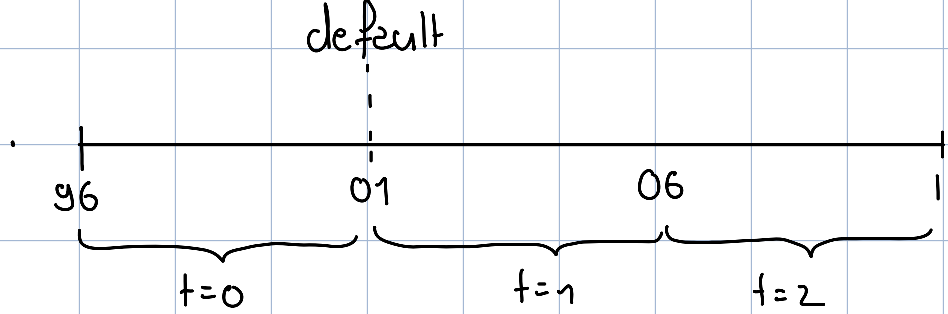
Uclare calibration

Set parameters

- risk aversion: $2 = \sigma$
- discount factor:
  - 5 years



  - $\beta = \frac{1}{(1+r)^{20}}$, r = risk-free rate = 0.01 (average quarterly interest rate on 3 month US treasury bill)
    $= \frac{1}{1.01^{20}} = 0.82$

- default rate: 40%
  - source: Sturzenegger and Zettlemeyer

- g_L and g_H
  - quarterly or annual real GDP from MECON
  - g_L -> 25 percentile: 0.0005
  - g_H -> 75 percentile: 0.007
  - 1980 or lower - 2001

- loss of endowment $K = 2\%$
  - source: Aguiar and Gopinath (2006)

Solving model: HH optimization

- Home HH FOC:

$$q_0 u'(y_0 + q_0 b_0) = \frac{1}{2}\Big[B(1-\delta_1(L))u'(y_1(L) + q_1(L)b_1(L) - (1-\delta_1(L))b_0)$$
$$+ B(1-\delta_1(H))u'(y_1(H) + q_1(H)b_1(H) - (1-\delta_1(H))b_0)$$

$$q_1(s) = B\frac{u'(c_2(s))}{u'(c_1(s))} \to c_1(s) = c_2(s)$$

- Foreign HH FOC:

$$q_0 = B\left[\frac{1}{2}(1-\delta_1(L)) + \frac{1}{2}(1-\delta_1(H))\right] = B\left[1 - \frac{\delta_1(L)}{2} - \frac{\delta_1(H)}{2}\right]$$

$$q_1(s) = B \to q_1(L) = q_1(H) = B$$

- pin down b_1:

from $c_1(s) + (1-\delta_1(s))b_0 = y_1(s) + q_1(s)b_1(s)$ ①

$\underbrace{c_1(s)}_{c = c_2(s)} + b_1(s) = \underbrace{y_1(s)}_{c = y_2(s)}$ ②

① $c_1(s) = y_1(s) + q_1(s)b_1(s) - (1-\delta_1(s))b_0$ in ②

② $b_1(s) = y_1(s) - c_1(s)$

$= -q_1(s)b_1(s) + (1-\delta_1(s))b_0$

$$b_1(s) = \frac{1-\delta_1(s)}{1+q_1(s)} b_0 = \frac{1-\delta_1(s)}{1+B} b_0$$

- from home HH FOC \ b_0 and exogenous $\delta_1(L)$ and $\delta_1(H)$ can solve for b_0

$$q_0 u'(y_0 + q_0 b_0) = \frac{1}{2}\Big[B(1-\delta_1(L))u'(y_1(L) + \overset{= B}{q_1(L)}b_1(L) - (1-\delta_1(L))b_0)$$
$$+ B(1-\delta_1(H))u'(y_1(H) + q_1(H)b_1(H) - (1-\delta_1(H))b_0)\Big]$$

$$q_0 u'(y_0+q_0 b_0) = B\left[1-\frac{\delta_1(L)}{2}-\frac{\delta_1(H)}{2}\right]\left[y_0+B\left(1-\frac{\delta_1(L)}{2}-\frac{\delta_1(H)}{2}\right)b_0\right]^{-\sigma}$$

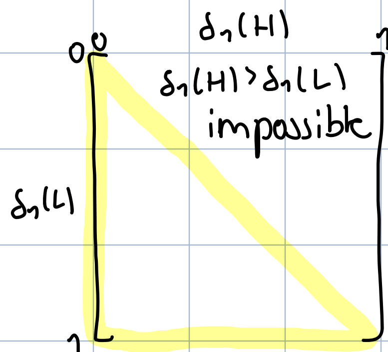
$$\frac{1}{2}\Big[B(1-\delta_1(L))u'(y_1(L)+q_1(L)b_1(L)-(1-\delta_1(L))b_0)$$
$$+B(1-\delta_1(H))u'(y_1(H)+q_1(H)b_1(H)-(1-\delta_1(H))b_0)\Big]$$
$$=\frac{1}{2}\Big\{B(1-\delta_1(L))\Big[y_1(L)+B\frac{1-\delta_1(L)}{1+B}b_0-(1-\delta_1(L))b_0\Big]^{-\sigma}$$
$$+B(1-\delta_1(H))\Big[y_1(H)+B\frac{1-\delta_1(H)}{1+B}b_0-(1-\delta_1(H))b_0\Big]^{-\sigma}\Big\}$$

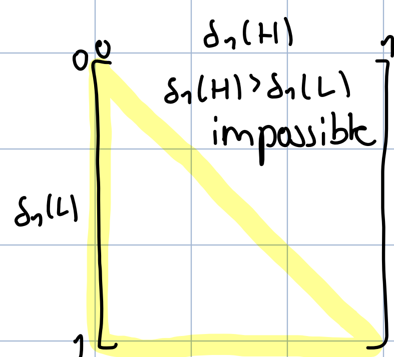
=

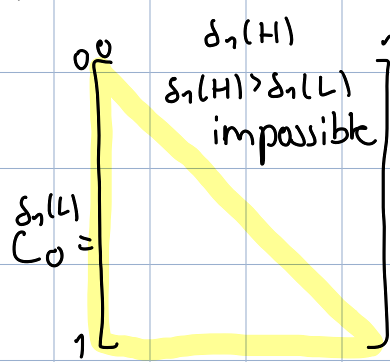
$$B\left[1-\frac{\delta_1(L)}{2}-\frac{\delta_1(H)}{2}\right]\left[y_0+B\left(1-\frac{\delta_1(L)}{2}-\frac{\delta_1(H)}{2}\right)b_0\right]^{-\sigma}$$
$$=\frac{1}{2}\Big\{B(1-\delta_1(L))\Big[y_1(L)+B\frac{1-\delta_1(L)}{1+B}b_0-(1-\delta_1(L))b_0\Big]^{-\sigma}$$
$$+B(1-\delta_1(H))\Big[y_1(H)+B\frac{1-\delta_1(H)}{1+B}b_0-(1-\delta_1(H))b_0\Big]^{-\sigma}\Big\}$$

$$\left.\begin{aligned}&[2-\delta_1(L)-\delta_1(H)]\left[y_0+B\left(1-\frac{\delta_1(L)}{2}-\frac{\delta_1(H)}{2}\right)b_0\right]^{-\sigma}\\ &=(1-\delta_1(L))\Big[y_1(L)+B\frac{1-\delta_1(L)}{1+B}b_0-(1-\delta_1(L))b_0\Big]^{-\sigma}\\ &+(1-\delta_1(H))\Big[y_1(H)+B\frac{1-\delta_1(H)}{1+B}b_0-(1-\delta_1(H))b_0\Big]^{-\sigma}\end{aligned}\right\}\ \text{solve numerically for } b_0$$

- for any pair $\delta_1(L)$ and $\delta_1(H)$, I would have q_0, b_0, c_0

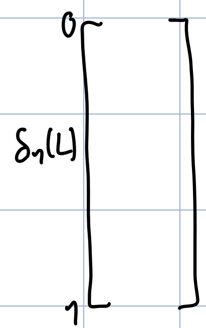
$q_0 =$ 

, $b_0 =$ 

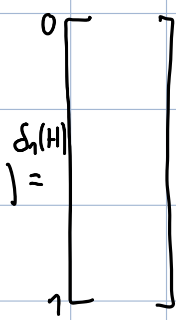
, $c_0 =$ 

- for any pair $\delta_1(L)$ and $\delta_1(H)$ and realization of $s = \{L, H\}$, I would have $b_1(s), c_1(s)$: (maybe wrong because requires δ_1^*)

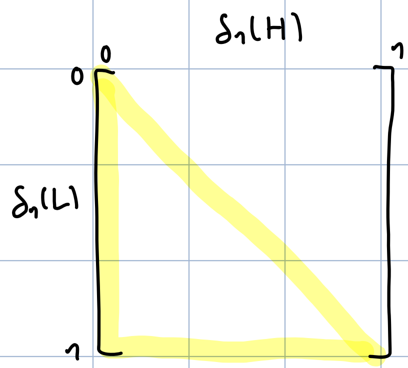
$b_1(L) =$



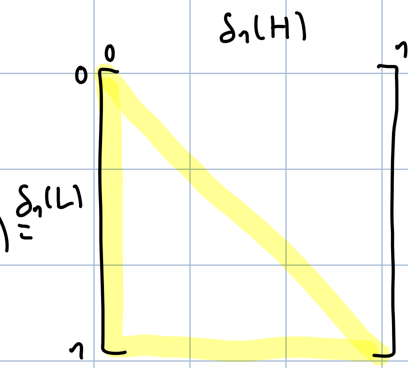
, $b_1(H) =$



$c_1(L) =$



, $c_1(H) =$



Solving model: Nash bargaining

- problem: for every $b_0(\delta_1(L), \delta_1(H))$ and realization of $s = \{L, H\}$

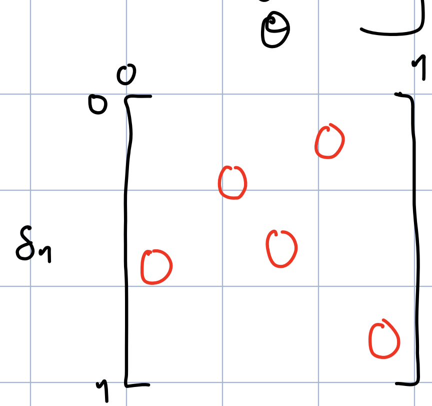
$$\begin{aligned}
\delta_1 &= \arg\max \, \Omega(\delta_1, b_0, y_1) \\
&= \arg\max \, [\Delta^h(\delta_1)]^{\theta} [\Delta^f(\delta_1)]^{1-\theta} \\
&= \arg\max \, [(1+B)u(c_1(s)) - u(y_1(s)) - Bu((1-K)y_1(s))]^{\theta} \\
&\quad \cdot [b_0(1-\delta_1(s))]^{1-\theta} \\
&= \arg\max \, [(1+B)u(y_1(s) - \tfrac{1-\delta_1(s)}{1+B} b_0) - u(y_1(s)) - Bu((1-K)y_1(s))]^{\theta} \\
&\quad \cdot [b_0(1-\delta_1(s))]^{1-\theta}
\end{aligned}$$

It feels weird to me that b_0 depends on $\delta_1(L)$ and $\delta_1(H)$ and that bargaining problem gives optimal $\delta_1(L)$ and $\delta_2(H)$

- compute numerically Ω for all pairs $(\delta_1(s), \theta)$

$\underbrace{\delta_1(L) \times \delta_1(H)}_{\text{determine } b_0} \times \delta_1 \times \theta$

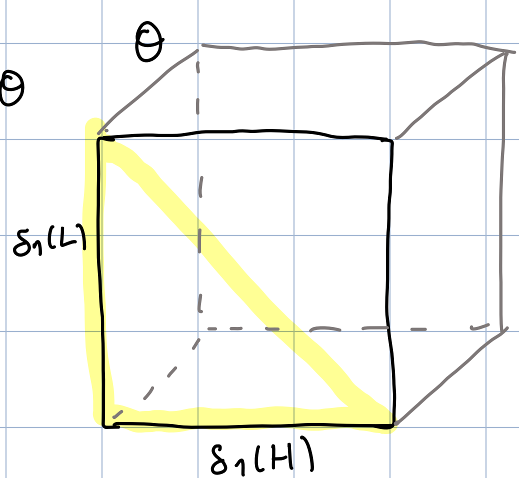
$\Omega = $



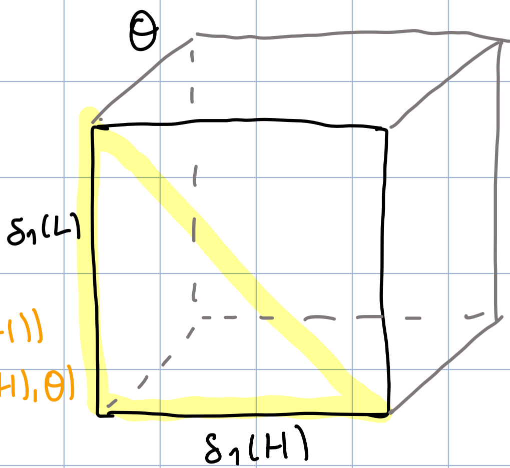
optimal $\delta_1 \; \forall \theta$

$\delta_1^* =$

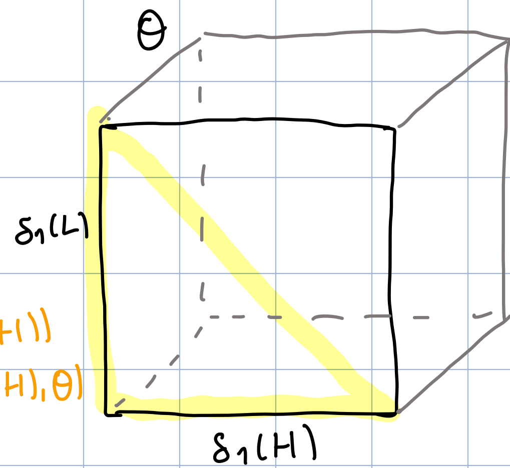
$\delta_1(L) \times \delta_1(H) \times \theta$



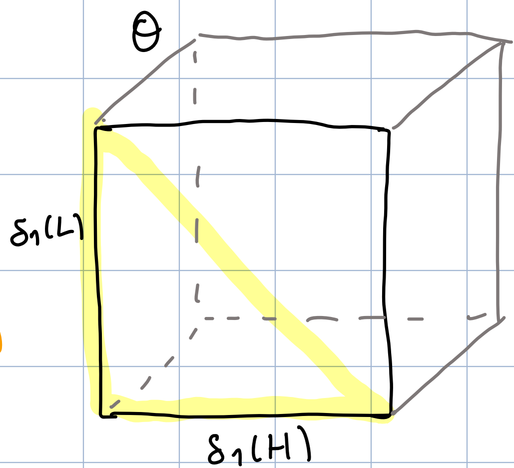
$\underbrace{b_1(L)}_{\substack{b_0(\delta_1(L),\delta_1(H)) \\ \delta_1^*(\delta_1(L),\delta_1(H),\theta)}} =$



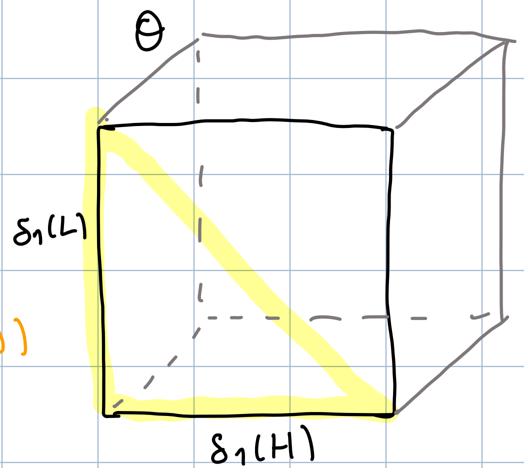
$\underbrace{b_1(H)}_{\substack{b_0(\delta_1(L),\delta_1(H)) \\ \delta_1^*(\delta_1(L),\delta_1(H),\theta)}} =$



$\underbrace{c_1(L)}_{y_1(L) - b_1(L)} =$

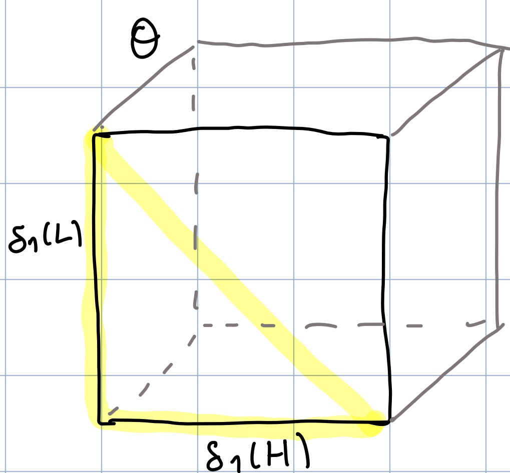


$\underbrace{c_1(H)}_{y_1(s) - b_1(s)} =$



- compute $U = \max\{u^r, u^d\} \ \forall\ \theta, \underbrace{\delta_1(L), \delta_1(H)}_{b_0}$
  - $u^r = (1+B)\,u\left(y_1(s) - \frac{1-\delta_1^*(s)}{1+B}\, b_0\right)$
  - $u^d = u(y_1(s)) + B\,u((1-K)y_1(s))$

$\underset{\delta_1(L)\times\delta_1(H)\times\theta}{U} =$



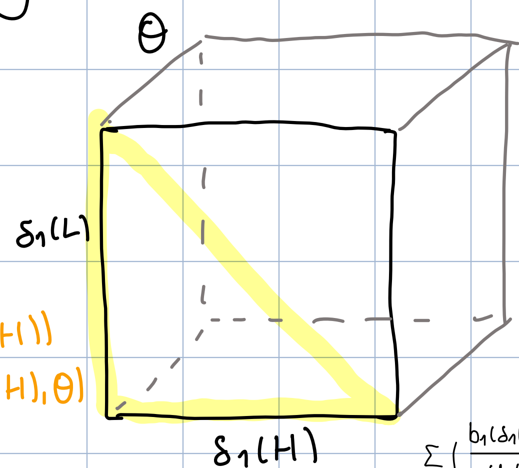
- calibrate θ to target $\frac{b_1}{y_1}$? $\frac{b_0}{y_0}$ does not depend on θ

• compute $b_1(s)$ matrix considering the renegotiation vs default decision

→ if default, $b_1(\delta_1(L), \delta_1(H), \theta) = 0$

→ if renegotiation, $b_1(\delta_1(L), \delta_1(H), \theta) \geq 0$

$$\underbrace{b_1^{rd}(s)}_{\substack{b_0(\delta_1(L),\delta_1(H))\\ \delta_1^*(\delta_1(L),\delta_1(H),\theta)}} =$$



$$\sum_{\substack{\text{entry}\\ \text{of } b_1^{rd} \text{ matrix}\\ \text{for given } \theta}} \left(\frac{b_1(\delta_1(L),\delta_1(H),s=L)}{y_1(L)} - \frac{b_1^{target}}{y_1^{target}}\right)^2 + \sum_{\substack{\text{entry}\\ \text{of } b_1^{rd} \text{ matrix}\\ \text{for given } \theta}} \left(\frac{b_1(\delta_1(L),\delta_1(H),s=H)}{y_1(H)} - \frac{b_1^{target}}{y_1^{target}}\right)^2$$

• for each θ, compute deviation of

$$\sum_{\substack{\text{entry of } b_1^{rd} \text{ matrix}\\ \text{for given } \theta}} \left(\frac{b_1(\delta_1(L),\delta_1(H)|s=L)}{y_1(L)} - \frac{b_1^{target}}{y_1^{target}}\right)^2 + \sum_{\substack{\text{entry of } b_1^{rd} \text{ matrix}\\ \text{for given } \theta}} \left(\frac{b_1(\delta_1(L),\delta_1(H)|s=H)}{y_1(H)} - \frac{b_1^{target}}{y_1^{target}}\right)^2$$

$$\underset{1\times\theta}{\text{deviation}} = \begin{array}{ccc} 0 & \theta & 1 \\ [& &] \end{array}$$

→ lowest deviation pins down θ

Algorithm: parameter bargaining power to target $\frac{debt}{GDP}$

0. grid on b
   value for y_0
1. guess Θ