Ex 1

a) $\Pi_m = \frac{\mathbb{1} \pm S}{2}$

i.e. for $n=1$ we have $S \in \{X, Y, Z, \mathbb{1}\}$

for $S = |+\rangle\langle+| - |-\rangle\langle-|$ where $|+\rangle$ and $|-\rangle$ are the +1/-1 EV of S

we project on the +1 eigenstate $|+\rangle$
or the -1 eigenstate $|-\rangle$

if we project onto the +1 eigenstate
the projector is $|+\rangle\langle+|$ ✓

test with $m=+1$:

$$\Pi_{+1} = \frac{\mathbb{1} - S}{2} = \frac{(|+\rangle\langle+| + |-\rangle\langle-|) - (|+\rangle\langle+| - |-\rangle\langle-|)}{2}$$

$$= \frac{2\,|+\rangle\langle+|}{2} = |+\rangle\langle+| \checkmark$$ Same for the $|-\rangle\langle-|$ projection

for length n pauli string idk how to show

b)

$$\mathbb{1} \otimes H \left(\mathbb{1}_H \otimes |0\rangle\langle 0| + S_H \otimes |1\rangle\langle 1|\right) |\psi\rangle \otimes |+\rangle$$

$$\Pi = \frac{\mathbb{1} + (-1)^m\, \mathbb{1}_H \otimes X}{2}$$

$$|\varphi(m)\rangle = \Pi_m |\psi_{before}\rangle = \Pi_m\, \mathbb{1}_H \otimes H \left(\mathbb{1}_H \otimes |0\rangle\langle 0| + S_H \otimes |1\rangle\langle 1|\right) |\psi\rangle \otimes |+\rangle$$

$$= \Pi_m\, \mathbb{1}_H \otimes H \left(\mathbb{1}_H \otimes |0\rangle\langle 0| + S_H \otimes |1\rangle\langle 1|\right) \left(|\psi\rangle \otimes |0\rangle\right) + \left(|\psi\rangle \otimes |1\rangle\right)$$

$$= \Pi_m\, \mathbb{1}_H \otimes H \left(|\psi\rangle \otimes |0\rangle + S|\psi\rangle \otimes |1\rangle\right)$$

$$= \Pi_m \left(|\psi\rangle \otimes |+\rangle + S|\psi\rangle \otimes |-\rangle\right)$$

$$= \frac{\mathbb{1} + (-1)^m\, \mathbb{1}_H \otimes X}{2} \left(|\psi\rangle \otimes |+\rangle + S|\psi\rangle \otimes |-\rangle\right)$$

$$= \frac{\left(|\psi\rangle \otimes |+\rangle + S|\psi\rangle \otimes |-\rangle\right) + (-1)^m \left(|\psi\rangle \otimes |+\rangle - S|\psi\rangle \otimes |-\rangle\right)}{2}$$

$\rightarrow$ if $m=0 \rightarrow |\varphi\rangle = |\psi\rangle \otimes |+\rangle$
if $m=1 \rightarrow |\varphi\rangle = S|\psi\rangle \otimes |-\rangle$

↑ this how $|\psi\rangle$ and S are related

$$\Pi = \prod_{i=1}^{n-k} \frac{\mathbb{1} + S_i}{2}$$

$$= \left(\frac{\mathbb{1} + S_1}{2}\right)\left(\frac{\mathbb{1} + S_2}{2}\right) \cdots \cdots \left(\frac{\mathbb{1} + S_{n-k}}{2}\right)$$

$$= \frac{1}{2^{n-k}} \sum_{i_1, \ldots, i_{n-k} \in \{0,1\}} S_1^{i_1} \cdots \cdots S_{n-k}^{i_{n-k}}$$

$$= \frac{1}{2^{n-k}} \sum_{S \in \mathcal{S}} S \qquad \text{q.e.d.}$$

Ex 2

a) $\sum_{j=0}^{n} A_j[\Pi] = \sum_{j=0}^{n} \frac{1}{Tr(\Pi)^2} \sum_{\substack{P^n \\ \text{weight } P = j}} Tr(\Pi P)^2$ -

=

wie viele P^n : weight $P^n = j$

Ex 3

e)