete, and whether the debt market can be supported in equilibrium. This is related to the properties of models of debt and default in infinite horizon where the insurance opportunities themselves may support debt (Bloise et al. 2017).

2 The model

In this section we present an endowment economy that features decentralized borrowing by private agents and collective default. When we refer to partial default or default rates, we mean haircuts that are a successful outcome of bargaining where debtors agree to repay a portion of outstanding debt. When we refer to full or complete default, we mean the breakdown of renegotiation resulting in creditors receiving nothing. We consider a setup along the lines of Yue (2010), in which the borrowers bargain with risk-neutral lenders over default rates, and the outcome is determined via a Nash Bargaining Solution.

We model two countries, Home and Foreign. Home is inhabited by a unit measure of identical households endowed with a single homogenous good. There are three time periods, $t = \{0, 1, 2\}$. Home households' endowments in periods $t = 1$ and $t = 2$ are uncertain. This uncertainty is resolved between $t = 0$ and $t = 1$, when one of two states, $s \in \{H, L\}$, is selected with probability $\frac{1}{2}$ and the corresponding growth rate in endowment $g_1(s)$ becomes known. There is no further uncertainty after $t = 1$, but the state in $t = 2$ depends on the prior realization of uncertainty. Therefore, in total there are 5 date events, 1 in the first period, 2 in the second period and 2 in the last period (see Fig. 1).

At date 0 Home households can issue debt b_0 that would be purchased by the Foreign households at a price q_0—to be repaid in the second period. We follow Yue (2010) in assuming that foreign lenders are risk-neutral. In period 1, households not wishing to honor the full debt due may send a request to the government to bargain with lenders on a reduced final repayment. These are what would be seen in practice as direct or indirect "bail-outs" of the financial sector by the sovereign.[5] If households did not transfer their indebtedness to the government, they would not be able to negotiate independently with external creditors. The government then initiates renegotiation, bargaining with the external creditors to reduce the total amount of debt owed. The outcome of this debt renegotiation, the default rate $\delta_1(s)$, is determined via the Nash Bargaining solution. When bargaining is concluded, the default rates are reported to households who decide whether to accept (in which case they repay the agreed rate) or decline (in which case they default fully but incur additional costs). Households that fully default cannot access the foreign debt market in period 1 and incur a loss of endowment. Given the finite horizon of the economy, we assume that in period 2 all debts are honored and no bargaining takes place.

nobody wants to repay debt, shouldn't we impose a condition on income?

The timing of events is depicted on Fig. 1.[6] We restrict our attention to rational expectations equilibria.

2.1 Home households

Households are identical; their lifetime welfare is given by:

$$V_0 = \max_{c_0, c_1(s), c_2(s), b_0, b_1(s)} \left\{ u(c_0) + \frac{\beta}{2} \sum_{s \in \{1,2\}} [u(c_1(s)) + \beta u(c_2(s))] \right\}, \tag{1}$$

where $\beta < 1$ is a common discount factor between domestic and foreign households. The budget constraints are:

$$c_0 = y_0 + q_0 b_0, \tag{2}$$

[5] It also has parallels with multilateral lenders bailing out sovereigns such as in Kirsch and Rühmkorf (2017).

[6] Formally speaking there are three possibilities: full default, in which case there would be nothing lent *ex ante*; partial repayment, in which case the default rate is priced into the interest rate; and full repayment. As only the latter two are possible in equilibrium, we ignore the case in which a country enters financial autarky in period 1.

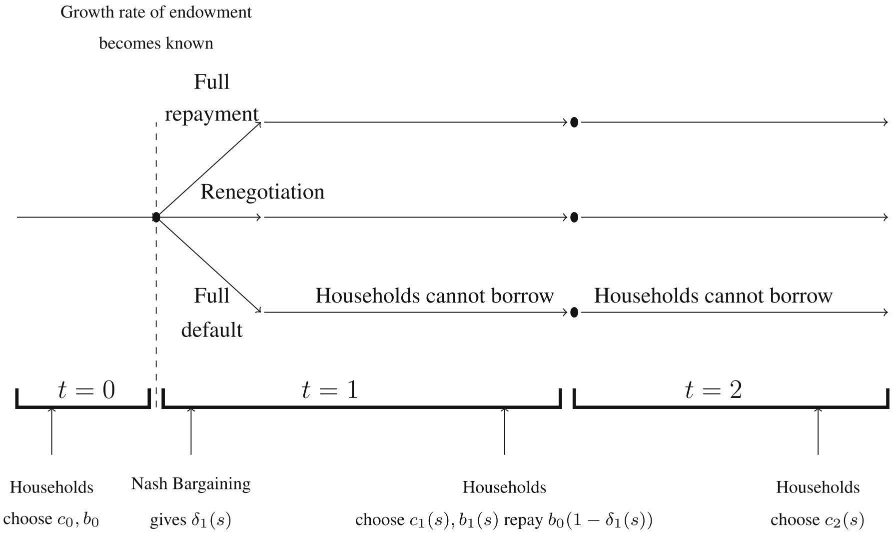


Fig. 1 Timeline

$$c_1(s) + (1 - \delta_1(s))b_0 = y_1(s) + q_1(s)b_1(s), \tag{3}$$

$$c_2(s) + b_1(s) = y_2(s), \tag{4}$$

where $y_1(s) = y_0(1 + g_1(s))$ and $y_2(s) = y_1(s)$,[7] $c_t(s)$ is consumption in period t state s, $b_t(s)$ is the amount of debt issued at price $q_t(s)$, $\delta_1(s)$ is the haircut or rate of default on debt owed, $y_t(s)$ is the endowment, and $g_1(s)$ is the net growth rate in the endowment between the first and second period. We assume $y_1(s) > y_0$, and choose $g_1(H) > g_1(L)$ and β such that home households prefer to borrow if they could.

If there is full default, then home households would not be able to borrow from abroad in subsequent periods, $b_1(s) \leq 0$, and there would be an exogenous endowment loss of $\kappa > 0$ such that the available endowment in the second and third period is reduced to $(1 - \kappa)y_1(s)$. On the other hand, when debt $(1 - \delta_1(s))b_0$ is repaid, the home households may borrow freely in the subsequent periods and enjoy their full endowments.[8]

The first order conditions that solve households' problem are:

$$q_0 u'(c_0) = .5 \sum_{s \in \{1,2\}} \left\{ \beta(1 - \delta_1(s))u'(c_1(s)) \right\}, \tag{5}$$

$$q_1(s) = \beta \frac{u'(c_2(s))}{u'(c_1(s))}. \tag{6}$$

[7] i.e. the endowment in the third period is the same as the endowment in the immediately preceding date-event.

[8] In the subsequent sections we will show that autarky will not occur in equilibrium: the haircut would be chosen such that the households accept the outcome of Nash Bargaining.

Condition (5) describes households' choice of debt b_0. Condition (6) refers to the choice of $b_1(s)$, provided that debt is partially repaid and the households can both borrow and lend in period 1—otherwise it is only relevant if the households choose to lend and not borrow.

2.2 Foreign households

Foreign country is populated by identical households with lifetime utility:

$$V_0^f = \max_{c_0^f, c_1^f(s), c_2^f(s), b_0^f, b_1^f(s)} \left\{ c_0^f + \sum_{s=1}^{2} \frac{\beta}{2} \left[c_1^f(s) + \beta c_2^f(s) \right] \right\}. \tag{7}$$

Their budget constraints are:

$$c_0^f + q_0 b_0^f = y_0^f \tag{8}$$

$$c_1^f(s) + q_1(s) b_1^f(s) = y_1^f(s) + (1 - \delta_1(s)) b_0^f \tag{9}$$

$$c_2^f(s) = y_2^f(s) + b_1^f(s) \tag{10}$$

where b are bonds traded with Home households at price q, and $y_t^f(s)$ is the endowment process.[9] Utility maximization demands:

$$q_0 = \beta \sum_{s=1}^{2} \frac{1}{2} (1 - \delta_1(s)), \tag{11}$$

$$q_1(s) = \beta \tag{12}$$

The default-free price of bonds is therefore β.

2.3 Nash bargaining

Our Nash Bargaing setup follows closely that of Yue (2010) featuring bargaining with foreign risk-neutral lenders over debt redemption. The Nash Bargaining problem determines the default rate outcome by maximizing a weighted product of welfare surpluses gained by each party participating in bargaining. Formally, one may think of a global planner or adjudicator that governs and implements the distribution of payments such as in Araujo et al. (2017). As such an agent has no further impact on our analysis, we do not describe them further for the sake of brevity.

We abstract from fiscal considerations and assume that all debt that is owed by the households is transferred to the sovereign who bargains on their behalf; the outstanding

[9] The endowments of Foreign households are permitted to depend on state s; we do not specify the precise relationship here because, since foreign households are risk-neutral, it has no bearing on the model solution.

debt is returned to the households after the bargaining is concluded.[10][11] As we have a representative agent at home, the distinction between the sovereign and private sector is only important in the bond price schedule: ex-ante households take bond prices as given, whereas a sovereign would internalize the relationship between the bond price and anticipated default rates.

In state $s \in \{H, L\}$, the Nash bargaining problem takes the following form:

$$\delta_1(s) = \begin{cases} \arg\max_{\delta \in [0,1]} \left[\left(\Delta_1^h(s) \right)^{\theta} \left(\Delta_1^f(s) \right)^{1-\theta} \right] & \text{if } \Delta_1^h(s) > 0, \text{ and } b_0 > 0, \\ 1, \ \text{else.} \end{cases}$$
$$s.t. \quad \Delta_1^h(s) \geq 0,$$
$$\Delta_1^f(s) \geq 0. \tag{13}$$

if debt <= 0 why would the default rate be 1

Note that the default rate is taken to be 1 if there is no outstanding debt. The functions $\Delta_1^h(s)$ and $\Delta_1^f(s)$ give welfare gains from participating in renegotiation, for Home and Foreign. These surpluses are assigned weights that represent relative bargaining power of each party, with $0 < \theta < 1$ corresponding to the bargaining power of the borrower, and $(1-\theta)$ to the bargaining power of the lender. For each party, the surplus $\Delta_1(s)$ is given by the difference between the value function of repayment, and the value function that would arise if the agent decided to exit renegotiation (invoking full default). Specifically, for Home (the borrower):

$$\Delta_1^h(s) = \{VR\}_1^h(s) - \{VA\}_1^h(s), \tag{14}$$

where $\{VR\}_1^h(s)$ gives value function under successful renegotiation:

$$\{VR\}_1^h(s) = u(c_1(s)) + \beta u(c_2(s)), \tag{15}$$

and $\{VA\}_1^h(s)$ gives value function under financial autarky:

$$\{VA\}_1^h(s) = u(y_1(s)) + \beta u((1-\kappa) y_1(s)). \tag{16}$$

Similarly, for the Foreign (the lender):

$$\Delta_1^f(s) = \{VR\}_1^f(s) - \{VA\}_1^f(s), \tag{17}$$

where $\{VR\}_1^f(s)$ is foreign households' value of renegotiation:

$$\{VR\}_1^f(s) = c_1^f(s) + \beta c_2^f(s), \tag{18}$$

[10] Foreign lenders also bargain collectively. In reality this represents the typical collective bargaining that occurs through bond-holder groups in sovereign debt restructuring events.

[11] We assume that this type of collective bargaining is incentive-compatible for an individual household. We believe this to be the case, as the Nash Bargaining protocol is set up to guarantee a non-negative surplus for the domestic households—but we leave the formal proof of this point to future research.

and $\{VA\}_1^f(s)$ is their value of autarky:

$$\{VA\}_1^f(s) = y_1^f(s) + \beta y_2^f(s). \tag{19}$$

Properties of the Nash bargaining protocol

Suppose we have arrived in some state s with income $y_1 = y_2$ and debt $b_0 > 0$. Here, we will drop the state index for brevity (i.e. use y_1 instead of $y_1(s)$, etc.). Given debt b_0, the Nash Bargaining surplus is maximized at

$$\delta_1 = \text{argmax}_{\delta_1 \in [0,1]} \Omega(\delta_1, b_0, y_1) \tag{20}$$

where

$$\Omega(\delta_1, b_0, y_1) = [\Delta^h(\delta_1)]^\theta [\Delta^f(\delta_1)]^{1-\theta} \tag{21}$$

and $\Delta^h = (1+\beta)u(c_1) - VA(y_1)$ with $c_1 = y_1 - \frac{1}{1+\beta}(1-\delta_1)b_0$ and $\Delta^f = b_0(1-\delta_1)$.

Before solving problem (20), we will first establish some properties of the function Ω that is being maximized. The foreign surplus given by the function Δ^f is positive everywhere where $\delta_1 \in (-\infty, 1)$; it reaches zero at $\delta_1 = 1$. The home surplus Δ^h is positive if $\delta_1 \in (\tilde{\delta}, \infty)$, where $-\infty < \tilde{\delta}_1 \leq 1$ is a default rate at which the borrower is indifferent between partial repayment and autarky, i.e. $\tilde{\delta} : \Delta^h(\delta_1 = \tilde{\delta}_1) = 0$. Combining these insights we note that Ω, the weighted product of the two surpluses, has the following properties:

1. $\Omega > 0$ for $\delta_1 \in (\tilde{\delta}, 1)$;
2. $\Omega = 0$ for $\delta = 1$;
3. $\Omega = 0$ for $\delta = \tilde{\delta}$.

It follow that the solution of the maximization problem (20) lies in $\delta_1 \in (\tilde{\delta}, 1)$.

Now consider the slope of $\Omega(\delta_1)$ and its second derivative:

$$\begin{aligned}
\frac{\partial \Omega(\delta_1, b_0, y_1)}{\partial \delta_1} &= \Omega \left[\theta \frac{1}{\Delta^h} \frac{\partial \Delta^h}{\partial \delta_1} + (1-\theta) \frac{1}{\Delta^f} \frac{\partial \Delta^f}{\partial \delta_1} \right] \\
&= \Omega \left[\theta \frac{u'(c_1) b_0}{(1+\beta)u(c_1) - VA(y_1)} - (1-\theta) \frac{1}{(1-\delta_1)} \right] \\
\frac{\partial^2 \Omega(\delta_1, b_0, y_1)}{\partial \delta_1^2} &= \Omega \Bigg[\theta \frac{1}{\Delta^h} \frac{\partial^2 \Delta^h}{\partial \delta_1^2} - \theta(1-\theta) \frac{1}{(\Delta^h)^2} \left(\frac{\partial \Delta^h}{\partial \delta_1} \right)^2 - \theta(1-\theta) \frac{1}{(\Delta^f)^2} \left(\frac{\partial \Delta^f}{\partial \delta_1} \right)^2 \\
&\quad + 2\theta(1-\theta) \frac{\partial \Delta^h}{\partial \delta_1} \frac{1}{\Delta^h} \frac{\partial \Delta^f}{\partial \delta_1} \frac{1}{\Delta^f} \Bigg]
\end{aligned} \tag{22}$$

should I be able to derive this expression?

In the interval $(\tilde{\delta}, 1)$, $\frac{\partial^2 \Delta^h}{\partial \delta_1^2} < 0$ (decreasing marginal utility); $\frac{\partial \Delta^h}{\partial \delta_1} > 0$ (home surplus increases in default rate); $\frac{\partial \Delta^f}{\partial \delta_1} < 0$; (foreigners' surplus decreases in default rate). With this, for $\delta_1 \in (\tilde{\delta}, 1)$ we get

$$\frac{\partial^2 \Omega}{\partial \delta_1^2} < 0. \tag{23}$$

The function $\Omega(\delta_1, b_0, y_1)$ therefore has a single peak, and the Nash Bargaining surplus is maximized at a $\delta_1 \in (\tilde{\delta}, 1)$ that solves problem (20). As noted before, the solution of

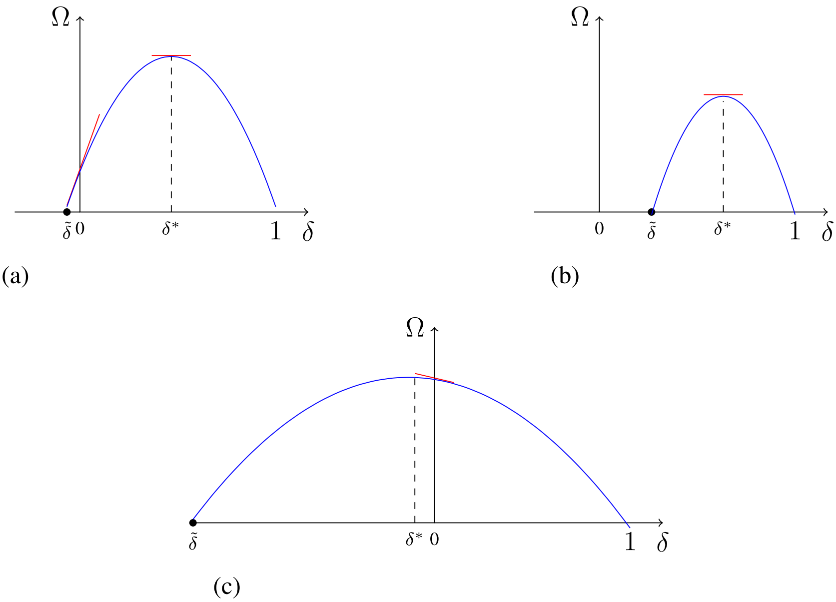


Fig. 2 Ω. **a** $\tilde{\delta} < 0$, $\frac{\partial \Omega}{\partial \delta}(\delta = 0) > 0$. Solution: $\delta = \delta^*$. **b** $\tilde{\delta} \geq 0$. Solution: $\delta = \delta^*$. **c** $\tilde{\delta} < 0$, $\frac{\partial \Omega}{\partial \delta}(\delta = 0) < 0$. Solution: $\delta = 0$

(20) is in $(\tilde{\delta}, 1)$. But because (20) is a constrained optimization, it does not necessarily lie at $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$. Specifically, there are three possible scenarios as depicted on Fig. 2. If $\tilde{\delta} \geq 0$, then the solution of (20) is achieved at $\delta_1 = \delta^*$ such that $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$, see Fig. 2b. If $\tilde{\delta} < 0$, then 1) if $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0, y_1) \geq 0$ then the solution is $\delta_1 = \delta^*$ such that $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$—see Fig. 2a; 2) if $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0, y_1) < 0$ then the solution is $\delta_1 = 0$ (i.e. corner), see Fig. 2c.

2.4 Equilibrium

In addition to individual optimization, and maximizing the Nash Bargaining surplus, equilibrium requires market clearing:

$$c_0 + c_0^f = y_0 + y_0^f \tag{24}$$

$$c_t(s) + c_t^f(s) = y_t(s) + y_t^f(s) \tag{25}$$

$$b_0 = b_0^f \tag{26}$$

$$b_1(s) = b_1^f(s) \tag{27}$$

Assumption 1 *The preferences of households over consumption paths commencing at date 0 are described by the separable, von Neumann-Morgenstern, intertemporal utility functions with differentiable cardinal utilities satisfying standard monotonicity, curvature and boundary conditions, and in addition,*

(a) $\frac{u'(y_1(L))}{u'(y_1(H))} > \frac{u(y_1(L))-u([1-\kappa]y_1(L))}{u(y_1(H))-u([1-\kappa]y_1(H))}$.
(b) $u(.)''' \geq 0$.

Not sure I fully understand

The first part of Assumption 1 places a requirement on the curvature of the utility function and model parameters. Suppose households choose to consume their endowments in period 1. Then, dividing the left–hand side of the above expression by $u(y_1(L))/u(y_1(H))$ would get us the ratio of utility function semi-elasticities in low and high states. Dividing the right-hand side by the same expression would give us the difference in percentage declines in utility following a drop in endowments due to the autarky cost. The first part of Assumption 1 then demands that the ratio of utility function elasticities at low and high states must exceed the difference in percentage drops in utility from autarky losses in those states. Note that this requirement is not especially restricting: for example, it will hold automatically for a CRRA utility—regardless of other model parameters. If under full default the home surplus is higher in the high state than in the low state, this assumption would hold for any utility function. The second part of Assumption 1 requires that marginal utility is convex, and therefore households would opt for precautionary saving when faced with uncertainty.

In what follows we will rely on Assumption 1 to prove statement (c) of Lemma 1, i.e. that, whenever there is partial default, haircuts would tend to be higher in states with lower endowments.

We can now define equilibrium in our economy.

Definition 1 Given Assumption 1, endowments $\left\{y_0, y_t(s), y_0^f, y_t^f(s)\right\}_{t=\{1,2\}}$, a set $\left\{\delta_1(s), q_0, q_1(s), c_0, c_t(s), b_0, b_1(s), c_0^f, c_t^f(s), b_0^f, b_1^f(s)\right\}_{t=\{1,2\}}$ is a competitive equilibrium with rational expectations if:

(a) Given bond prices and default rates, the Home household's problem is solved by $\{c_0, c_t(s), b_0, b_1(s)\}_{t=\{1,2\}}$, and $\left\{c_0^f, c_t^f(s), b_0^f, b_1^f(s)\right\}_{t=\{1,2\}}$ solves the Foreign household's problem
(b) Given bond prices and household demands, $\delta_1(s)$ solves Nash Bargaining Problem (13);
(c) Goods markets clear, $c_0 + c_0^f = y_0 + y_0^f$ and $c_t(s) + c_t^f(s) = y_t(s) + y_t^f(s)$
(d) Asset markets clear $b_0^f = b_0$ and $b_1^f(s) = b_1(s)$.

Our definition of equilibrium is susceptible to 'undue pessimism' about the expected default rates, as in Dubey et al. (2005). Agents may expect there to be complete default in the future, and hence not issue debt as the bond price would be zero. As a consequence, there is potentially two equilibria: one with bond trade and one without. However we avoid analysis of 'unduly pessimistic' equilibria and restrict our attention to equilibria where there is either trade, or when the equilbria with bond trade cannot be supported (the market freeze).

3 Equilibrium analysis

We start by characterizing the Nash bargaining solution given an outstanding level of debt (Lemma 1). We use these results to shows that equilibrium default can only occur in the low state (Proposition 1). Furthermore, in Proposition 1 we demonstrate that the existence of an equilibrium with borrowing crucially depends on the Home country's bargaining power: when bargaining power is sufficiently low, there is borrowing in equilibrium; when bargaining power is too high, the bond market freezes or ceases to operate. In Proposition 2 we consider the former case (i.e. "low" bargaining power) and show that, if the debt market functions, then there is underborrowing in the competitive equilibrium—compared to an allocation chosen by a domestic planner that takes into account the effect of debt on default rates. This is because households do not internalize the benefit of additional insurance opportunities from default. We propose a policy intervention that results in a Pareto improvement (by incentivizing the households to borrow more). In Proposition 3 we consider the case of the market freeze (i.e. "high" bargaining power). We show that when the debt market ceases to function, an intervention designed to curb households' borrowing appetites can enable borrowing and result in a Pareto improvement.

Lemma 1 *Given borrowing* $b_0 > 0$ *and endowments* $y_1(H) > y_1(L)$ *in the two states,*

a) The Nash Bargaining surplus is maximized by default rates that are $0 \leq \delta_1(s) < 1$*;*
b) Default rates $\delta_1(s) > 0$ *increase in* b_0*;*
c) If the Nash Bargaining protocol selects a positive default rate in the high endowment state, it would also select a greater rate of default in the low endowment state or, $\delta_1(L) \geq \delta_1(H)$.

The first part of Lemma 1 establishes that full default cannot happen in any state. When there is full default, the lender's surplus is zero, and so is the Nash Bargaining surplus. At the same time, there always exists a default rate below 1 that would result in a positive Nash Bargaining surplus—therefore, $\delta_1(s) = 1$ is never an equilibrium outcome. The second part of Lemma 1 claims that higher outstanding debt is associated with higher default rates. Suppose that, given some level of debt, the objective function in Nash bargaining is maximized at a positive default rate. If the outstanding debt increases, the value of the objective function would fall—unless there is a proportionate increase in the default rate. The third part of Lemma 1 relies on Assumption 1, which guarantees that, holding the default rate constant, the surplus of the borrower in the lower endowment state would be lower than in the high endowment state; this then implies that the Nash Bargaining protocol would select a higher default rate when endowment is low.

Next, in Proposition 1 below we show that there cannot be partial default in both states in equilibrium. Taken together with the first part of Lemma 1, this result implies that in equilibria with risky borrowing, there is full repayment in the state with the highest endowment and partial default in the other state. The second part of this proposition uses this argument to show that increasing the bargaining power of the borrower ultimately leads to the market for debt ceasing to function in equilibrium.

Proposition 1 *In equilibrium,*

a) Generically, partial default cannot occur in every state of the world at $t = 1$;
b) When bargaining power is sufficiently high, the bond market freezes at $t = 0$;
c) When bargaining power is sufficiently low, there is positive borrowing at $t = 0$.

not sure I fully get it

The intuition behind the first part of Proposition 1 can be summarized as follows. When households choose how much to borrow, they base their decisions on the *asset span* rather than the actual default rates in each state. Because of this, the first order condition characterizing the households' decisions can be re-framed in terms of giving the optimal relationship between repayment amounts in the two states. But when there is partial default in both states, the Nash bargaining protocol would pin down these same repayment amounts: each would be determined by the exogenous parameters of the model. Some of these parameters do not at all factor into the households' choice (e.g. the collective bargaining power). Under these conditions equilibrium with borrowing would generally not exist, because the repayment amounts prescribed by the Nash bargaining would generally not satisfy households' optimization.

The second part of Proposition 1 follows from the observation that as the borrowers gain more bargaining power, the weight of their surplus in the total Nash bargaining surplus increases. This then translates into higher default rates that maximize the joint surplus. When bargaining power is sufficiently high, full repayment in the higher-income state is no longer feasible. According to the logic of the first part of Proposition 1, this then leads to mismatch between the repayment amounts desired by the households (based on the asset span) and those that arise from renegotiation (as determined by the Nash Bargaining protocol). Because of this inconsistency, positive issuance of debt cannot be sustained in equilibrium.

Another way to think of this result would be to invoke an off-equilibrium argument. When bargaining power is high, repayment amounts set by Nash bargaining end up being low for both states; this then places an upper limit on the revenue from the bonds auction the households can extract. At the same time, the households would like to achieve revenue from auctioning bonds that exceeds this upper limit. Observing a low bond price, the households would choose a higher amount of outstanding debt to boost the revenue from the auction of bonds. But since the maximum revenue is limited from above by Nash bargaining, this would inevitably lead to an even lower bond price—an effect the households do not internalize. Households would then choose to borrow more to offset this effect, which would be followed by a further decrease in the bond price. This loop would eventually lead to the bond market imploding and the bond prices reaching zero.

The last part of the proposition notes that if bargaining power is sufficiently reduced, full repayment in the higher-income state becomes possible, allowing for an equilibrium where positive borrowing exists. This means that adjusting the bargaining power downwards can create conditions under which borrowers are both willing and able to fully repay their debts in the high income state, thereby supporting positive issuance of debt in equilibrium.

To sum up, giving borrowers more bargaining power can help provide more insurance opportunities through additional default in low-income states of the world. But there is a catch: if this bargaining power becomes excessive, it can actually hinder their ability to issue debt from the start. To better understand this issue, we are going to outline and examine the planner's problem. This step will allow us to establish a benchmark against which we can measure how well the market operates on its own.

Properties of the Planner's problem
Unlike the household, the planner internalizes how the choice of b_0 might affect the default rates. First, note that, because there is no uncertainty between periods 1 and 2 (and no bargaining), the planner's problem from period 1 onwards would obtain the same solution as in the decentralized economy above. Therefore,

$$c_1(H) = y_1(H) - \frac{1}{1+\beta}(1 - \delta_1(H))b_0 = c_2(H) \tag{28}$$
$$c_1(L) = y_1(L) - \frac{1}{1+\beta}(1 - \delta_1(L))b_0 = c_2(L) \tag{29}$$

The planner chooses b_0 to maximize:

$$EW = u(y_0 + q_0(b_0)b_0) + \frac{\beta}{2}\sum_{s=H,L}\left[(1+\beta)u(y_1(s) - \frac{1}{1+\beta}(1 - \delta_1(s)(b_0))b_0)\right] \tag{30}$$

where $\delta_1(s)(b_0)$ describes how the choice of b_0 affects default rates that come out of NBS, and $q_0(b_0) = \beta(2 - \delta_1(L)(b_0) - \delta_1(H)(b_0))/2.$ solved: intuitive but is there a mathematical derivation to get this formally?

$$\begin{aligned}\frac{\partial EW}{\partial b_0} &= u'(c_0)\frac{\beta}{2}\left[(2 - \delta_1(L) - \delta_1(H)) - \left(\frac{\partial\delta_1(H)}{\partial b_0} + \frac{\partial\delta_1(L)}{\partial b_0}\right)b_0\right] \\ &\quad -\frac{\beta}{2}\left[u'(c_1(H))\left(1 - \delta_1(H) - \frac{\partial\delta_1(H)}{\partial b_0}b_0\right) + u'(c_1(L))\left(1 - \delta_1(L) - \frac{\partial\delta_1(L)}{\partial b_0}b_0\right)\right]\end{aligned} \tag{31}$$

Rearranging, we would get:

$$\begin{aligned}\frac{\partial EW}{\partial b_0} &= \frac{\beta}{2}\big(u'(c_0)(2 - \delta_1(L) - \delta_1(H)) \\ &\quad - \left[u'(c_1(H))(1 - \delta_1(H)) + u'(c_1(L))(1 - \delta_1(L))\right]\big) \\ &\quad - b_0\frac{\beta}{2}\left[\frac{\partial\delta_1(H)}{\partial b_0}\left[u'(c_0) - u'(c_1(H))\right] + \frac{\partial\delta_1(L)}{\partial b_0}\left[u'(c_0) - u'(c_1(L))\right]\right]\end{aligned} \tag{32}$$

Planner's preference at market equilibrium with $\delta_1(H) = 0$ and $\delta_1(L) > 0$
Consider the decentralized equilibrium where, given some b_0, NBS produces corner solution in high state and interior solution in low state. In the proof of Proposition 2 we show that in this equilibrium $c_0 > c_1(L)$ (see Appendix). Furthermore, in this equilibrium $u'(c_0)(2 - \delta_1(L) - \delta_1(H)) - [u'(c_1(H))(1 - \delta_1(H)) + u'(c_1(L))(1 - \delta_1(L)] = 0$ (from combining the FOCs for Home and Foreign households). For a corner solution in high state, $\frac{\partial\delta_1(H)}{\partial b_0} = 0$; for interior solution in low state $\frac{\partial\delta_1(L)}{\partial b_0} > 0$.

Given all this, at equilibrium allocation we have:

$$\frac{\partial EW}{\partial b_0} > 0, \tag{33}$$

i.e. the planner would choose to increase b_0 relative to the decentralized economy solution. Because in the low state $\frac{\partial \delta_1(L)}{\partial b_0} > 0$, higher b_0 chosen by the planner would result in a higher haircut in the low state.

Proposition 2 below summarizes these results. It states that the competitive equilibrium is inefficient because there is underborrowing, and hence too small a haircut—compared to the equilibrium with a domestic Ramsey planner who internalizes the relationship between the quantity of debt issued, the default rate, and the bond price.

Proposition 2 *If bargaining power is sufficiently low, such that a competitive equilibrium with positive borrowing and default in low state exists, then at this equilibrium*

(a) There is underborrowing compared to an equilibrium with the Centralized borrower;
(b) The haircut is suboptimally low;
(c) The government can implement a debt subsidy (payed for with a lump-sum tax) that would results in a Pareto improvement.

The intuition for the underborrowing result is described in a numerical example in Fig. 7 in the Appendix. As default rates increase in the low endowment state, so too do the insurance opportunities from debt—and welfare improves. This continues until full repayment cannot be supported in the high endowment state and the market fails.

The Centralized borrower internalizes two externalities from debt. The first is the relationship between the debt issued and the effective *span* of debt. Given that markets are incomplete, default provides insurance opportunities for the borrower, and there is an optimal rate of default that allows for consumption smoothing across the "high" and the "low" states. The second externality arises from the relationship between the quantity of debt, the default rate and the bond price. As default rates increase with the amount of debt issued, the Centralized borrower can select a level of debt that optimally trades-off the *quantity of debt issued* with the *insurance opportunities from the rate of default* on that debt.

When bargaining power is too high, markets freeze and there is no borrowing in the competitive equilibrium (per Proposition 1). In this circumstance, the planner can implement a Pareto improvement by issuing a sufficiently small quantity of debt that guarantees full repayment in at least one state. In a decentralized economy, such an improvement can be implemented by a government that aims to limits borrowing through e.g. taxing debt inflows.[12] We summarize these points in Proposition 3 below.

Proposition 3 *If the bargaining power is sufficiently high, such that there is no competitive equilibrium with capital flows, then*

[12] The possibility of taxing debt inflows has been studied by, among others, Goodhart et al. (2013) and Goodhart and Tsomocos (2014). In practice, increasing the capital charge or requiring greater capital buffers for banks when they issue external debt would have a similar effect.

(a) *The Centralized borrower will choose a positive level of debt;*
(b) *The government can implement a proportional debt tax (reimbursed with a lump-sum transfer) that would results in a Pareto improvement.*

In the first part of Proposition 3 we establish that, when there is a market freeze, the potential gains to trade mean that any positive level of debt can improve the welfare of the borrower household. In the second part we correct the pecuniary externality that causes the debt market to fail in the first place: the off-equilibrium excess issuance of debt. In the third part of we show that a proportional tax on debt issued can jump-start the market—by reducing the marginal benefits from issuing debt, and thereby supporting full repayment in the high endowment state.

Proposition 3 extends to many situations where there is a market freeze or no borrowing in the competitive equilibrium because households cannot commit to limit the quantity of debt they would issue. In this sense, there is a *desire* to overborrow to the extent that the market fails. Interestingly, a tax on capital inflow has a similar effect to reducing the bargaining power of borrowers. In both cases it serves to reduce default rates, and possibly result in full repayment in the high endowment state.

Our argument contributes to the literature on macroprudential regulation, particularly to the discussion about over- and underborrowing. Bianchi and Mendoza (2018) and Jeanne and Korinek (2019) show that the fact that private agents do not internalize the effect their borrowing has on collateral prices amplifies booms and busts along the business cycles. They conclude that policies that combat overborrowing are welfare-improving. By contrast, Schmitt-Grohe and Uribe (2016) show that in a framework where multiple equilibria are possible a similar environment with collateral constraints will generate underborrowing, as agents will attempt to self-insure against bad times. Our result is that the market may both underborrow and have a *desire* to overborrow - the inefficiency goes in both directions. When the market functions, households underborrow, but when it does not, it is because they desire to borrow more than what the market can sustain.

Propositions 2 and 3 state that the planner can *improve* upon the market allocation, but not that it can implement the (constrained) *Pareto optimal* allocation. As the allocations here depend entirely on the *asset span*, there will be a span (and hence default rates) that will result in a highest level of welfare. While the government would *ex-ante* wish to implement such an allocation by committing to such default rates, *ex-post* it has no means of committing to repayment. Nevertheless, a debt contract with ex-ante specified stochastic rates of return and sufficiently low bargaining power of the sovereign (which would ensure full repayment of the stochastic rates of return) could implement a desired allocation.[13]

4 Concluding remarks

In this paper we studied the nexus between private borrowing and centralized renegotiation and concluded that in such an environment bargaining generates suboptimal outcomes and even market failure or freezing. The problem is twofold. First, when

[13] We thank an anonymous referee for pointing out this argument.

there is borrowing in equilibrium, expected default rates are inefficient as households do not internalize how their debt decisions affect their subsequent default and *insurance* opportunities. Second, when the government's bargaining power is too high, capital flows may cease and there may not be an equilibrium with borrowing—despite the need for consumption smoothing across periods. We show that both these issues can be addressed with macroprudential policies in the form of capital flow taxes.

Appendix A Decentralized borrowing, centralized renegotiation: Ireland 2008–2012

In the period preceding 2008, Ireland experienced a construction boom. Irish banks funded the growing demand for loans by borrowing from abroad, raising net foreign borrowing to 60 percent of GDP, compared to 10 percent in 2003.[14] In 2008, following a sharp decline in house prices, the largest Irish bank, Anglo Irish Bank, began rapidly losing funds. Fears of an impending banking crisis prompted the Irish government to take action: on September 30, 2008, it announced that it had extended guarantees to the deposits of the six largest Irish banks. Following this announcement, the Credit Default Swap (CDS) spreads on government bonds soared. In an effort to save the banks, the government took further actions that included buying non-performing loans from the banks via the National Asset Management Agency (NAMA), established in 2009, and eventually nationalizing five of the six largest banks. In 2010, following further increases in spreads on sovereign bonds, the Irish government lost access to financial markets and in November 2010 was forced to request assistance from the 'troika' (the EC, the ECB, and the IMF), reaching an agreement on a 67.5 billion euro financial support package. In July 2012, Ireland regained partial access to the financial market. In 2013, Ireland exchanged 25 billion euros in high-interest (about 8%) promissory notes used to bail out Anglo Irish Bank for lower-cost (about 3%) long-term government debt.[15]

Appendix B Proof of Propositions

Proof of Lemma 1

We will first consider the Nash Bargaining protocol and establish its properties (with basic properties found in the main body of the paper), showing that, given some b_0 and y_1, there is a solution for $\delta_1(s)$. This solution may be interior ($\delta_1(s) > 0$) or corner ($\delta_1(s) = 0$). We will show that, if the solution is interior, the default rate that maximizes the Nash Bargaining surplus is increasing in the outstanding debt. We will then use Assumption 1 to show that, if the Nash Bargaining protocol assigns a positive default rate in the high endowment state, it would also assign a greater default rate in the low state.

[14] For detailed accounts of the Irish crisis, see Whelan (2014) and Honohan (2009).

[15] Source: OECD Sovereign Borrowing Outlook 2014.

(a) Nash bargaining protocol: properties

Suppose we have arrived in some state s with income $y_1 = y_2$ and debt $b_0 > 0$. For the proofs in Lemma 1 we will drop the state index for brevity (i.e. use y_1 instead of $y_1(s)$, etc.). As we posit in the main body of the paper, then, given debt b_0, the Nash bargaining surplus is maximized at

$$\delta_1 = \text{argmax}_{\delta_1 \in [0,1]} \Omega(\delta_1, b_0, y_1) \tag{34}$$

where

$$\Omega(\delta_1, b_0, y_1) = [\Delta^h(\delta_1)]^{\theta}[\Delta^f(\delta_1)]^{1-\theta} \tag{35}$$

and $\Delta^h = (1+\beta)u(c_1) - VA(y_1)$ with $c_1 = y_1 - \frac{1}{1+\beta}(1-\delta_1)b_0$ and $\Delta^f = b_0(1-\delta_1)$. In the main body of the paper (Sect. "2.3") we show that this Nash bargaining surplus is maximized at $0 \leq \delta_1 < 1$, thereby proving the first statement of Lemma 1.

(b) Interior Nash bargaining solution and debt b_0

We will now consider how debt b_0 affects the Nash Bargaining solution. First, consider the case presented on Fig. 2b characterized by $\tilde{\delta} \geq 0$. The value of $\tilde{\delta}_1$ is non-negative if $\Delta^h(\delta_1 = 0) \leq 0$, i.e. autarky is preferred to full repayment (or household is indifferent). This will happen if $(1+\beta)u\left(y_1 - \frac{1}{1+\beta}b_0\right) \leq VA(y_1)$, which would be the case if b_0 is too high relative to the autarky loss κ. Denote $\tilde{b}_0$ such that $(1+\beta)u\left(y_1 - \frac{1}{1+\beta}\tilde{b}_0\right) = VA(y_1)$; for all $b_0 \geq \tilde{b}_0$, we have $\tilde{\delta} \geq 0$ and problem (20) is described by Fig. 2b yielding an interior solution δ^* such that $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$.

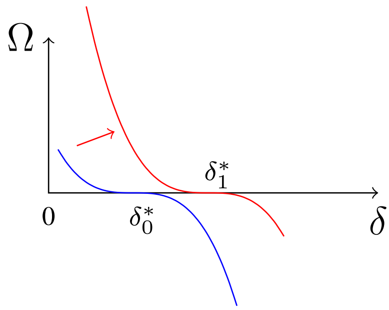
Now consider $b_0 < \tilde{b}_0$, the cases depicted in Fig. 2a and c. The interior solution (Fig. 2a) is achieved if $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0, y_1) \geq 0$; at the interior solution $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$. The corner solution (Fig. 2b) with $\delta_1 = 0$ is achieved if $\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0, y_1) < 0$. Note that $\frac{\partial \Omega}{\partial \delta_1}(\delta_1, b_0, y_1)$ is increasing in b_0; therefore, there exists a $\hat{b}_0(y_1) : \frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, \hat{b}_0, y_1) = 0$ such that 1) if $b_0 \geq \hat{b}_0$, then the solution is interior; 2) if $b_0 < \hat{b}_0$, the solution is corner.

Combining the above insights we can conclude that:

1. The Nash bargaining problem has a corner solution (i.e. $\delta_1 = 0$) if debt is small. Specifically, if $b_0 < \hat{b}_0$ where $\hat{b}_0 : \frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, \hat{b}_0, y_1) = 0$.
2. The Nash bargaining problem has an interior solution if debt is large. Specifically, if $b_0 \geq \hat{b}_0$ where $\hat{b}_0 : \frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, \hat{b}_0, y_1) = 0$. At the interior solution $\delta_1 = \delta^* : \frac{\partial \Omega}{\partial \delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$.

Suppose $b_0 \geq \hat{b}_0$, i.e. the Nash Bargaining problem has an interior solution. At the interior solution, we have $\frac{\partial \Omega}{\partial \delta_1}(\delta_1, b_0, y_1) = 0$. Note that on the relevant interval $\frac{\partial \Omega}{\partial \delta_1}(\delta_1, b_0, y_1)$ is decreasing in δ_1, but increasing in b_0. This means that, at the interior

Fig. 3 Shift in $\frac{\partial\Omega}{\partial\delta}(\delta_1, b_0, y_1)$ following an increase in b_0. Red line is higher level of b_0 and blue line is lower (color figure online)



solution, higher b_0 is associated with higher $\delta_1 = \delta^* : \frac{\partial\Omega}{\partial\delta_1}(\delta_1 = \delta^*, b_0, y_1) = 0$, see Fig. 3.

Intuitively, suppose that given some quantity of debt the Nash Bargaining surplus is maxi- mized at a positive default rate; then, as the debt increases, the value of the surplus would stay the same if the repayment rate (one minus the default rate) falls proportionately to the debt increase.

(c) Comparing interior solutions across states

We will use Assumption 1 to compare the outcomes of Nash Bargaining across states.

Consider some positive number A such that $u(y_1(L) - A) - u([1-\kappa]y_1(L)) > 0$ and $u(y_1(H) - A) - u([1-\kappa]y_1(H)) > 0$. Because $u''(.) < 0$, the following is true (see Fig. 4a):

$$\frac{u(y_1(L)) - u([1-\kappa]y_1(L))}{u(y_1(H)) - u([1-\kappa]y_1(H))} > \frac{u(y_1(L) - A) - u([1-\kappa]y_1(L))}{u(y_1(H) - A) - u([1-\kappa]y_1(H))}. \tag{36}$$

Because $u'''(.) \geq 0$, it is also true that (see Fig. 4b):

$$\frac{u'(y_1(L) - A)}{u'(y_1(H) - A)} \geq \frac{u'(y_1(L))}{u'(y_1(H))}. \tag{37}$$

Combining these insights with Assumption 1, we get

$$\begin{aligned}\frac{u'(y_1(L) - A)}{u'(y_1(H) - A)} \geq \frac{u'(y_1(L))}{u'(y_1(H))} &> \frac{u(y_1(L)) - u([1-\kappa]y_1(L))}{u(y_1(H)) - u([1-\kappa]y_1(H))} \\ &> \frac{u(y_1(L) - A) - u([1-\kappa]y_1(L))}{u(y_1(H) - A) - u([1-\kappa]y_1(H))}.\end{aligned} \tag{38}$$

or

$$\frac{u'(y_1(L) - A)}{u'(y_1(H) - A)} > \frac{u(y_1(L) - A) - u([1-\kappa]y_1(L))}{u(y_1(H) - A) - u([1-\kappa]y_1(H))} \tag{39}$$

or

$$\frac{u'(y_1(L) - A)}{u(y_1(L) - A) - u([1-\kappa]y_1(L))} > \frac{u'(y_1(H) - A)}{u(y_1(H) - A) - u([1-\kappa]y_1(H))}. \tag{40}$$

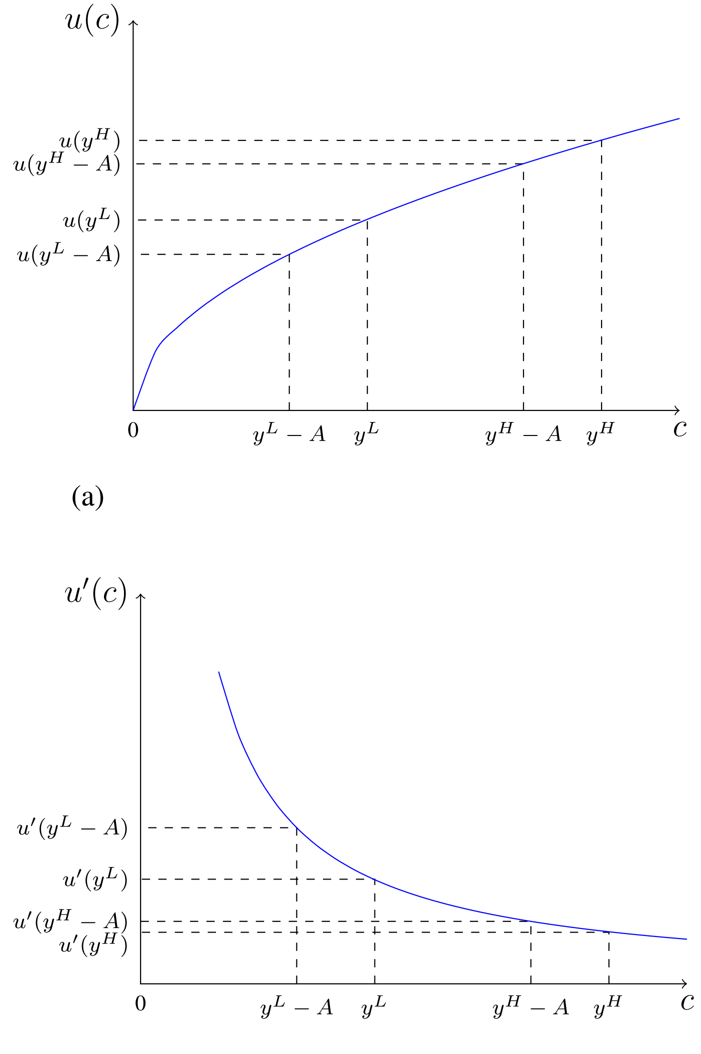


Fig. 4 **a** $u\,(y - A) - u\,(y)$ depending on state. **b** $u'\,(y - A) - u'\,(y)$ depending on state

We will now use this result to show that, if Assumption 1 holds, then whenever the Nash bargaining protocol selects default in high state, it would also select default in low state, with a higher default rate.

Suppose that, given some level of outstanding debt b_0', the Nash bargaining protocol selects default in the high state, and the corresponding default rate is δ_1'. In the high state, the derivative of the Nash Bargaining surplus with respect to δ_1 is:

$$\frac{\partial\Omega}{\partial\delta_1}(\delta_1, b_0, y_1(H))$$
$$= \Omega\left[\theta\frac{u'\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1)b_0\right)b_0}{(1+\beta)\left[u\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1)b_0\right) - u\left([1-\kappa]y_1(H)\right)\right]} - \frac{1-\theta}{1-\delta_1}\right].$$

Given $b_0 = b_0'$ this derivative must reach zero at $\delta_1 = \delta_1'$:

$$\frac{u'\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1')b_0'\right)b_0'(1-\delta_1')/(1+\beta)}{\left[u\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1')b_0'\right) - u\left([1-\kappa]y_1(H)\right)\right]} = \frac{1-\theta}{\theta}. \tag{41}$$

Substituting $A \equiv \frac{1}{1+\beta}(1-\delta_1')b_0'$, we rewrite this as:

$$\frac{u'\left(y_1(H) - A\right)A}{\left[u\left(y_1(H) - A\right) - u\left([1-\kappa]y_1(H)\right)\right]} = \frac{1-\theta}{\theta}. \tag{42}$$

Combining this result with the insight from (40), we note that:

$$\frac{u'(y_1(L) - A)A}{u(y_1(L) - A) - u([1-\kappa]y_1(L))} > \frac{u'(y_1(H) - A)A}{u(y_1(H) - A) - u([1-\kappa]y_1(H))} = \frac{1-\theta}{\theta}, \tag{43}$$

or, substituting back $A \equiv \frac{1}{1+\beta}(1-\delta_1')b_0'$:

$$\frac{u'\left(y_1(L) - \frac{1}{1+\beta}(1-\delta_1')b_0'\right)b_0'(1-\delta_1')/(1+\beta)}{\left[u\left(y_1(L) - \frac{1}{1+\beta}(1-\delta_1')b_0'\right) - u\left([1-\kappa]y_1(L)\right)\right]} - \frac{1-\theta}{\theta} > 0. \tag{44}$$

This latter result implies that in the low state, $\frac{\partial\Omega}{\partial\delta_1}(\delta_1', b_0', y_1(L)) > 0$—that is, the derivative of the Nash Bargaining surplus at $\delta = \delta_1'$ is positive; this in turn means that the bargaining surplus in the low state is maximized at a default rate higher than δ_1'. Therefore, whenever there is default in the high state, there would also be default in the low state with a higher corresponding default rate.

Proof of Proposition 1

In equilibrium, the default rates must maximize the Nash Bargaining surplus. Simultaneously the allocations must be consistent with household optimization. We will first show that, in general, there is no allocation consistent with both the above conditions and default in both states—i.e. that equilibrium with default in both states generally does not exist.

(a) No equilibrium with partial default in both state

Suppose that interior solutions are achieved in both states for some $b_0, \delta_1(L), \delta_1(H)$. In that case, the Nash Bargaining protocol requires that both of the following optimality conditions hold (see proof of Lemma 1 for further details):

$$\theta \frac{u'\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1(H))b_0\right) b_0}{(1+\beta)u\left(y_1(H) - \frac{1}{1+\beta}(1-\delta_1(H))b_0\right) - VA(y_1(H))} - \frac{1-\theta}{1-\delta_1(H)} = 0. \tag{45}$$

$$\theta \frac{u'\left(y_1(L) - \frac{1}{1+\beta}(1-\delta_1(L))b_0\right) b_0}{(1+\beta)u\left(y_1(L) - \frac{1}{1+\beta}(1-\delta_1(L))b_0\right) - VA(y_1(L))} - \frac{1-\theta}{1-\delta_1(L)} = 0. \tag{46}$$

Note that each of the above equations defines $b_0(1-\delta_1(s))$ as a function of period 1 parameters, pinning down the amounts of dedt recovered. Denote:

$$b_0(1-\delta_1(L)) \equiv \bar{b}(y_1(L), \theta, \kappa)$$
$$b_0(1-\delta_1(H)) \equiv \bar{b}(y_1(H), \theta, \kappa)$$

where $\bar{b}(y_1(L), \theta, \kappa)$ solves (45) and $\bar{b}(y_1(H), \theta, \kappa)$ solves (46). Furthermore, it follows that the NBS fixes the ratio of the repayment rates in the two stats:

$$\frac{1-\delta_1(L)}{1-\delta_1(H)} = \frac{\bar{b}(y_1(L), \theta, \kappa)}{\bar{b}(y_1(H), \theta, \kappa)}. \tag{47}$$

Now consider the households' optimization problem in period 0. The market allocation has to comply with households' FOCs. Combining the FOCs of domestic and foreign households, we obtain:

$$u'(c_0)(2-\delta_1(H)-\delta_1(L)) = u'(c_1(H))(1-\delta_1(H)) + u'(c_1(L))(1-\delta_1(L)) \tag{48}$$

Substituting solutions for c_0, $c_1(H)$ and $c_1(L)$ given b_0, obtain:

$$\begin{aligned} &u'\left(y_0 + \frac{\beta}{2}\left(2-\delta_1(H)-\delta_1(L)\right)b_0\right)(2-\delta_1(H)-\delta_1(L)) \\ &= u'\left(y_1(H) - \frac{1}{1+\beta}\left(1-\delta_1(H)\right)b_0\right)(1-\delta_1(H)) \\ &\quad + u'\left(y_1(L) - \frac{1}{1+\beta}\left(1-\delta_1(L)\right)b_0\right)(1-\delta_1(L)) \end{aligned} \tag{49}$$

This equation can be rewritten in terms of debt recovery amounts. Denoting $\tilde{b}_1(L) \equiv (1-\delta_1(L))b_0$ and $\tilde{b}_1(H) \equiv (1-\delta_1(H))b_0$, obtain:

$$u'\left(y_0 + \frac{\beta}{2}\tilde{b}_1(H) + \frac{\beta}{2}\tilde{b}_1(L)\right)(\tilde{b}_1(L) + \tilde{b}_1(H))$$
$$= u'\left(y_1(H) - \frac{1}{1+\beta}\tilde{b}_1(H)\right)\tilde{b}_1(H) + u'\left(y_1(L) - \frac{1}{1+\beta}\tilde{b}_1(L)\right)\tilde{b}_1(L) \quad (50)$$

Equilibrium allocation has to be consistent with households' choice—therefore, in equilibrium the above equation must hold. This equation gives a relationship between debt recovery amounts, $(1-\delta_1(L))b_0$ and $(1-\delta_1(H))b_0$.

However, as discussed above, in an equilibrium with default in both states these same values are independently pinned down by the Nash Bargaining protocol. Therefore, in general, market equilibrium with default in both states does not exist—except in a knife-edge case in which the model parameters happen to be such that

$$u'\left(y_0 + \frac{\beta}{2}\bar{b}(y_1(L),\theta,\kappa) + \frac{\beta}{2}\bar{b}(y_1(H),\theta,\kappa)\right)(\bar{b}(y_1(H),\theta,\kappa) + \bar{b}(y_1(L),\theta,\kappa)) \quad (51)$$

$$= u'\left(y_1(H) - \frac{1}{1+\beta}\bar{b}(y_1(H),\theta,\kappa)\right)\bar{b}(y_1(H),\theta,\kappa)$$
$$+ u'\left(y_1(L) - \frac{1}{1+\beta}\bar{b}(y_1(L),\theta,\kappa)\right)\bar{b}(y_1(L),\theta,\kappa) \quad (52)$$

In other words, household optimization is inconsistent with NBS protocol that selects default in both states. Therefore, we reach a contradiction. This implies that, if there is borrowing in equilibrium, then there must be full repayment in the high endowment state.

(b) No equilibrium with borrowing when θ is high

We now turn to prove the second half of the statement of Proposition 1. As we showed above, the households' problem can be re-framed in terms of solving for debt recovery amounts $\tilde{b}_1(L)$ and $\tilde{b}_1(H)$; the Eq. (50) then gives pairs of recovery amounts that are consistent with households' optimization. Within this set, we will now consider just the pairs $(\tilde{b}_1(L), \tilde{b}_1(H))$ associated with positive repayment in at least one of the states (thereby excluding $(0,0)$). At $\tilde{b}_1(L) = 0$ Eq. (50) results in $\tilde{b}_1(H) = (y_1(H) - y_0)\frac{2(1+\beta)}{\beta^2+\beta+2} > 0$; when $\tilde{b}_1(H) = 0$ it yields $\tilde{b}_1(L) = (y_1(L) - y_0)\frac{2(1+\beta)}{\beta^2+\beta+2} > 0$; in-between these two points pairs $(\tilde{b}_1(L), \tilde{b}_1(H))$ are linked by a continuous relation. Under these conditions, there exists a neighborhood around $(0,0)$ such that none of the points in the neighborhood are consistent with household optimization. At the same time, as θ increases, the limits $\bar{b}(H)$ and $\bar{b}(L)$ set by the Nash Bargaining protocol move closer to the point $(0,0)$—therefore, for θ sufficiently high there are no pairs $(\tilde{b}_1(L), \tilde{b}_1(H))$ that are both 1) consistent with household optimization and 2) are within the repayment limits set by the Nash Bargaining protocol.

Figure 5 illustrates this point. On Fig. 5, the blue line depicts pairs $(\tilde{b}_1(L), \tilde{b}_1(H))$ consistent with household optimization under log utility. Red and green lines show the upper bounds on debt repayments set by the Nash Bargaining protocol; the higher the θ, the closer the two bounds are to zero. Figure 5a shows equilibrium with full repayment (given by the green dot on the 45-degree line); here, full repayment is consistent with the Nash Bargaining protocol, as debt repayment amounts are within the bounds set by the Nash Bargaining protocol in both states. But as θ increases, the bounds $\bar{b}(H)$ and $\bar{b}(L)$ move inwards. When θ is too high, there are no pairs $(\tilde{b}_1(L), \tilde{b}_1(H))$ that are both consistent with household optimization (i.e. lie on the blue line) and are within the repayment limits prescribed by Nash Bargaining (i.e. are below $\bar{b}(H)$ and $\bar{b}(L)$)—see Fig. 5b.

(c) There is equilibrium with borrowing when θ is low

We will now show that, whenever the bargaining power is 'low enough', there is equilibrium with positive borrowing. First, consider $\theta = 0$, i.e. the case when the government has no bargaining power. In this extreme case, the Nash Bargaining Problem would select full repayment in every state—no matter the level of debt. Then, from the households' point of view, the problem of selecting debt b_0 would reduce to solving for borrowing in a risk-free asset. Given that we have assumed that income in period 1 exceeds income in period 0 in every state of the world, this problem would result in the households choosing some positive level of borrowing $b_0^* > 0$ that would allow for better consumption smoothing across periods.[16]

Now suppose that the government's bargaining power is slightly higher than 0, and $\theta = \epsilon$. Would the borrowing level b_0^* from before still be an equilibrium outcome? Consider the derivative of the Nash Bargaining surplus with respect to δ_1 in state S, assuming the level of debt b_0^* and full repayment:

$$\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0^*, y_1(S))$$
$$= \Omega \left[\theta \frac{u'\left(y_1(S) - \frac{1}{1+\beta} b_0^*\right) b_0^*}{(1+\beta)\left[u\left(y_1(S) - \frac{1}{1+\beta} b_0^*\right) - u\left([1-\kappa] y_1(S)\right)\right]} - (1-\theta) \right].$$

Note that we can always find θ 'small enough', such that this derivative is negative, in which case (as discussed in the proof of Lemma 1), the Nash Bargaining protocol would result in a corner solution $\delta_1(S) = 0$, and full debt repayment would be an equilibrium outcome.

To sum up, when θ is low enough, an equilibrium with positive borrowing and full debt repayment exists. This is because, on the one hand, conditional on full repayment in every state, households would like to borrow a positive amount b_0^*; on the other hand, conditional on debt b_0^*, the Nash Bargaining protocol selects full repayment in every state. Once again, Fig. 5a depicts such an equilibrium.

[16] We have assumed that the loss from financial autarky, κ, is high enough—such that, when debt is b_0^*, the household would prefer full repayment to financial autarky.

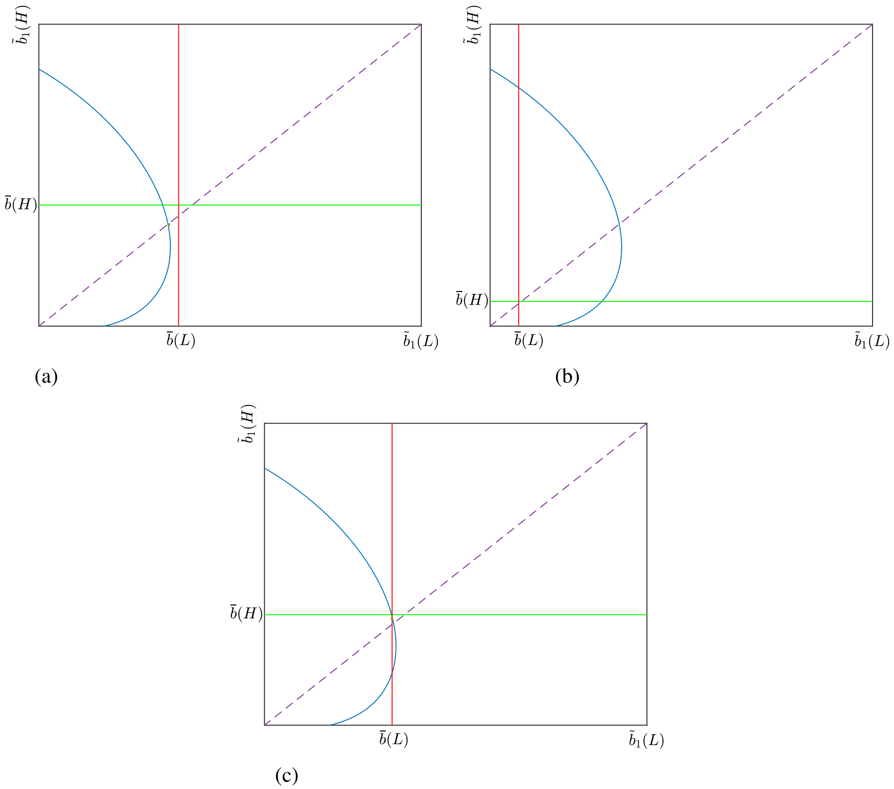


Fig. 5 Values of θ and equilibrium existence. **a** Equilibrium with full repayment (low θ). **b** No equilibrium (high θ). **c** Equilibrium with default in low state

Relationship to the non-pecuniary penalties for default in Dubey et al. (2005)

Proposition 1 can be explained through an alternate but equivalent representation of the economy. First, note that the following conditions hold for a solution of the Nash bargaining problem:

$$\begin{cases} u'(c_1(s))b_0 - \lambda_1(s) \leq 0 & for \quad \delta_1(s) = 0, \\ u'(c_1(s))b_0 - \lambda_1(s) = 0 & for \quad 1 > \delta_1(s) \geq 0, \\ u'(c_1(s))b_0 - \lambda_1(s) > 0 & for \quad \delta_1(s) = 1. \end{cases} \tag{53}$$

where $\lambda_1(s) = \frac{1-\theta}{\theta} \frac{\{VR\}_1^h(s) - \{VA\}_1^h(s)}{1-\delta_1(s)}$ and the last inequality when $\delta_1(s) = 1$ does not hold in equilibrium as this would imply the surplus for foreign households is zero. The above expressions arise from the first order condition with respect to the default rate that maximizes the joint surplus in (13).

Consider now an alternative version of our model, in which the households choose both the quantity of debt and the default rate in each of the states. Partial default is associated with a non-pecuniary cost $\lambda_1(s)$ that households take into account when choosing $\delta_1(s)$:

$$u(c_0) + \sum_{s=1}^{S} \pi(s) \left\{ \sum_{\tau=1}^{2} \beta^{\tau} u(c_{\tau}(s)) - \beta\lambda_1(s) \cdot \max\{\delta_1(s), 0\} \right\}. \qquad (54)$$

When the cost $\lambda_1(s)$ is defined as above, this augmented model yields the same equilibrium allocation as our original model with Nash bargaining. Therefore, our Nash bargaining model can be thought of as a micro-founded version of this alternative approach featuring non-pecuniary losses from default.

The non-pecuniary penalty approach has previously been explored in Dubey et al. (2005), and Peiris and Tsomocos (2015) in an international context. In contrast to these papers, the penalty for default $\lambda_1(s)$ featured here is state dependent, which means that the resulting default rate will depend on the wealth of the lender. Using the terminology in Dubey et al. (2005), the expressions in (53) are the *on-the-verge conditions*, in which the first term on the left-hand side represents the marginal cost of repayment and the second term is the marginal cost of default.

We now turn to a numerical example—to show how this augmented model can be used to interpret the results listed in Proposition 1. Figure 6 plots the zero equations in (53) for the high and the low states, and illustrates how agents adjusts their expectations about default rates until equilibrium is reached. Suppose that the bargaining power is low, and that the agents originally expect a pair of positive default rates $\delta_1(L) = \tilde{\delta}_L > 0$ and $\delta_1(H) = \tilde{\delta}_H > 0$ corresponding to the origin point of the first black arrow on Fig. 6a. For these default rates, the marginal gain from defaulting in low state equals the marginal cost, whereas in high state the loss exceeds the gain. That means that the default rate expected in the high state is 'too high'. In consequence, $\delta_1(H)$ falls to where in high state the marginal loss from defaulting equals the gain (see destination point for the first black arrow on Fig. 6a). But at this new coordinate the default rate in low state turns out to be 'too high', as the associated loss from defaulting is higher than the gain, causing expected $\delta_1(L)$ to fall, and so on. This process continues until the equilibrium default rates $\delta_1(L) = 1 - (1 - \tilde{\delta}_L)/(1 - \tilde{\delta}_H)$ and $\delta_1(H) = 0$ are reached.

Now let us compare this economy with the one in which the borrower has higher bargaining power (high θ), Fig. 6b. As before, zero level lines of marginal gains minus losses do not intersect, indicating there is no equilibrium with partial defaults in both state. But unlike on Fig. 6a, we now observe that the zero level line for default in high state is above that corresponding to the low state. For every combination of $\delta_1(L), \delta_1(H) < 1$ marginal gain from defaulting more in one of the states (or in both) is higher than the loss. Figure 6b shows that in such a setup, given any starting pair of default rates agents' expectations will adjust toward higher default rates in both states, and at the limit both expected default rates will approach 1. But if agents rationally expect full default in both states, then borrowing ex ante would not be possible. Therefore, when the borrowers' bargaining power is high, there is no equilibrium with positive borrowing. Therefore, an increase in the borrowers' bargaining power may compromise their ability to borrow.

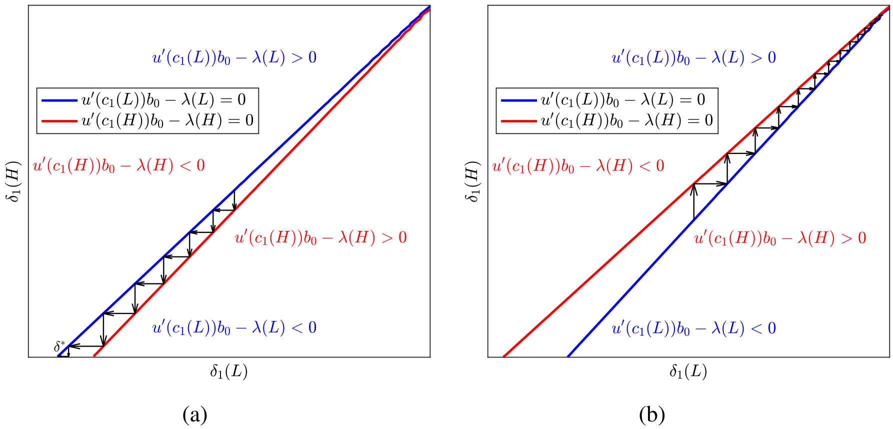


Fig. 6 Marginal gains and losses: equilibrium. **a** Low θ. **b** High θ (color figure online)

Numerical example of default in a decentralized equilibrium

On Fig. 7 we plot equilibrium values of Home households expected utility, default rate and the level of borrowing that arise in the model with bargaining, depending on the Home's bargaining power. When the borrowers' bargaining power is low, they are forced to fully repay the debt in both states, and borrowing is risk-free. When the bargaining power increases, the default rate in the low state rises. At higher $\delta_1(L)$ households wish to borrow more, as the repayment on the bonds now correlates negatively with the endowment, and issuing more debt promotes consumption smoothing across states L and H. As a result, as θ rises, consumers' welfare increases. This effect remains in place as long as θ is sufficiently low, but once it increases beyond some threshold, positive borrowing becomes unsustainable and at $t = 0$ capital flows cease.[17]

[17] It appears that the range of bargaining power that supports risk-free borrowing is wider compared to the range for which there is risky borrowing in equilibrium. With the parametrization we use to construct Fig. 7, the latter appears quite narrow. We suspect that the width of the risky borrowing range depends on the parametrization and functional form assumptions. For example, we speculate that this range depends on assumptions made about the endowment loss in the event of financial autarky. We have assumed that, following default, there is a fractional loss κ of endowment, the fraction that does not depend on whether the state is high or low. Suppose, instead, that the fraction of endowment lost in high state is higher (i.e. $\kappa(H) > \kappa(L)$)—due to e.g. a reputation loss associated with defaulting in high state. This assumption is in line with e.g. Arellano (2008), where the fractional loss of endowment increases with the endowment size. Under this assumption, the range of bargaining powers supporting risky borrowing would likely be wider. This is because, to sustain risky borrowing, Nash bargaining needs to deliver a haircut in the low state, while maintaining full repayment in the high state. The wedge between bargaining outcomes in the two states would be greater if the borrowers' reserve option in the high state was markedly worse than in the low state. At the same time, the households' incentives to borrow (as descibed by 50) would remain unchanged. Therefore, the range of bargaining powers supporting risky borrowing would likely be wider for $\kappa(H) > \kappa(L)$.

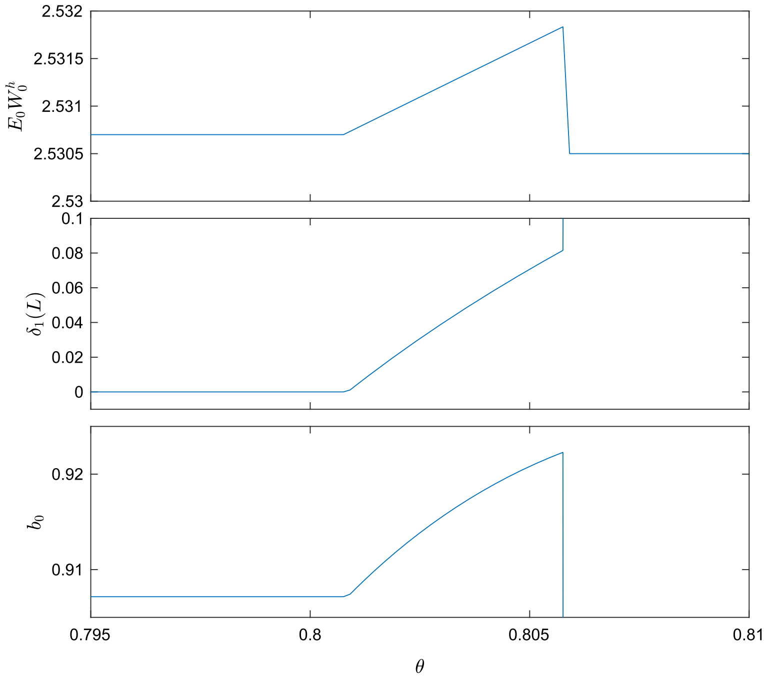


Fig. 7 Decentralized equilibrium and bargaining power. The figure plots equilibrium variables in the model with decentralized borrowing, depending on the borrowers' bargaining power. The plots depict expected welfare, default rate and the interest rate

Proof of Proposition 2

We will first consider the market equilibrium of interest and establish its properties. We will then compare the allocation chosen by the market to that preferred by the Planner; the planner who can choose borrowing directly, but is constrained by the Nash Bargaining protocol—i.e. we will compare market outcomes to the 'second best' allocation.

(a, b) There is underborrowing in competitive equilibrium; the haircut is too low

We have shown that whenever the Nash Bargaining protocol selects default in high state, it also selects default in low state (see Lemma 1). We have also established that default cannot occur in both states (see Proposition 1). Therefore, the only possible scenario that involves default is that with default in low state and full repayment in high state. In what follows we will consider the associated equilibrium in the decentralized version of the model.

Equilibrium with $\delta_1(H) = 0$ and $\delta_1(L) > 0$: proof that $c_1(H) > c_1(L)$.

Consider an equilibrium where, given some b_0, NBS produces corner solution in high state and interior solution in low state. If that is the case, then:

$$\frac{\partial\Omega}{\partial\delta}(\delta_1(H) = 0, b_0, y_1(H)) < 0 \tag{55}$$
$$\frac{\partial\Omega}{\partial\delta}(\delta_1(L) \geq 0, b_0, y_1(L)) = 0 \tag{56}$$

This holds if:

$$\left[\theta\frac{u'(c_1(H))b_0}{(1+\beta)u(c_1(H)) - VA(y_1(H))} - \frac{1-\theta}{1}\right] < 0. \tag{57}$$
$$\left[\theta\frac{u'(c_1(L))b_0}{(1+\beta)u(c_1(L)) - VA(y_1(L))} - \frac{1-\theta}{1-\delta_1(L)}\right] = 0. \tag{58}$$

Expanding the first statement we get:

$$\theta\frac{u'(c_1(H))b_0}{(1+\beta)u(c_1(H)) - VA(y_1(L))} - \frac{1-\theta}{1-\delta_1(L)} < \theta\frac{u'(c_1(H))b_0}{(1+\beta)u(c_1(H)) - VA(y_1(H))} - \frac{1-\theta}{1} < 0. \tag{59}$$
$$\theta\frac{u'(c_1(L))b_0}{(1+\beta)u(c_1(L)) - VA(y_1(L))} - \frac{1-\theta}{1-\delta_1(L)} = 0. \tag{60}$$

Combining the two statements we obtain:

$$\phi(c_1(H)) \equiv \frac{u'(c_1(H))}{(1+\beta)u(c_1(H)) - VA(y_1(L))} < \frac{u'(c_1(L))}{(1+\beta)u(c_1(L)) - VA(y_1(L))} \equiv \phi(c_1(L)) \tag{61}$$

Because $\phi(c)$ is decreasing, it follows that $c_1(H) > c_1(L)$.
Equilibrium with $\delta_1(H) = 0$ **and** $\delta_1(L) > 0$: **proof that** $c_0 > c_1(L)$.
Combining the FOCs for b_0 of the home and the foreign households we get (where the foreign household FOC is used to substitute the bond price function):

$$u'(c_0)(2 - \delta_1(H) - \delta_1(L)) = u'(c_1(H))(1 - \delta_1(H)) + u'(c_1(L))(1 - \delta_1(L)) \tag{62}$$

Substituting equilibrium $\delta_1(L) > 0$ and $\delta_1(H) = 0$ we get

$$[u'(c_0) - u'(c_1(L))](1 - \delta_1(L)) = [u'(c_1(H)) - u'(c_0)] \tag{63}$$

If $c_0 \leq c_1(L)$, then $c_1(H) \leq c_0$, and therefore $c_1(H) \leq c_1(L)$—a contradiction. Therefore, $c_0 > c_1(L)$.

Properties of the Planner's problem

We have defined the planner's problem in the body of the paper (see Eq. 30). We then characterized the derivative of households' expected welfare with respect to b_0—as perceived by the planner (see Eq. 31).

Planner's preferences at market equilibrium

In the body of the paper we have shown that, at the market allocation, the derivative of the planner's objective function over debt b_0 is positive. This implies that, compared to the market allocation, the planner would prefer a higher level of borrowing. This is because the planner internalizes the link between the level of borrowing and the haircut in the low state: at higher b_0, the default rate $\delta_1(L)$ would be higher—this would allow for better consumption smoothing across states of period 1.

(c) Borrowing subsidy

We have previously shown that in the competitive equilibrium with positive borrowing and default in the low state, the households borrow too little—in the sense that a marginally higher b_0 would result in a Pareto improvement. Here, we show that there is a balanced budget policy that the government can implement to improve competitive equilibrium outcomes. Particularly, in period $t = 0$ the government could subsidize household borrowing and pay for it with a lump-sum tax:

$$c_0 = y_0 + q_0(1 + \tau^s)b_0 - tr_0 \tag{64}$$

and

$$q_0 \tau^s b_0 = tr_0. \tag{65}$$

A small subsidy $\tau^s = \epsilon > 0$ would distort household optimization, and result in a marginally higher level of borrowing b_0. In the previous subsection we have shown that this, in turn, would improve household welfare. □

Proof of Proposition 3

Markets fail because home households wish to borrow beyond the level that ensures full repayment in the high endowment state. A proportional tax on debt issuance will limit the quantity of debt issued. As some debt issued is better than none, it follows that capital flow taxes can stimulate the market and, given the risk-neutrality of the lender, cause a Pareto improvement.

First, we show that an allocation in which households borrow and fully repay a small amount of debt $b_0^* = \epsilon > 0$ is preferred to an allocation with no capital flows. Second, we demonstrate that for any bargaining power $0 < \theta < 1$ we can find a 'small enough' positive level of debt b_0^* such that the Nash Bargaining protocol would elect full repayment in both states. Third, we propose a policy of taxing debt that would ensure that the optimizing households choose this small positive debt level b_0^* in a competitive equilibrium.[18]

(a) Gains to trade

Consider first the allocation with no capital flows. With this allocation, the households would choose to consume their endowment in each of the states of the world: $c_0 = y_0$; $c_1(s) = c_2(s) = y_1(s)$ (note that we have assumed $y_1(s) = y_2(s)$). Now suppose we perturb this allocation, allowing the households to borrow a small amount of debt $b_0^* = \epsilon > 0$ in period 0 that would be fully repaid in period 1. The households would now choose $c_0 = y_0 + \beta b_0^*$; $c_1(s) = c_2(s) = y_1(s) - \frac{1}{1+\beta} b_0^*$. The associated expected welfare would be $EW = u(y_0 + \beta b_0^*) + \frac{\beta(1+\beta)}{2}[u(y_1(L) - \frac{1}{1+\beta} b_0^*) + u(y_1(H) - \frac{1}{1+\beta} b_0^*)]$. Taking the derivative of the expected welfare over b_0^*, we obtain:

$$\frac{\partial EW}{\partial b_0^*} = \beta \left[u'(y_0 + \beta b_0^*) - \frac{1}{2} \left(u'(y_1(L) - \frac{1}{1+\beta} b_0^*) + u'(y_1(H) - \frac{1}{1+\beta} b_0^*) \right) \right] \tag{66}$$

If there are no capital flows (i.e. $b_0^* = 0$), this derivative becomes

$$\frac{\partial EW}{\partial b_0^*} = \beta \left[u'(y_0) - \frac{1}{2} \left(u'(y_1(L)) + u'(y_1(H)) \right) \right] > 0, \tag{67}$$

a value that is positive because, by assumption, $y_0 < y_1(L) < y_1(H)$ and $u''(.) < 0$. Intuitively, because households expect higher endowments in period 1, they prefer to smooth consumption fluctuations between periods through borrowing—rather than just consume endowments. Therefore, the allocation with small positive debt b_0^* is preferred to the allocation with no capital flows.

Implementing borrowing tax for Pareto improvement

For small enough debt, full repayment is the bargaining outcome.
We will now show that for small enough levels of debt, Nash Bargaining would indeed select full repayment in both state. To see this, consider the derivative of the function Ω derived under Lemma 1 assuming full repayment:

$$\begin{aligned} &\frac{\partial \Omega}{\partial \delta_1}(\delta_1 = 0, b_0^*, y_1(S)) \\ &= \Omega \left[\theta \frac{u'\left(y_1(S) - \frac{1}{1+\beta} b_0^*\right) b_0^*}{(1+\beta)\left[u\left(y_1(S) - \frac{1}{1+\beta} b_0^*\right) - u\left([1-\kappa] y_1(S)\right)\right]} - (1-\theta) \right]. \end{aligned}$$

[18] Here, we choose to only show that a small Pareto improvement can be implemented, and abstract from the discussion of the constraint optimal allocation. We do this for brevity, as this does not necessitate proving that the constraint optimal allocation exists—a task complicated by the presence of Nash Bargaining and the regime switch that occurs when equilibrium with positive borrowing ceases to exist.

As discussed in Lemma 1, $\Omega > 0$ for $b_0^* > 0$. The expression in brackets is an increasing function of b_0^*; given $0 < \theta < 1$, this function is negative at $b_0^* = 0$ and is continuously increasing as b_0^* goes up. This means that we can always find a small but positive b_0^* such that the derivative of Ω would be negative in both states. As discussed in Lemma 1, for such a b_0^* the Nash Bargaining protocol would then select full repayment in both states. We have thus shown that, on the one hand, a small enough level of borrowing would result in full repayment in both states; on the other hand, the corresponding allocation would be Pareto improving compared to the allocation with no capital flows.

Borrowing tax

We have previously shown that, when bargaining power θ is too high, there is no competitive equilibrium with borrowing. We have also demonstrated that the corresponding allocation is inferior compared to the allocation in which the households are allowed to borrow a small positive amount b_0^* that would be fully repaid under Nash Bargaining. What remains to show is that we can find policy tools that would influence the households' decisions such that the latter would choose to borrow b_0^* in a competitive equilibrium.

There are several balanced budget policies that the government can implement to reduce households' borrowing levels. First, the government could place an outright constraint on the households' choice of b_0, requiring $b_0 \leq b_0^*$. Alternatively, the government could implement a tax on borrowing that would be returned to the households as a lump-sum transfer:

$$c_0 = y_0 + q_0(1 - \tau^t)b_0 + tr_0 \tag{68}$$

and

$$q_0 \tau^t b_0 = tr_0. \tag{69}$$

A large enough tax on borrowing would stir the households towards choosing small positive level of debt b_0^*, a level that can be supported as equilibrium with full repayment in both states under Nash Bargaining.[19] Thus, such a policy would result in a Pareto improvement compared to the allocation with no capital flows.

[19] Specifically, the tax rate should be such that the households' first order condition is satisfied at the borrowing level b_0^*, i.e. τ^t should solve $u'(y_0 + \beta b_0^*)(1 - \tau^t) = \frac{1}{2}\left(u'(y_1(L) - \frac{1}{1+\beta} b_0^*) + u'(y_1(H) - \frac{1}{1+\beta} b_0^*)\right)$.

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