Detection et Estimation

TP:
Detecttion

Master SISEA

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Chapter 1

Introduction

1.1 Objectif

1.2 Write the log likelihoood funciton $L(A, \phi)$ for a given observation vector

We have the signal

$$x(n) = Acos(2\pi f_0 n + \phi) + w(n) \tag{1.1}$$

The non deterministic part of the signal is the noise. A White Gaussian noise with variance σ^2, wich is common in many fields. So the log-likelihood function is evaluated in function of this noise with the next expression.

$$w(n) = x(n) - Acos(2\pi f_0 n + \phi) \tag{1.2}$$

The WGN distributions are iid so

$$P(w(n)|A, \theta) = \prod_{n=0}^{N-1} P_w(w(n)) \tag{1.3}$$

So the log-likelihood function $L(A, \phi)$ is

$$L(A, \phi) = logP(w(n)|A, \theta) \tag{1.4}$$

$$L(A, \phi) = log \prod_{n=0}^{N-1} P_w(w(n)) \tag{1.5}$$

$$L(A, \phi) = \sum_{n=0}^{N-1} logP_w(w(n)) \tag{1.6}$$

$$L(A,\phi) = \sum_{n=0}^{N-1} log(\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-1/2(\frac{x(n) - Acos(2\pi f_0 n + \phi) - \mu}{\sigma})^2\right] \tag{1.7}$$

$$L(A,\phi) = \sum_{n=0}^{N-1} B + ([-1/2(\frac{x(n) - Acos(2\pi f_0 n + \phi) - \mu}{\sigma})^2]a \tag{1.8}$$

1.3 Show that the maximun likelihood estimators are the solution of the following equations

$$\frac{\partial L(A,\phi)}{\partial A} = \sum_{n=0}^{N-1} (\frac{x(n) - Acos(2\pi f_0 n + \phi) - \mu}{\sigma})(\frac{cos(2\pi f_0 n + \phi}{\sigma}) \tag{1.9}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = \sum_{n=0}^{N-1} -(\frac{x(n) - Acos(2\pi f_0 n + \phi) - \mu}{\sigma})(\frac{Asin(2\pi f_0 n + \phi}{\sigma}) \tag{1.10}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} -(\frac{Ax(n)sin(2\pi f_0 n + \phi)}{\sigma^2} - \frac{A^2 cos(2\pi f_0 n + \phi)sin(2\pi f_0 n + \phi)}{\sigma^2}) = 0 \tag{1.11}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} -(\frac{Ax(n)sin(2\pi f_0 n + \phi)}{\sigma^2} - \frac{A^2 cos(2\pi f_0 n + \pi)sin(2\pi f_0 n + \phi)}{\sigma^2}) = 0 \tag{1.12}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \frac{A}{\sigma^2} \sum_{n=0}^{N-1} -(\frac{Ax(n)sin(2\pi f_0 n + \phi) - A^2 cos(2\phi f_0 n + \pi)sin(2\phi f_0 n + \phi)}{\sigma^2}) = 0 \tag{1.13}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} (x(n)sin(2\pi f_0 n + \phi) - Acos(2\pi f_0 n + \pi))sin(2\phi f_0 n + \phi)) = 0 \tag{1.14}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} (x(n)sin(2\pi f_0 n + \phi) - A \sum_{n=0}^{N-1} cos(2\pi f_0 n + \phi)sin(2\pi f_0 n + \phi)) = 0 \tag{1.15}$$

Using the entity

$$\sum_{n=0}^{N-1} cos(2\pi f_0 n+\phi)sin(2\pi f_0 n+\phi)) \approx 0 \tag{1.16}$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1}(x(n)sin(2\pi f_0 n+\phi) = 0 \tag{1.17}$$

Developing the sum of angles

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} x(n)(sin(2\pi f_0 n+\phi)cos(2\pi f_0 n+\phi) + cos(2\pi f_0 n+\phi)sin(2\pi f_0 n+\phi)) = 0$$

$$\frac{\partial L(A,\phi)}{\partial \phi} = 0 = \sum_{n=0}^{N-1} x(n)(sin(2\pi f_0 n+\phi)cos(\phi) = -\sum_{n=0}^{N-1} x(n)cos(2\pi f_0 n+\phi)sin(\phi)) \tag{1.18}$$

$$\frac{\sum_{n=0}^{N-1} x(n)sin(2\pi f_0 n)}{\sum_{n=0}^{N-1} x(n)cos(2\pi f_0 n)} = \frac{sin(\phi)}{cos(\phi)} = tg(\phi) \tag{1.19}$$

$$\hat{\Phi}_{ML} = arctg(-\frac{\sum_{n=0}^{N-1} x(n)sin(2\pi f_0 n)}{\sum_{n=0}^{N-1} x(n)cos(2\pi f_0 n)}) \tag{1.20}$$

Making the derivation to the other parameter

$$\frac{\partial^2 L(A,\phi)}{\partial \phi^2} = \sum_{n=0}^{N-1} -(\frac{Asin(2\pi f_0 n+\phi)}{\sigma})(\frac{Asin(2\pi f_0 n+\phi)}{\sigma})+$$
$$\sum_{n=0}^{N-1} -(\frac{x(n) - Acos(2\pi f_0 n+\phi)}{\sigma})(\frac{Acos(2\pi f_0 n+\phi)}{\sigma})$$

$$\frac{\partial^2 L(A,\phi)}{\partial \phi^2} = \sum_{n=0}^{N-1} -(\frac{A^2 sin^2(2\pi f_0 n+\phi)}{\sigma^2})+$$
$$\sum_{n=0}^{N-1} -(\frac{x(n)Acos(2\pi f_0 n+\phi) - A^2 cos^2(2\pi f_0 n+\phi)}{\sigma^2})$$

$$\frac{\partial^2 L(A,\phi)}{\partial \phi^2} = \sum_{n=0}^{N-1} -(\frac{A^2 sin^2(2\pi f_0 n+\phi)}{\sigma^2})+$$
$$\sum_{n=0}^{N-1} (\frac{w(n)}{\sigma^2})$$

1.4 To compute the term of Fisher Matrix

$$-\sum_{n=0}^{N-1} E[\frac{\partial^2 L(A,\phi)}{\partial \phi^2}] = \sum_{n=0}^{N-1} \frac{A^2 sin^2(2\pi f_0 n + \phi)}{\sigma^2} \tag{1.21}$$

$$-\sum_{n=0}^{N-1} E[\frac{\partial^2 L(A,\phi)}{\partial \phi^2}] = \frac{A^2 N}{\sigma^2 2} \tag{1.22}$$

$$[\frac{\partial^2 L(A,\phi)}{\partial \phi^2}] = -\sum_{n=0}^{N-1} \frac{-cos(2\pi f_0 n + \phi)}{\sigma} \frac{Asin(2\pi f_0 n + \phi)}{\sigma}$$
$$+\sum_{n=0}^{N-1} -\frac{x(n) - Acos(2\pi f_0 n + \phi)}{\sigma} \frac{sin(2\pi f_0 n + \phi)}{\sigma}$$

$$-\sum_{n=0}^{N-1} E[\frac{\partial^2 L(A,\phi)}{\partial \phi^2}] = -\sum_{n=0}^{N-1} \frac{-cos(2\pi f_0 n + \phi)}{\sigma} \frac{Asin(2\pi f_0 n + \phi)}{\sigma} \tag{1.23}$$

Using the simplification in equation 1.16

$$-\sum_{n=0}^{N-1} E[\frac{\partial^2 L(A,\phi)}{\partial \phi^2}] = 0 \tag{1.24}$$

1.5 Verify thath thw maximun-likelihood estimator of the amplitude is unbiase if $\phi = \hat{\Phi}_{ML}$

Condition pour $\hat{\Phi}_{ML} = \Phi$

$$tan(\hat{\Phi}_{ML}) = tan(\Phi) \tag{1.25}$$

$$\frac{\sum_{n=0}^{N-1} x(n) sin(2\pi f_0 n + \phi)}{\sum_{n=0}^{N-1} x(n) cos(2\pi f_0 n + \phi)} =$$
$$\frac{\sum_{n=0}^{N-1} Acos(2\pi f_0 n + \phi) + w(n) sin(2\pi f_0 n + \phi)}{\sum_{n=0}^{N-1} Acos(2\pi f_0 n + \phi) + w(n) cos(2\pi f_0 n + \phi)}$$

$$\frac{\sum_{n=0}^{N-1} x(n) sin(2\pi f_0 n + \phi)}{\sum_{n=0}^{N-1} x(n) cos(2\pi f_0 n + \phi)} =$$
$$\frac{\sum_{n=0}^{N-1} Acos(2\pi f_0 n + \phi) + w(n) sin(2\pi f_0 n + \phi)}{\sum_{n=0}^{N-1} Acos(2\pi f_0 n + \phi) + w(n) cos(2\pi f_0 n + \phi)}$$

$$\frac{AN/2sin(\phi)\sum_{n=0}^{N-1}w(n)sin(2\pi f_0 n)}{AN/2sin(\phi)\sum_{n=0}^{N-1}w(n)cos(2\pi f_0 n)} \approx tan(\Phi)$$

This term will be next to $tan(\Phi)$ when

$$\sum_{n=0}^{N-1}|w(n)sin(2\pi f_0 n)| << |AN/2sin(\phi)| \quad (1.26)$$

and when we have

$$\sum_{n=0}^{N-1}|w(n)cos(2\pi f_0 n)| << |AN/2cos(\phi)| \quad (1.27)$$

The noise have a normal distribution so the sum of noise have the next distribution

$$\sum_{n=0}^{N-1}|w(n)sin(2\pi f_0 n)| \sim N(0, N\sigma^2/2) \quad (1.28)$$

We take a probabilité of 3σ

$$3\sqrt{\sigma^2 N/2} = 3\sqrt{N/2}\sigma << AN/2sin(\phi) \quad (1.29)$$

$$3 << \frac{A\sqrt{N}}{\sigma}sin(\phi) \quad (1.30)$$

For larges values of N $\hat{\Phi}_M L \approx \phi$

We cans observe that the parameters A, σ and N can modified the estimator properties.

The ratio $\frac{A^2}{\sigma^2}$ represent the RSB

1.6 Give the condition under which $\hat{\Phi}_M L \approx \phi$ different from $E|\hat{\Phi}_M L| \approx \phi$

Using

$$sen(\alpha)cos(\alpha) = 1/2sin(2\alpha)$$
$$cos^2(\alpha) = \frac{1+cos(2\alpha)}{2}$$
$$sin^2(\alpha) = \frac{1-cos(2\alpha)}{2}$$

In the entities of the copy, we obtain

$$\sum_{n=0}^{N-1} cos(4\pi f_0 n) \approx 0 \tag{1.31}$$

$$\sum_{n=0}^{N-1} sin(4\pi f_0 n) \approx 0 \tag{1.32}$$

Chapter 2

Implémentation en MATLAB®

2.1 Estimation de l'information mutuelle

2.2 Conclusion