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Research Article

ODE Versus Petri Net Implementation of Identical SEIRS Model

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Abstract

Differential equation; more commonly, ordinary differential equation (ODE); and Petri Net are complementary methods commonly used in dynamic systems modelling. However, the differences between ODE models and Petri Net models have not been adequately studied. In this study, we implement a closed 4-compartment SEIRS infectious disease model in both ODE and Petri Net, to examine the differences by comparing their simulation results. Our simulation results suggest that although there are differences between the simulation results across various ODE solvers, the differences between ODE or Petri Net implementations are significant differences ($t \geq 15.34$, p-value $\leq$ 1.59E-12) as a whole; but these differences may not be significant across all compartments. This suggests that ODE model and Petri Net model may reveal different insights into the same problem; hence, supporting the view that ODE model and Petri Net model are complementary.

Keywords: Dynamic Systems Modelling (DSM); Ordinary Differential Equation (ODE); Petri Net; Epidemiological Model

Introduction

Dynamic systems modelling (DSM) is the conversion of real entities into mathematical form [1] and using it to find solutions to or to study real world issues [2]. Irwin and Wang [3] define DSM as description of multiple components of a phenomenon that are viewed as a system, which can be used to predict the interactions over time. DSM has been used to in a wide variety of fields; including, economics [4], biology [5], social science [6], crisis management [7], and even history [8]. Mathematical models are instrumental tools in the field of epidemiology [9]; especially in infectious disease epidemiology [10,11], and substance abuse epidemiology [12-14]. The two common methodologies for modelling dynamic systems [15] are differential equations, more commonly ordinary differential equation (ODE), and Petri Net; and has been shown to be complementary to each other [16].

However, there has been little studies examining the differences between ODE models and Petri Net models. The only notable study to date is by Zhao and Krishnan [17] whom modelled mRNA translation and protein synthesis using ODE, and Petri Net; and found that small differences between the simulation results from ODE model and Petri Net model. In this study, we implement a closed 4-compartment SEIRS model in ODE and Petri Net, to examine the differences in simulation results. SEIRS model is a well-known infectious disease epidemiological model [18-20]. Our simulation results suggest that there are overall significant differences ($t \geq 15.34$, p-value $\leq$ 1.59E-12) between ODE or Petri Net implementations of SEIRS model but these differences may not be significant across all compartments. This further suggests that ODE model and Petri Net model may reveal different insights into the same problem, which supports the view that ODE model and Petri Net model are complementary [16].

Method

A closed 4-compartment SEIRS model, without births and deaths, were adapted from Bjørnstad., *et al*. [21], and implemented

in both ODE and Petri Net. The ODE equations are (a) $^{dS}/_{dt} = (^{R}/_{365}) - (0.21SI)$, (b) $^{dE}/_{dt} = (0.21SI) - (^{E}/_{7})$, (c) $^{dI}/_{dt} = (^{E}/_{7}) - (^{I}/_{14})$, and (d) $^{dR}/_{dt} = (^{I}/_{14}) - (^{R}/_{365})$. The correspondence between ODE model and Petri Net model is based on Soliman and Heiner [22]. The ODE model was implemented and simulated using all 11 ODE solvers described in Ling [23], for 100 days using 0.1 and 1 day as time steps. The Petri Net model was implemented and simulated using PNet [24] for 100 days using 1 day as time step. Differences within different ODE solvers, and between ODE solvers and Petri Net, were determined using root mean square error (RMSE) [25] and Pearson's correlation [26,27], and statistically analyzed using t-test where p-value of less than 0.05 is considered significant.

Results and Discussion

Time step is important in ODE solver

The choice of time step in ODE solver is a balance between accuracy and computational efficiency [28]. As an exploratory analysis and baseline for comparing between ODE and Petri Net models, we examine the differences between two different time steps (time step of 0.1 and 1.0) across the 11 ODE solvers. For time step of 0.1 (Figure 1), our results show that the mean RMSE across the ODE solvers (n = 55) is 0.004096 with standard deviation of 0.0080220 whereas the mean Pearson's correlation is 0.9998 with standard deviation of 0.00049051. For time step of 1.0 (Figure 2), our results show that the mean RMSE across the ODE solvers (n = 55) is 0.006649 with standard deviation of 0.0099896 whereas the mean Pearson's correlation 0.9995 with standard deviation of 0.0010202. Paired t-tests show insignificant differences between the 2 different time steps for both RMSE (t = 1.393, p-value = 0.169) and Pearson's correlation (t = 1.779, p-value = 0.0809). Despite so, our results suggest that time step may affect accuracy in a non-uniform manner. For example, simulation results from RK4(3/8) solver are substantially different compared to other solvers when time step is 0.1 (Figure 1) but when time step is 1, RK4 solver is substantially different compared to other solvers (Figure 2). The impact of time step on ODE simulation has been studied [29-32] for more than 30 years, and our exploratory examination suggests that time step may play a role in ODE solver accuracy.

Overall significant differences between ODE and petri net implementations

Using the same time step (time step = 1) for both ODE and Petri Net, the mean RMSE between the 11 ODE solvers and Petri Net is 0.04201 with standard deviation of 0.0029335 whereas the mean Pearson's correlation is 0.9956 with standard deviation of 0.00070189. The mean RMSE (t = 21.95, p-value = 7.08E-29) and mean Pearson's correlation (t = 15.34, p-value = 1.59E-12) between ODE solvers and Petri Net are significant compared to within ODE solvers, assuming unequal variances. This suggests that there are significant differences considering the simulation results in entirety. Of the 11 ODE solvers using time step of 1.0, the results from CK4 solver show largest RMSE (RMSE = 0.02692) and lowest Pearson's correlation (r = 0.9973) to the results from RK4. Hence, by graphing the results from CK5, RK4, and Petri Net together (Figure 3), the differences between ODE-implemented and Petri Net-implemented SEIRS are visible.

	Euler	Heun's	RK3	RK4	RK4(3/8)	RKF4	RKF5	CK4	CK5	DP4	DP5
Euler		1.00000	1.00000	1.00000	0.99874	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
Heun's	5.98E-04		1.00000	1.00000	0.99878	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
RK3	0.00E+00	5.98E-04		1.00000	0.99874	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
RK4	0.00E+00	5.98E-04	0.00E+00		0.99874	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
RK4(3/8)	2.00E-02	1.96E-02	2.00E-02	2.00E-02		0.99874	0.99874	0.99874	0.99874	0.99874	0.99874
RKF4	2.00E-02	5.99E-04	1.05E-06	1.05E-06	2.00E-02		1.00000	1.00000	1.00000	1.00000	1.00000
RKF5	1.05E-06	5.99E-04	1.05E-06	1.05E-06	2.00E-02	3.26E-13		1.00000	1.00000	1.00000	1.00000
CK4	1.05E-06	5.99E-04	1.05E-06	1.05E-06	2.00E-02	2.45E-13	8.10E-14		1.00000	1.00000	1.00000
CK5	1.05E-06	5.99E-04	1.05E-06	1.05E-06	2.00E-02	2.45E-13	8.10E-14	0.00E+00		1.00000	1.00000
DP4	1.05E-06	5.99E-04	1.05E-06	1.05E-06	2.00E-02	1.29E-13	1.97E-13	1.17E-13	1.17E-13		1.00000
DP5	1.05E-06	5.99E-04	1.05E-06	1.05E-06	2.00E-02	1.29E-13	1.97E-13	1.17E-13	1.17E-13	0.00E+00	

Figure 1: RMSE and Pearson's correlation of simulation results using different ODE solvers with Time Step = 0.1. The lower triangular matrix shows RMSE while the upper triangular matrix shows Pearson's correlation.

	Euler	Heun's	RK3	RK4	RK4(3/8)	RKF4	RKF5	CK4	CK5	DP4	DP5
Euler		1.00000	0.99990	0.99741	0.99990	0.99990	0.99990	0.99990	0.99990	0.99990	0.99990
Heun's	0.00E+00		0.99990	0.99741	0.99990	0.99990	0.99990	0.99990	0.99990	0.99990	0.99990
RK3	6.04E-03	6.04E-03		0.99733	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
RK4	2.68E-02	2.68E-02	2.69E-02		0.99733	0.99733	0.99733	0.99733	0.99733	0.99733	0.99733
RK4(3/8)	6.04E-03	6.04E-03	0.00E+00	2.69E-02		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
RKF4	6.04E-03	6.04E-03	1.55E-06	2.69E-02	1.55E-06		1.00000	1.00000	1.00000	1.00000	1.00000
RKF5	6.04E-03	6.04E-03	1.57E-06	2.69E-02	1.57E-06	2.23E-08		1.00000	1.00000	1.00000	1.00000
CK4	6.04E-03	6.04E-03	1.57E-06	2.69E-02	1.57E-06	2.23E-08	0.00E+00		1.00000	1.00000	1.00000
CK5	6.04E-03	6.04E-03	1.57E-06	2.69E-02	1.57E-06	2.23E-08	0.00E+00	0.00E+00		1.00000	1.00000
DP4	6.04E-03	6.04E-03	1.57E-06	2.69E-02	1.57E-06	2.23E-08	0.00E+00	0.00E+00	0.00E+00		1.00000
DP5	6.04E-03	6.04E-03	1.57E-06	2.69E-02	1.57E-06	2.23E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00	

Figure 2: RMSE and Pearson's correlation of simulation results using different ODE solvers with Time Step = 1.0. The lower triangular matrix shows RMSE while the upper triangular matrix shows Pearson's correlation.

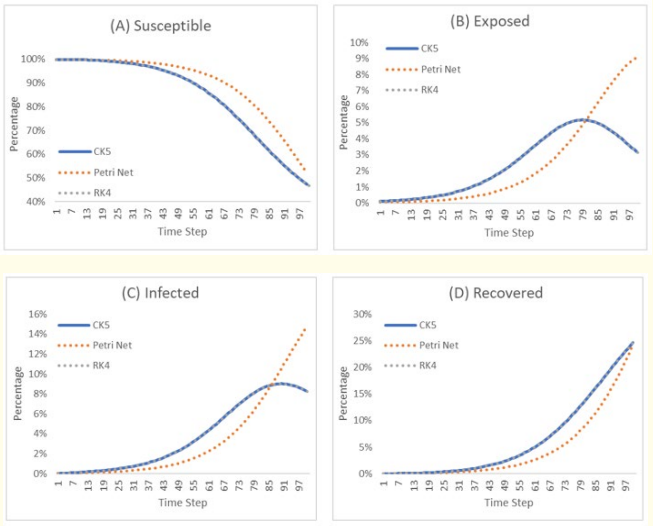

Figure 3: Comparison between simulation results from ODE solvers (RK4 and CK5) and Petri Net. Panels A, B, C, and D illustrate susceptible (S), exposed (E), infected (I), and recovered (R), respectively.

However, statistical comparisons by compartment using paired t-test (Table 1), our analysis show that the simulation results from either ODE solvers are not significantly different compared to the simulation results from Petri Net at 95% confidence for exposed populations (t = 0.15, p-value = 0.878), and significantly different at 95% confidence for infected populations (t = 2.26, p-value = 0.0258). Yet, the simulation results from the 2 ODE solvers, RK4 and CK5, are significant for every compartment (t ≥ 3.62, p-value ≤ 4.71E-04). Hence, our results support previous study [17] showing that small differences between the simulation results from ODE model and Petri Net model. More importantly, our results demonstrate that these differences may not be uniform and/or significant across all compartments despite overall significant differences if all compartments are taken in entirety. This further suggests that ODE model and Petri Net model may reveal different insights into the same problem; therefore, supporting the view that ODE model and Petri Net model are complementary [16].

Compartment	Measure	Comparison		
		RK4 / CK5	RK4 / PN	CK5 / PN
Susceptible (S)	t	12.83	11.12	11.12
	p-value	8.98E-23	3.95E-19	3.94E-19
Exposed (E)	t	3.62	0.15	0.15
	p-value	4.71E-04	8.78E-01	8.78E-01
Infected (I)	t	9.86	2.26	2.26
	p-value	2.25E-16	2.58E-02	2.58E-02
Recovered (R)	t	11.26	10.28	10.28
	p-value	1.99E-19	2.64E-17	2.64E-17

Table 1: Paired t-test analysis of simulation results from ODE solvers (RK4 and CK5) and Petri Net by compartments.

Conclusion

In this study, we implement a closed SEIRS model in ODE and Petri Net and show that there are significant differences (t ≥ 15.34, p-value ≤ 1.59E-12) between ODE or Petri Net implementations of SEIRS model but these differences may not be significant across all compartments. Hence, ODE model and Petri Net model may reveal different insights into the same problem, supporting the view that ODE model and Petri Net model are complementary.

Supplementary Materials

Data set for this study can be downloaded at https://bit.ly/ODE_vs_PNet.

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Conflict of Interest

The authors declare no conflict of interest.

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