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Chapter 4

Section 1

№ 6

a) $12345 = -39 \cdot (-316) + 21$

$q = -316$

$r = 21$

b) $-27361 = -977 \cdot 29 + 972$

$q = 29$

$r = 972$

Section 2

№ 12

d) $1575 = 231 \cdot 6 + 189$

$231 = 189 \cdot 1 + 42$

$189 = 42 \cdot 4 + 21$

$42 = 21 \cdot 2 + 0$

$21 = 189 - 42 \cdot 4 = 189 - 4(231 - 189) = -4 \cdot 231 + 5 \cdot 189 =$

$= -4 \cdot 231 + 5 \cdot (1575 - 231 \cdot 6) = -4 \cdot 231 + 5 \cdot 1575 - 30 \cdot 231 =$

$= \cancel{-34 \cdot} \; 5 \cdot 1575 - 34 \cdot 231$

$\underbrace{5}_{m} \cdot 1575 - \underbrace{34}_{n} \cdot 231$

Section 3

№4

a) $856 = 2 \cdot 2 \cdot 2 \cdot 107 = 2^3 \cdot \cancel{8} \cdot 107$

б) ~~8)~~ $6647 = 17 \cdot 17 \cdot 23$

в) $2323 = 23 \cdot 101$

d) $9970 = 5 \cdot 2 \cdot 997$

e) $(2^8 - 1)^{20} = 5^{20} \cdot 17^{20} \cdot 3^{20}$

f) ~~5511~~ $55551111 = 11 \cdot 3 \cdot 7 \cdot 101 \cdot 2381$

№17

c) $d(n) = 5$ if n will be number in power 4

№18

a) No. Because we can use $a^3 - b^3$ properties so $2^{15} - 1$ have divisors

b) No, because we can use properties $a^n - b^n$, so $(2^{13})^7 - 1^7$ isn't prime

c) ~~if $n = a$~~ we ~~write~~ $n = ~~ab~~$ ~~where~~ ~~a, b~~

~~$2^{ab} - 1$~~ we again get formula $a^n - b^n$ so $2^n - 1^n$ isn't prime

d) c isn't true

Section 4

№7

b) $17891 + 14485 \pmod{143} = 32376 \pmod{143} = 58$

$32376 = 143 \cdot 226 + 58$ $(a+b \pmod n)$

~~$143 = 58 \cdot 2 + 27$~~ ~~$17891 \pmod{143}$~~

~~$58 = 27 \cdot 2 + 4$~~ $17891 = 143 \cdot 125 + 16$ $\equiv 4$

~~$27 = 4 \cdot 6 + 3$~~ $14485 = 143 \cdot 101 + 42$ $\equiv 4$

~~$4 = 3 \cdot 1 + 1$~~

~~$3 = 3 \cdot 1 + 0$~~

$ab \pmod n = 16 \cdot 42 = 672$

$(a+b)^2 \pmod n = (16+42)^2 = 3364$

c) $-389221 = 10000 \cdot 39 + 779$ $\equiv 4$

$123450 = 10000 \cdot 12 + 3450$ $\equiv 4$

$a+b \pmod n = 779 + 3450 = 4229$

$ab \pmod n = \cancel{268755}\ 779 \cdot 3450 = 2687550$

$(a+b)^2 \pmod n = 4229^2 = 17884,441$

№9

a) $5 \pmod{25}$ $\quad 5 = 25 \cdot 0 + 5$

$5x = 5$

$x = 1$

№ 9

(g) $5x - 5 = 25k$ | :5

$x - 1 = \cancel{25}\,5k$

$x = 5k + 1$

So $x = 1, 6, 11, 16, 21$

h) $4x \equiv 301 \pmod{n}$

$\cancel{4x - 301}$

$301 = 592 \cdot 0 + 301$

$4x - 301$ is even. This is impossible, for any x, $4x$ is even

№ $\cancel{508}$ 22

c) $3^{508} \equiv 1 \pmod{509}$

$3^{509} \equiv 3 \pmod{509}$

$\cancel{3^{512} \equiv 3^4 \equiv 81 \pmod{509}}$

$3^{512} \equiv 3^4 = 81 \pmod{509}$

Section 5

№ 18

b) $1 = -2 \cdot 4 + 1 \cdot 9$

$x = 8 \cdot (-2) \cdot 4 + 1 \cdot 1 \cdot 9 = -55 \equiv 17 \pmod{36}$. So $x = 17$

g) $1 = -2 \cdot 4 + 1 \cdot 9$; $x = -55 \equiv 17 \pmod{36}$

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$1 = 13 \cdot 25 - 9 \cdot 36$

$x = 10 \cdot 13 \cdot 38 + 17 \cdot 13 \cdot 25 = 2285 \equiv 485 \pmod{900}$

$x = 485$

Chapter 5

Section 1

№ 4

b) $n = 1$

$1^3 + (1+1)^3 + (1+2)^3 = 1 + 8 + 27 = 36$ is divided by 9

$n = k$

$k^3 + (k+1)^3 + (k+2)^3$

$n = k+1$

$(k+1)^3 + (k+2)^3 + (k+3)^3 = (k+1)^3 + (k+2)^3 + k^3 + 9k^2 + 9k + 27 =$

$= (k^3 + (k+1)^3 + (k+2)^3) + (9k^2 + 9k + 27)$ is divided by 9

№ 6

d) $n = 1$

$1(1+1) \cdot (1+2) = 1 \cdot 2 \cdot 3 = \frac{1 \cdot 2 \cdot 3 \cdot 4}{4}$

$n = k+1$

$(k+1)(k+2)(k+3)(k+4) = \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

$\frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3) =$

$= (k+1)(k+2)(k+3)\left(\frac{k}{4}+1\right) = \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

№ 9

б) $n = -3$

$2^{-3} \geqslant \frac{1}{8}$

$\frac{1}{8} \geqslant \frac{1}{8}$

$n = k$

$2^{k} \geqslant \frac{1}{8}$

$n = k+1$

$\cancel{2^{k+1} \geqslant \frac{1}{8}}$

$2^{k+1} = 2 \cdot 2^{k} \geqslant 2 \cdot \frac{1}{8}$

$\frac{1}{4} > \frac{1}{8}$

Section 2

№ 5

$0;\ \frac{1}{2};\ \frac{1}{4};\ \frac{3}{8};\ \frac{5}{16};\ \frac{11}{32};\ \frac{21}{64}$

$n = 1$

$\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{1-1}\right) = 0$

$n = 2$

$\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{2-1}\right) = \frac{1}{2}$

$n = k$

$\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{k-1}\right)$

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$a_k = \frac{1}{2}(a_{k-2} + a_{k-1})$

$k = k-1$

$a_k = \frac{1}{2}\left(\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{k-3}\right) + \frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{k-2}\right)\right) =$

$= \frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{k-3}\left(-\frac{1}{2}\right)^{2}\right) = \frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{k-1}\right)$

№2

$a_0 = 2,\ a_1 = 3,\ a_2 = 5,\ a_3 = 9,\ a_4 = 17,\ a_5 = 33$

$a_n = \cancel{a_{n-2}}\ 2^n + 1$

$n = 0$

$a_0 = 2^0 + 1 = 2$

$n = 1$

$a_1 = 2^1 + 1 = 3$

$a_n = 3a_{n-1} - 2a_{n-2}$ if $n \geq 2$

Change a_{n-1} on $2^{n-1}+1$ and a_{n-2} on $2^{n-2}+1$

$a_n = 3(2^{n-1}+1) - 2(2^{n-2}+1) = 3(2^{n-1}) + 3 - 2(2^{n-2}) - 2 =$

$= 3(2^{n-1}) - 2^{n-1} + 1 = 2^n + 1$

Section 3

№4

$x^2 - 7x + 10 = 0 \qquad x = \frac{7 \pm 3}{2} = 5; 2$

$D = 49 - 40 = 9$

$\cancel{a_n =}\ C_1(2) + C_2(5) = 10$

$\cancel{a}\ C_1(2^2) + C_2(5^2) = 29$

$C_1 = \frac{7}{2}$; $C_2 = \frac{3}{5}$

$a_n = \frac{7}{2}(2^n) + \frac{3}{5}(5^n) = 7(2^{n-1}) + 3(5^{n-1})$

№8

$x^2 + 5x - 6 = 0$

$D = 25 + 24 = 49$

$x = \frac{-5 \pm 7}{2} = 1; -6$

~~$a_n = t$~~

$\cancel{C_1(1^n}$ $C_1(1) + C_2(-6) = 5$

$C_1(1) + C_2(-6) = 19$

$C_1 = 7$; $C_2 = -2$

$a_n = 7 - 2(-6)^n$

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