Name: ______________________ Period: ____________

Instructor: Mr. Rodriguez Course: Conceptual Physics A

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Algebra Foundations for Science

"Mathematics is the alphabet with which God has written the universe."

—Galileo Galilei

1 Single Variable Equations

In your algebra course, you have learned how to solve equations for a particular variable. For example, you have undoubtedly encountered many problems of the form,

Example

Problem: *Solve the following equation for x:*

$$2x + 6 = 10.$$

Solution: How do we do it? Well, we may either stare at the equation long enough and guess the solution (Think: *"What number when doubled and added to six gives ten?"*), or we may fall back and the tried and true algebraic manipulations you have mastered in your math courses. The fundamental rule in solving any equation is, as always:

1. **All mathematical operations must be applied to both sides of the equation.**

To wit:

$$\begin{aligned} 2x + 6 &= 10 && \text{(write down the problem)} \\ 2x &= 4 && \text{(subtract 6 from both sides)} \\ \boxed{x = 2.} & && \text{(divide both sides by 2)} \end{aligned}$$

It is always good practice to box your final answers.

Now give it a try yourself:

1. Solve $2x - 4 = 2$ for x.

2. Solve $4x - 9 = 7$ for x.

3. Solve $6x - 3 = 3x + 9$ for x.

4. Solve $3y + 2 = 14$ for y.

5. Solve $5z - 8 = 27$ for z.

2 Double Variable Equations

Oftentimes in physics problems, there is more than one variable involved. If there are two — call them x and y — we might refer to them as *independent* and *dependent* variables, respectively. The naming convention reminds us that y depends on x; if we change x, we expect y to change as well.

In a science experiment, for example, x might represent the amount of water given each day to a plant in mL (milliliters), while y might represent the height of the plant in cm (centimeters).

Example

Problem: *Solve for y. Then interpret the physical meaning of the equation:*

$$2y - 10x = 20.$$

Solution:

$$\begin{aligned} 2y - 10x &= 20 && \text{(write down the problem)} \\ 2y &= 10x + 20 && \text{(add } 10x \text{ to both sides)} \\ \boxed{y = 5x + 10.} &&& \text{(divide both sides by 2)} \end{aligned}$$

Because the equation takes the familiar $y = mx + b$ form, we could say that "y increases **linearly** with x." In the context of the scientific situation described above the example box, this equation might then be interpreted to be

$$\boxed{(\text{height of plant in cm}) = 5 \times (\text{water given to plant in mL}) + 10.}$$

If we knew that a particular plant was given $x = 5\,\text{mL}$ of water, say, we could plug this into our equation to find the expected height of the plant:

$$\begin{aligned} y &= 5(5) + 10 && \text{(plug in } x = 5\text{)} \\ y &= 25 + 10 && \text{(add the numbers)} \\ \boxed{y = 35\,\text{cm}} &&& \text{(the expected height of the plant is 35 cm)} \end{aligned}$$

1. (a) Solve $2x + 3y = 12$ for y.

   (b) If $x = 4$, what is y?

2. (a) Solve $5a - 2b = 20$ for b.

   (b) If $a = 6$, what is b?

3. (a) Solve $\frac{1}{f} = g$ for f.

   (b) If $g = 2$, what is f?

4. (a) Say that you are (quickly) walking to class. Every 1 second, you walk 2 meters. Could you write an equation that gives you the distance d that you walk in terms of the seconds that you are walking (call the number of seconds t)?

   (b) Using your equation from the previous part, calculate far would you be able to walk in 60 seconds.

3 Full-Variable Equations

It is also possible (and quite common) for physical equations to contain more than two variables. For example, say that the average speed (velocity) of your car v is equal to the total distance d traveled divided by the time t of travel. The equation that describes this situation would be

$$v = \frac{d}{t}.$$

If you were to travel $d = 100\,\text{mi}$ in $t = 2\,\text{hr}$, your average speed would be

$$v = \frac{100\,\text{mi}}{2\,\text{hr}} = 50\,\text{mph}.$$

Example

`Problem:` Say that you are on the freeway going $v = 65\,\text{mph}$. How far will you travel in $0.5\,\text{hr}$?

`Solution:` To solve this problem, we need to rearrange the velocity equation $v = {}^{d}/_{t}$ for our desired variable, namely the distance d:

$$\begin{aligned}
v &= \frac{d}{t} && \text{(start with the given equation)}\\
t \times v &= \frac{d}{t} \times t && \text{(multiply both sides by t)}\\
t \times v &= \frac{d}{\cancel{t}} \times \cancel{t} && \text{(cancel the t's)}\\
d &= v\,t && \text{(swap left and right sides of the equation)}\\
d &= (65\,\text{mph})(0.5\,\text{hr}) && \text{(plug in the given values)}\\
&\boxed{d = 32.5\,\text{mi}} && \text{(box final answer)}
\end{aligned}$$

1. Given the equation $F = ma$,

   (a) Solve for m in terms of F and a. Your answer will be a fraction.

   (b) Solve for a in terms of F and m. Your answer again will be a fraction.

2. (a) Solve the velocity equation $v = d/t$ for the time t.

   (b) Use your result from the previous part to find how long would it take you to travel $d = 200\,\text{mi}$ if you are going $v = 65\,\text{mph}$.

3. The speed of light can be written as $c = \lambda f$, where c is the speed of light, λ is the wavelength of light, and f is the frequency of the light. (By the way, the Greek letter λ is pronounced "lamb-dah".)

   (a) Solve for the frequency f in terms of c and λ.

   (b) If the speed of light is $c = 300\,000\,000\,\text{m/s}$ and the wavelength of a beam of light is $\lambda = 10\,\text{m}$, what is the frequency f of the beam of light?

4. *Challenge Problem:* Consider the equation

$$\frac{F}{A} = P + \frac{B}{C}.$$

How would you go about solving for C in terms of the other 4 variables?

5. *Challenge Problem:* Consider the following two equations:

$$E = hf \qquad\qquad c = \lambda f.$$

The first equation represents the energy of a packet of light in terms of its frequency and a constant h. Use both equations to solve for E in terms of h, c, and λ.