

Assembly of Drug Epidemiological Models – Supplementary Materials

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S1. Description of Tang and Ling's Model

Tang and Ling's model [1] is an adaptation of Njagarah and Nyabadza's model [2] by adding 2 relapse processes. The compartments of the model are susceptible (S), light drug user (L), heavy drug user (H), drug user in treatment (T), drug mules/pushers (D), and removed by various causes (R). In the simplified model of Njagarah and Nyabadza [2], relapse from treatment (T) into heavy user (H) is defined as $g_3(T)$. Tang and Ling [1] added 2 relapse processes – from treatment (T) to susceptible (S) as $g_5(T)$, and from treatment (T) to light drug user (L) as $g_4(T)$.

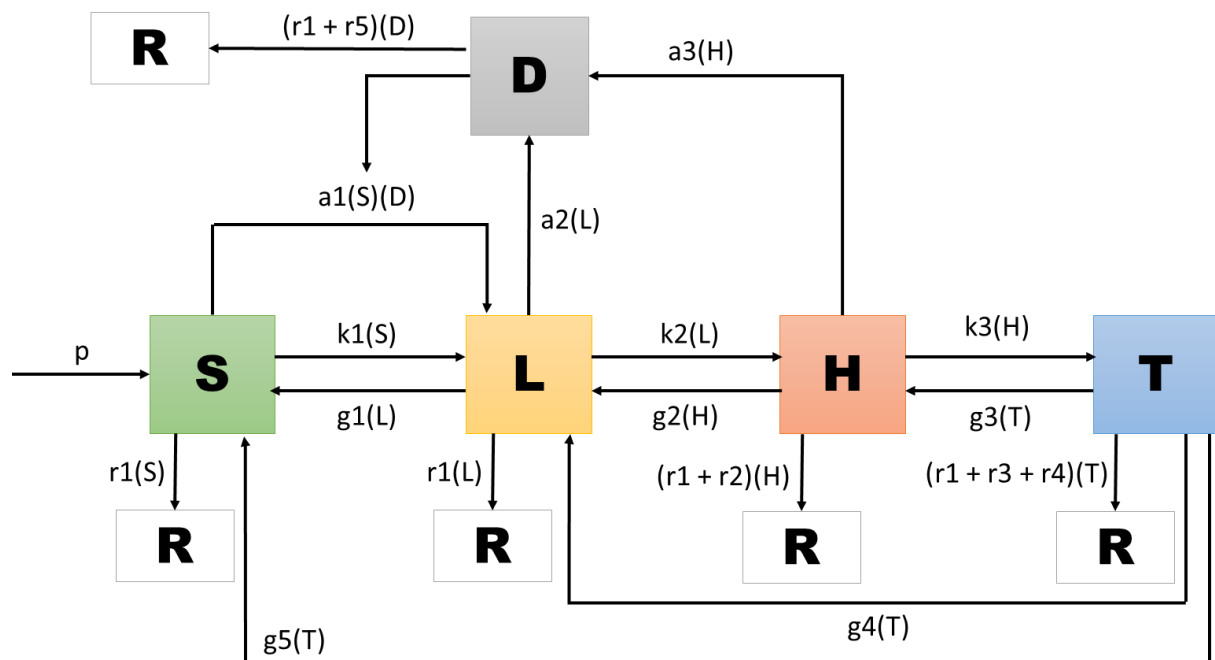


Figure S1. Tang and Ling's model [1].

The parameters are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).
k1	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k2	0.56	Rate at which light users (L) escalates to heavy drug use (H).
k3	0.223	Rate at which heavy users (H) enters rehabilitation (T).
g1	0.2	Rate at which light users (L) quit and become susceptible (S) again.
g2	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.
g3	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
g4	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
g5	0.283	Rate at which rehabilitated users (T) reverted to susceptible (S).
a1	0.4	Effective contact rate between drug barons (D) and susceptible population (S).
a2	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.

S2. Description of Knolle's Model

Knolle's model [3] estimates the prevalence of drug use before being caught (X) based on the number of charged users (Y), given that a proportion of drug users that may never be caught (σX) or stopped illicit drug use after being caught (τY). The model acknowledges that the proportion of caught users is a product of police search intensity (π) and the exposed population of pre-caught users (f) or caught and charged users (g).

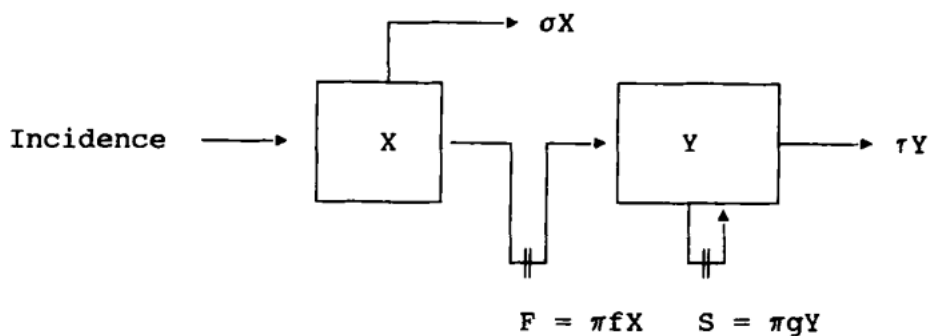


Figure S2. Knolle's model [3].

The parameters are as follow:

Parameter	Nominal Value	Description
σ	0.25	Proportion of drug users that may never be caught.
τ	0.1	Proportion of drug users quit drug use after being caught.
f	0.02	Proportion of pre-caught drug users (X) exposed to police search.
g	0.04	Proportion of post-caught drug users (Y) exposed to police search.
π	5	Intensity of policing / police search.

S3. Stage 1: Assemble Knolle's Model and Tang and Ling's Model into Model-1

Tang and Ling's model [1] did not cater to policing, which is present in Knolle's model [3]. Pre-caught drug users in Knolle's model [3] can be seen as summation of light (L) and heavy (H) drug users and post-caught drug users can be equivalent to rehabilitated users (T). However, the rate at which heavy users (H) enters rehabilitation (T), original k_3 (which equals to 0.223), can be seen as the exposure of heavy drug users (H) to police search. Hence, k_3 is redesignated as a product of k_3 [proportion of heavy drug users (H) exposed to policing / police search] and k_5 (intensity of policing). By extension, light drug users (L) can also be caught by policing activities to rehabilitated users (T), which can be modelled as the product of k_4 [proportion of light drug users (L) exposed to policing / police search] and k_5 (intensity of policing).

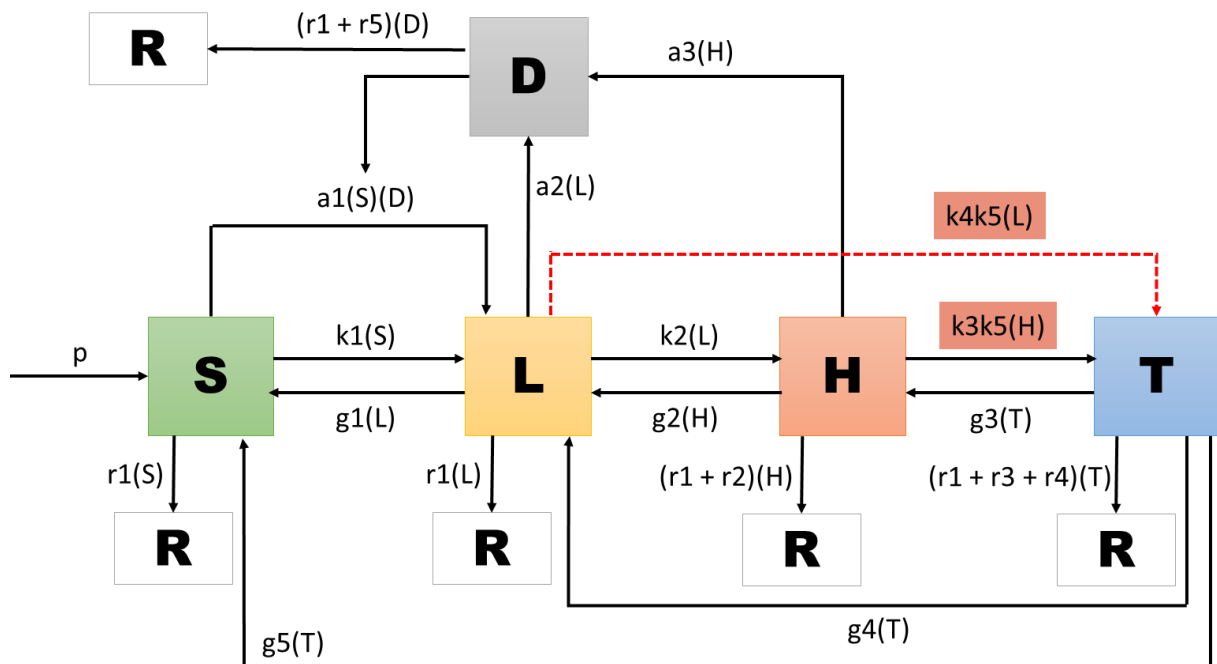


Figure S3. Model-1.

The revised parameters for Model-1 are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).

k1	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k2	0.56	Rate at which light users (L) escalates to heavy drug use (H).
k3	0.446	Proportion of heavy drug users (H) exposed to police search.
k4	0.223	Proportion of light drug users (L) exposed to police search.
k5	0.5	Intensity of policing / police search.
g1	0.2	Rate at which light users (L) quit and become susceptible (S) again.
g2	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.
g3	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
g4	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
g5	0.283	Rate at which rehabilitated users (T) reverted to susceptible (S).
a1	0.4	Effective contact rate between drug barons (D) and susceptible population (S).
a2	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.

S4. Description of Caulkin et al.'s 2009 Model

Caulkin et al.'s 2009 model [4] examines susceptibility into users. It also assumes that once a user, always a user; hence, there is no rehabilitation.

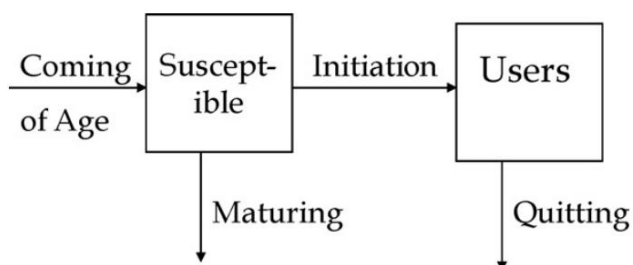


Figure S4. Caulkin et al.'s 2009 model [4].

S5. Stage 2: Assemble Caulkin et al.'s 2009 Model and Model-1 into Model-2

In Caulkin et al.'s 2009 model [4], maturing from susceptible and quitting from user are not of the same meaning as removal in Model-1. Assuming that quitting in Caulkin et al.'s 2009

model [4] is only possible for light users, then maturing from susceptible and quitting from users in Caulkin et al.'s 2009 model [4] have a different meaning to per capita mortality rate of population. In other words, Model-1 does not allow for maturing or quitting. Hence, two new compartments are added – M for maturing from susceptible, and Q for quitting from light users.

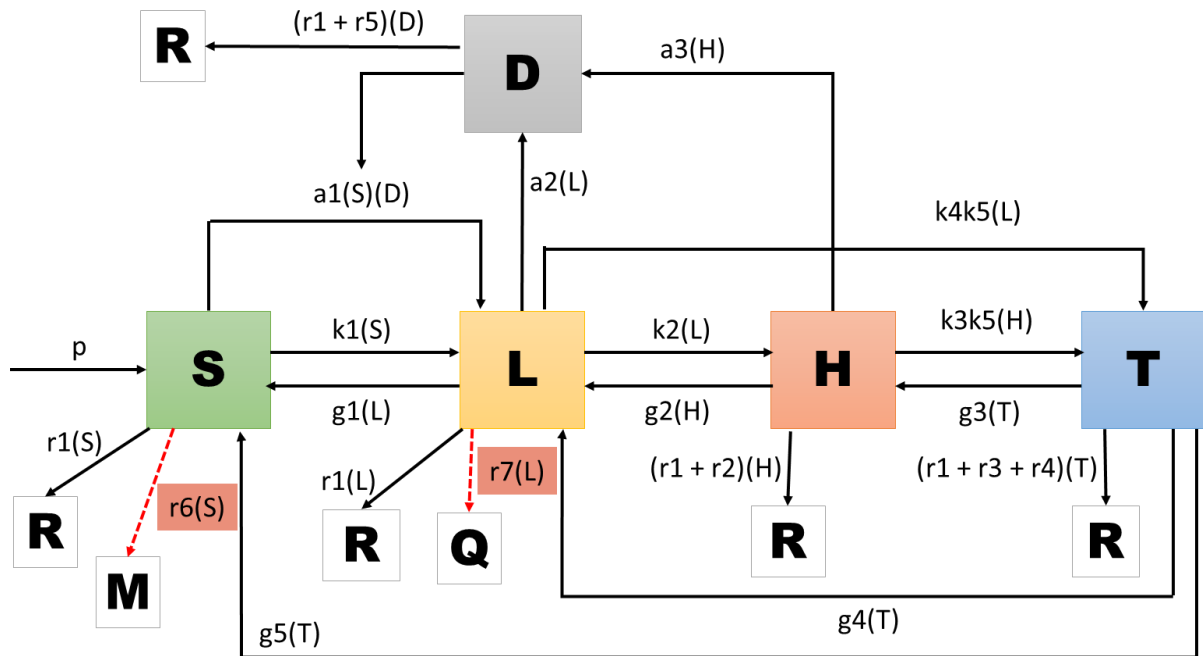


Figure S5. Model-2.

The revised parameters for Model-2 are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).
$k1$	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
$k2$	0.56	Rate at which light users (L) escalates to heavy drug use (H).
$k3$	0.446	Proportion of heavy drug users (H) exposed to police search.
$k4$	0.223	Proportion of light drug users (L) exposed to police search.
$k5$	0.5	Intensity of policing / police search.
$g1$	0.2	Rate at which light users (L) quit and become susceptible (S) again.
$g2$	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.
$g3$	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
$g4$	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
$g5$	0.283	Rate at which rehabilitated users (T) reverted to susceptible (S).
$a1$	0.4	Effective contact rate between drug barons (D) and susceptible population (S).
$a2$	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).

a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	0.01	Rate of susceptible (S) maturing into non-susceptible (M)
r7	0.01	Rate of light users (L) quitting drug use permanently (Q)

S6. Stage 3: Description of Caulkin et al.'s 2010 Model

Caulkin et al.'s 2010 model [5] examines the social cost of drug use using the compartments described in Caulkin et al.'s 2009 model [4]. Hence, Caulkin et al.'s 2010 model [5] is out of the scope of this study.

S7. Description of White and Comiskey's Model

In White and Comiskey's model [6], susceptible (S) represents the coming of age and U_1 represents the population of drug users, while U_2 represents the population of drug users in treatment. The likelihood of susceptible (S) entering drug use (U_1) is a probability of these two populations meeting. The rate of drug users (U_1) entering treatment (U_2) is presented by p . The relapse from treatment (U_2) to drug use (U_1) is represented by β_3 .

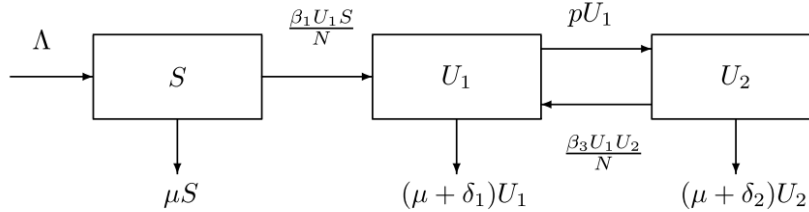


Figure S6. White and Comiskey's Model [6].

S8. Stage 4: Model-2 Incorporates White and Comiskey's Model

Light (L) and heavy (H) drug users in Model-2 are represented as drug users (U_1), and users under treatment (T) in Model-2 are represented as U_2 in White and Comiskey's model [6]. Therefore, Model-2 incorporates White and Comiskey's model [6].

S9. Stage 5: Model-2 Incorporates Mulone and Straughan's Model

Mulone and Straughan [7] use the model of White and Comiskey's model [6] without amendments. Hence, Model-2 also incorporates Mulone and Straughan's model [7].

S10. Description of Nyabadza and Hove-Musekwa's Model

Nyabadza and Hove-Musekwa [8] adapt White and Comiskey's model [6] for methamphetamines. However, drug users (U_1) in White and Comiskey's model [6] are split into light (I_1) and heavy (I_2) drug users in Nyabadza and Hove-Musekwa's model [8]. Users in treatment (T) in Nyabadza and Hove-Musekwa's model [8] are equivalent to U_2 in White and

Comiskey's model [6]. Moreover, Nyabadza and Hove-Musekwa [8] added a compartment, recovered users (R), who are both light drug users (I_1) and treated drug users (T) in remission.

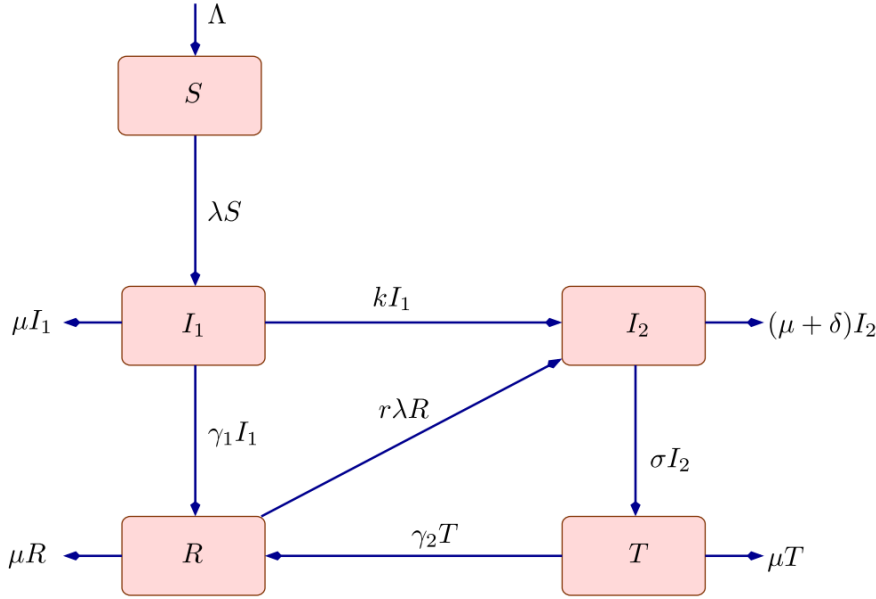


Figure S7. Nyabadza and Hove-Musekwa [8].

S11. Stage 6: Assemble Nyabadza and Hove-Musekwa's Model and Model-2 into Model-3

Differing from Tang and Ling's model [1], release from treatment (T) no longer reverts to susceptible (S) but redesignated as in remission (Re); hence, g₅ is redesignated as the proportion of rehabilitated users (T) entering remission (Re). Incorporating from Nyabadza and Hove-Musekwa's model [8], light users (L) can enter remission (Re) on their own accord through a period of non-usage. However, users in remission (Re) may revert to heavy use (H). In addition to Nyabadza and Hove-Musekwa's model [8], users in remission (Re) may also revert to light drug use (L).

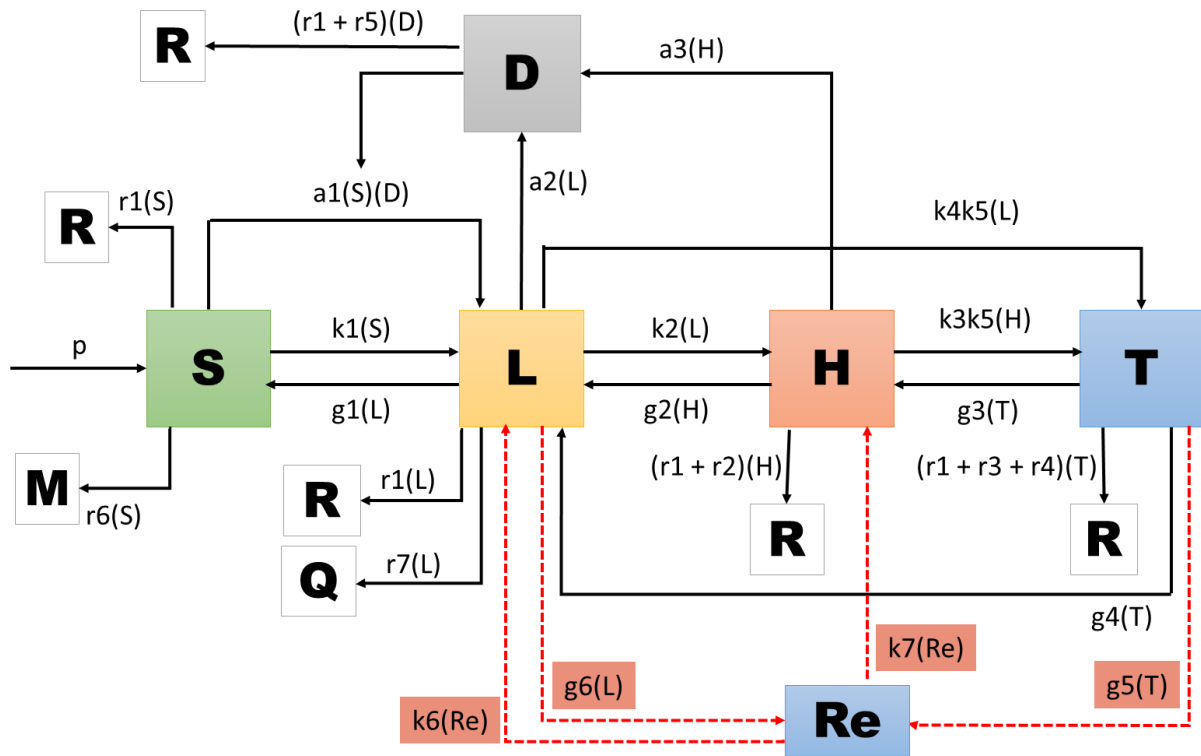


Figure S8. Model-3.

The revised parameters for Model-3 are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).
k1	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k2	0.56	Rate at which light users (L) escalates to heavy drug use (H).
k3	0.446	Proportion of heavy drug users (H) exposed to police search.
k4	0.223	Proportion of light drug users (L) exposed to police search.
k5	0.5	Intensity of policing / police search.
k6	0.05	Rate of relapse from remission (Re) to light drug use (L).
k7	0.01	Rate of relapse from remission (Re) to heavy drug use (H).
g1	0.2	Rate at which light users (L) quit and become susceptible (S) again.
g2	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.
g3	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
g4	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
g5	0.283	Rate at which rehabilitated users (T) enter remission (Re).
g6	0.005	Proportion of light drug users (L) entering remission (Re) on their own accord
a1	0.4	Effective contact rate between drug barons (D) and susceptible population (S).

a2	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	0.01	Rate of susceptible (S) maturing into non-susceptible (M)
r7	0.01	Rate of light users (L) quitting drug use permanently (Q)

S12. Stage 7: Model-3 Incorporates Wang et al.'s Model

Wang et al. [9] uses Mulone and Straughan [7], which is White and Comiskey's model [6] without amendments. Hence, Model-3 also incorporates Wang et al.'s model [9].

S13. Description of Kalula and Nyabadza's Model

Kalula and Nyabadza [10] use a closed model with known input (N_p) into susceptible population (S), which proceed to light (U_L) or heavy (U_H) drug use. Importantly, it is possible for susceptible population (S) to enter heavy drug use (U_H) without transiting through light drug use (U_L). However, light drug users (U_L) can quit (Q) without treatment (U_T) but not heavy users (U_H).

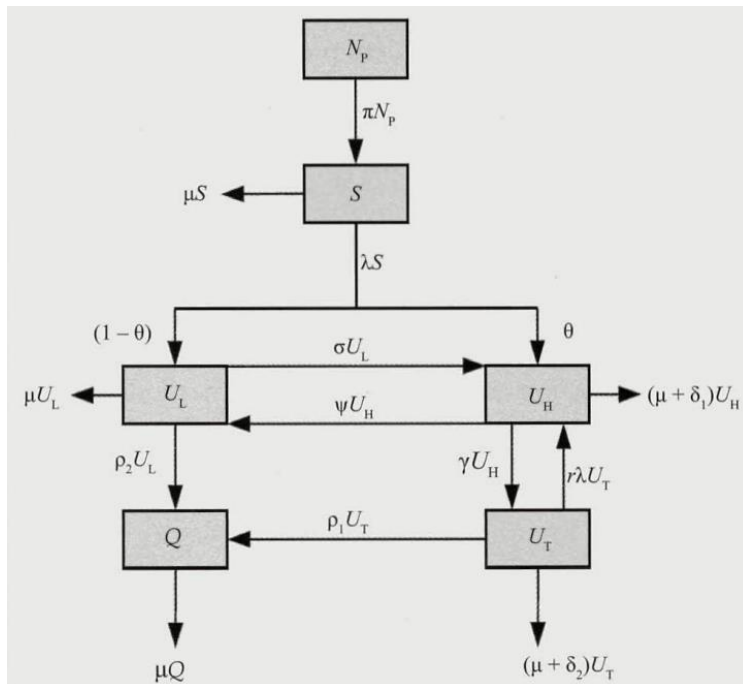


Figure S9. Kalula and Nyabadza's Model [10].

S14. Stage 8: Assemble Kalula and Nyabadza's Model and Model-3 into Model-4.

Two processes from Kalula and Nyabadza's model [10] were added. Firstly, a very small but important proportion of susceptible (S) may enter into heavy drug use (H) without going through light drug use (L). Secondly, a proportion of rehabilitated users (T) may quit permanently (Q).

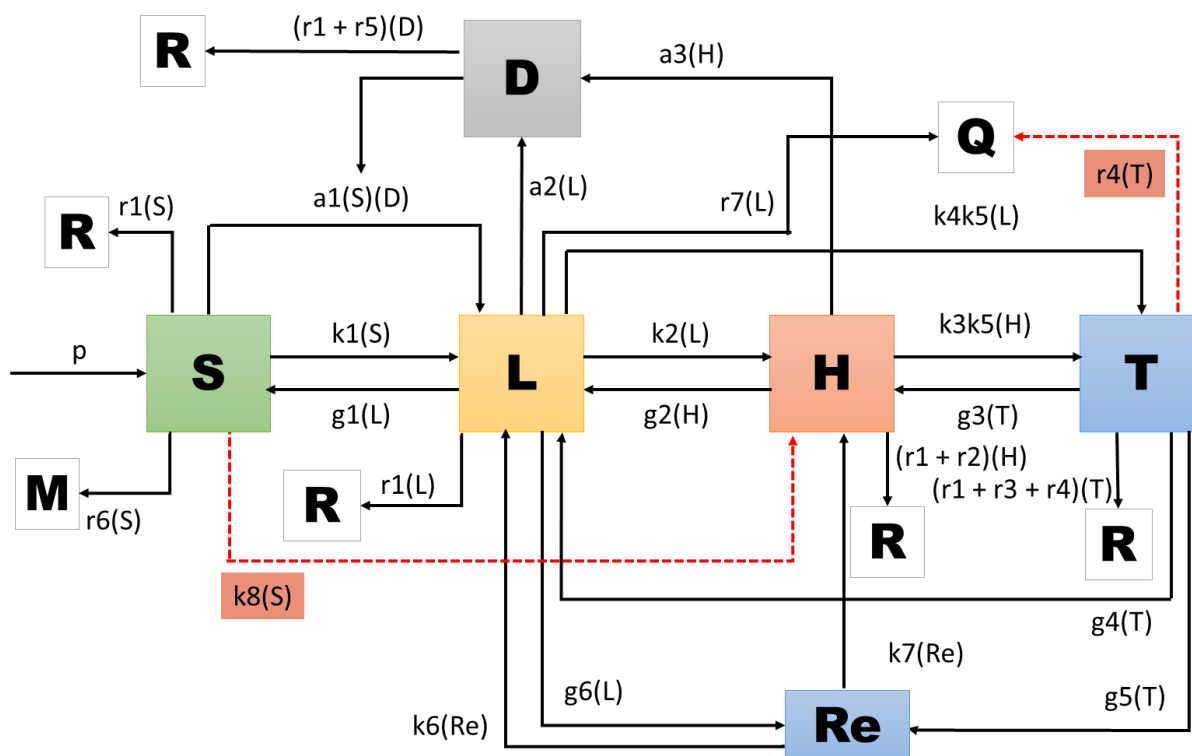


Figure S10. Model-4.

The revised parameters for Model-4 are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).
k_1	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k_2	0.56	Rate at which light users (L) escalates to heavy drug use (H).
k_3	0.446	Proportion of heavy drug users (H) exposed to police search.
k_4	0.223	Proportion of light drug users (L) exposed to police search.
k_5	0.5	Intensity of policing / police search.
k_6	0.05	Rate of relapse from remission (Re) to light drug use (L).
k_7	0.01	Rate of relapse from remission (Re) to heavy drug use (H).
k_8	0.001	Rate of susceptible population (S) become heavy drug users (H).
g_1	0.2	Rate at which light users (L) quit and become susceptible (S) again.
g_2	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.

g3	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
g4	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
g5	0.283	Rate at which rehabilitated users (T) enter remission (Re).
g6	0.005	Proportion of light drug users (L) entering remission (Re) on their own accord.
a1	0.4	Effective contact rate between drug barons (D) and susceptible population (S).
a2	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	0.01	Rate of susceptible (S) maturing into non-susceptible (M)
r7	0.01	Rate of light users (L) quitting drug use permanently (Q)

S15. Description of Nyabadza et al's Model

Nyabadza et al. [11] models the availability or density of drugs (D) as a means of susceptible population (S) entering light drug use (U_l) without coming into contact with light drug users (U_l). This can be seen as self-seeking behaviour.

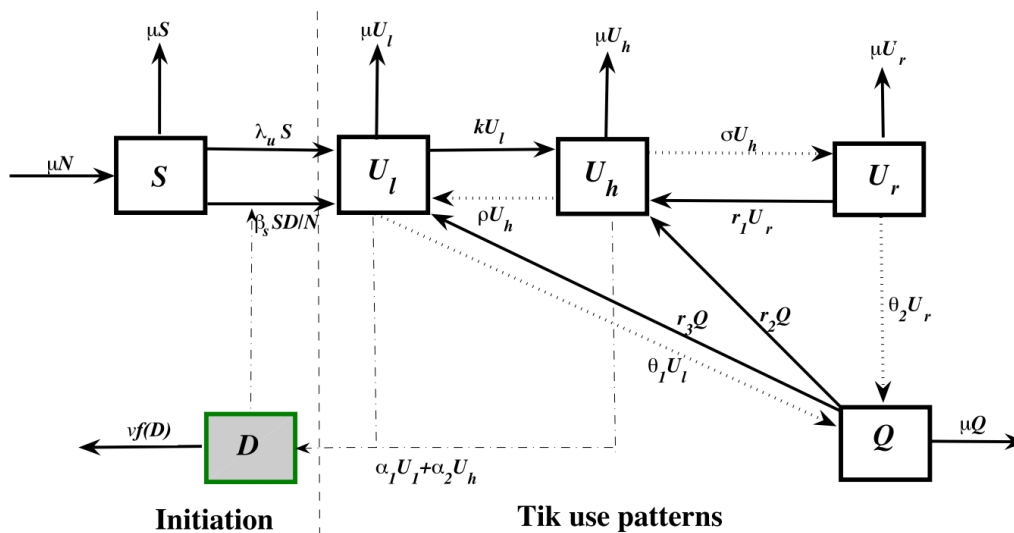


Figure S11. Nyabadza et al.'s Model [11].

S16. Stage 9: Assemble Nyabadza et al's Model and Model-4 into Model-5.

The availability of drugs in the system from Nyabadza et al.'s model [11] is incorporated into Model-4 as parameter k_9 .

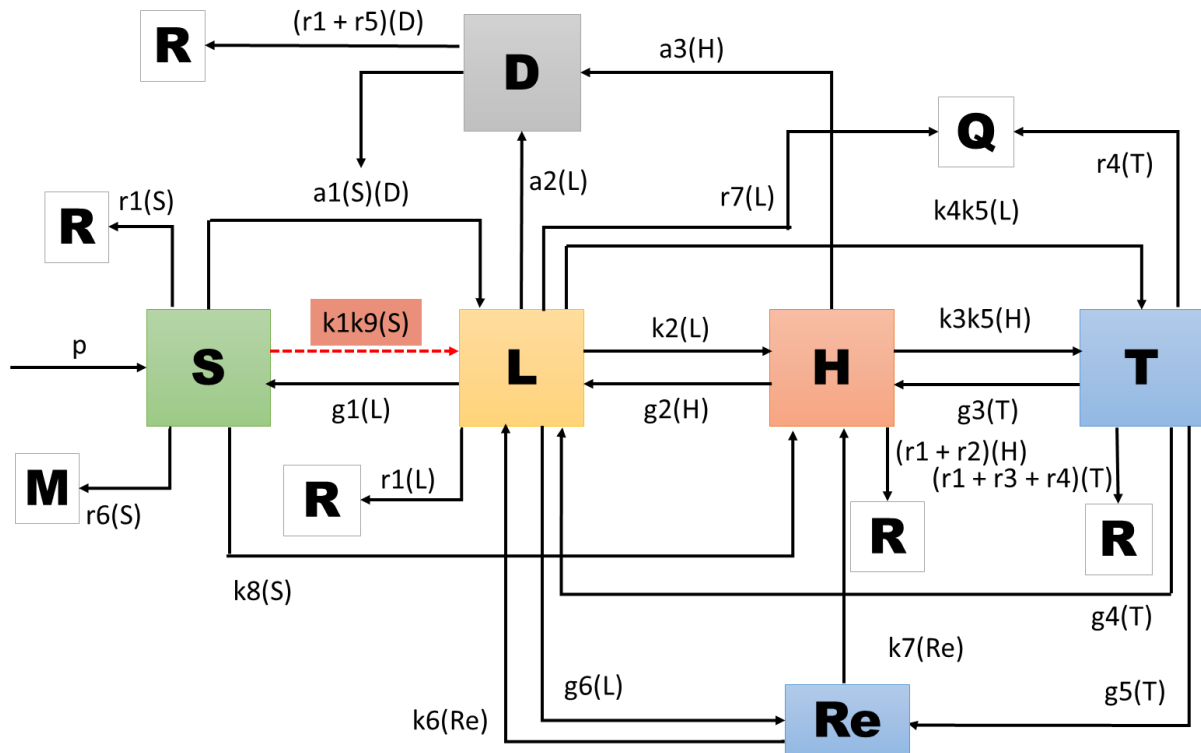


Figure S12. Model-5.

The revised parameters for Model-5 are as follow:

Parameter	Nominal Value	Description
p	0.02	Recruitment rate from general population into susceptible population (S).
k1	0.28	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k2	0.56	Rate at which light users (L) escalates to heavy drug use (H).
k3	0.446	Proportion of heavy drug users (H) exposed to police search.
k4	0.223	Proportion of light drug users (L) exposed to police search.
k5	0.5	Intensity of policing / police search.
k6	0.05	Rate of relapse from remission (Re) to light drug use (L).
k7	0.01	Rate of relapse from remission (Re) to heavy drug use (H).
k8	0.001	Rate of susceptible population (S) become heavy drug users (H).
k9	1	Availability of drugs in the system.
g1	0.2	Rate at which light users (L) quit and become susceptible (S) again.
g2	0.4	Rate at which heavy users (H) become light users (L), which includes amelioration.
g3	0.25	Rate at which rehabilitated users (T) reverted to heavy drug use (H).
g4	0.325	Rate at which rehabilitated users (T) reverted to light drug use (L).
g5	0.283	Rate at which rehabilitated users (T) enter remission (Re).
g6	0.005	Proportion of light drug users (L) entering remission (Re) on their own accord

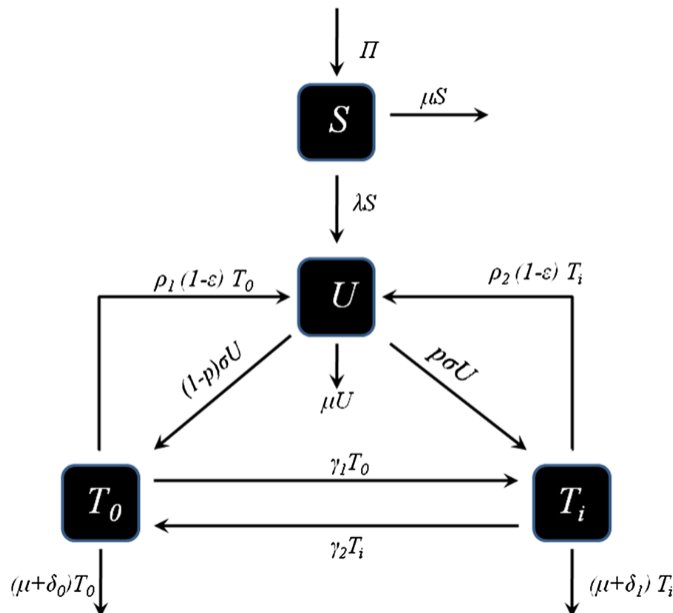
a1	0.4	Effective contact rate between drug barons (D) and susceptible population (S).
a2	0.04	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	0.08	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	0.02	Per capita mortality rate of population.
r2	0.0014	Removal rate of heavy users (H) due to events related to drug usage.
r3	0.003	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	0.2	Rate at which rehabilitated users (T) permanently quit.
r5	0.028	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	0.01	Rate of susceptible (S) maturing into non-susceptible (M)
r7	0.01	Rate of light users (L) quitting drug use permanently (Q)

S17. Stage 10: Model-5 Incorporates Muroya et al.'s Model

Muroya et al. [12] use White and Comiskey's model [6] without amendments. Hence, Model-5 also incorporates Muroya et al.'s [12].

S18. Description of Mushanyu et al. 2015's Model

Mushanyu et al. [13] define a 4-compartment model for methamphetamine usage as S being the population at risk of being initiated into methamphetamine abuse, U are those initiated into methamphetamine abuse, T_o are those in rehabilitation as out-patients, and T_i are those in rehabilitation as in-patients.



S19. Stage 11: Assemble Model-5 and Mushanyu et al. 2015's Model into Model-6

In Mushanyu et al. [13]; S, being the population at risk of being initiated into methamphetamine abuse, corresponds to susceptible (S) in Model-5. U, those initiated into methamphetamine abuse, corresponds to both light (L) and heavy (H) user in Model-5. T_o and T_i corresponds to treatment (T) in Model-5. However, to incorporate Mushanyu et al. [13] into Model-5, treatment (T) is split into in-patient treatment (T_i) and out-patient treatment (T_o). Hence, the following processes are added into Model-5 to form Model-6:

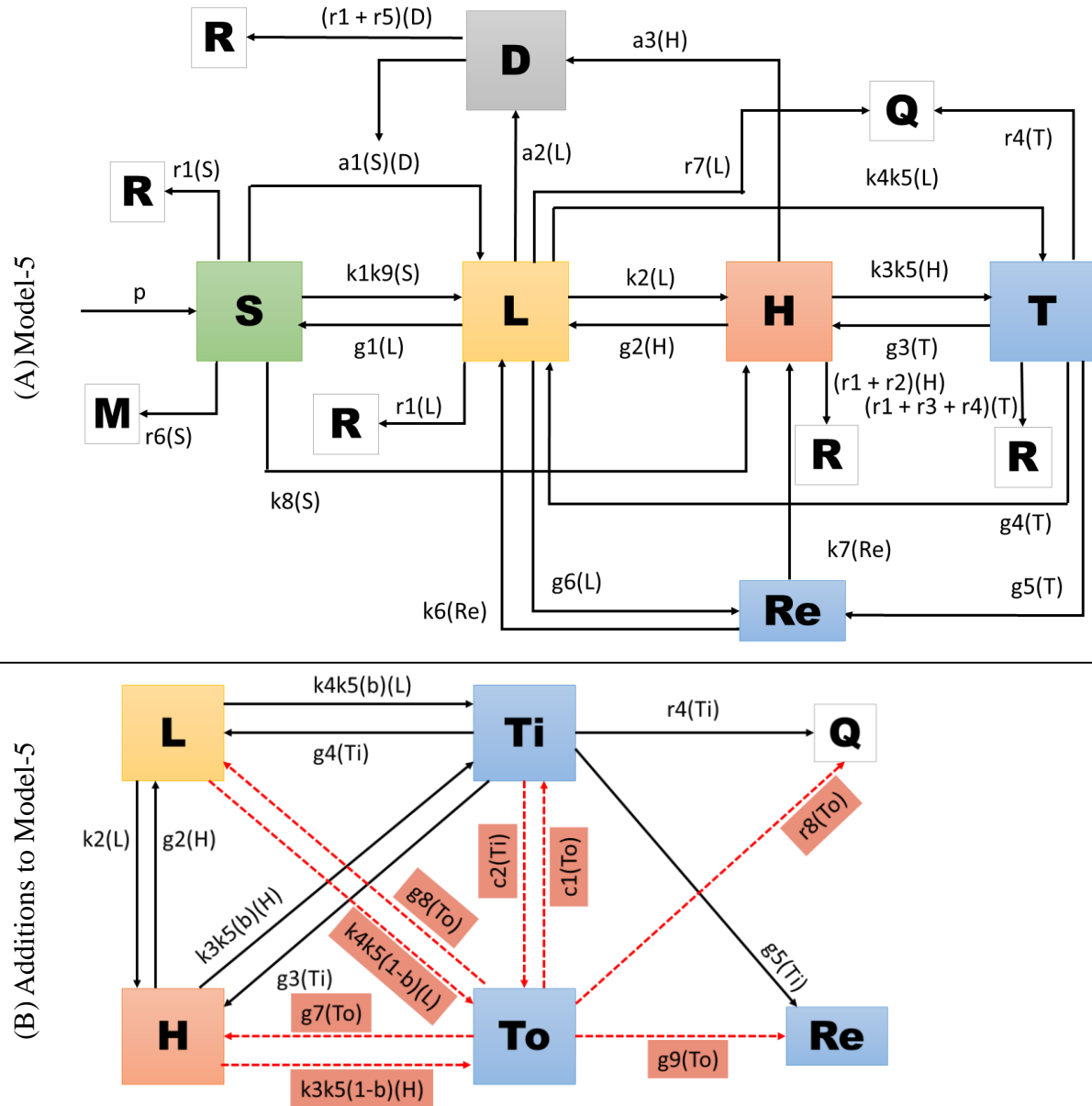


Figure S13. Model-6 Assembly. Panel A shows Model-5. Panel B shows the addition to Model-5 to form Model-6.

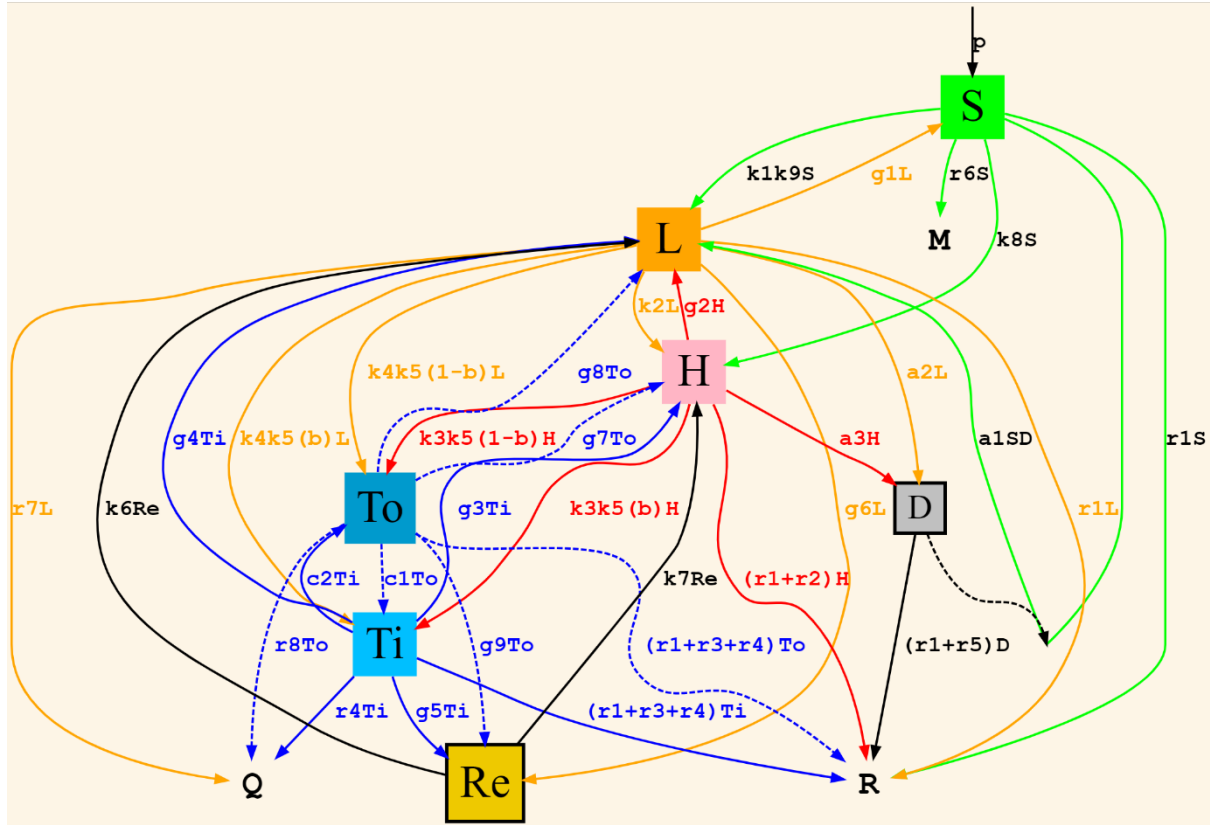


Figure S14. Model-6.

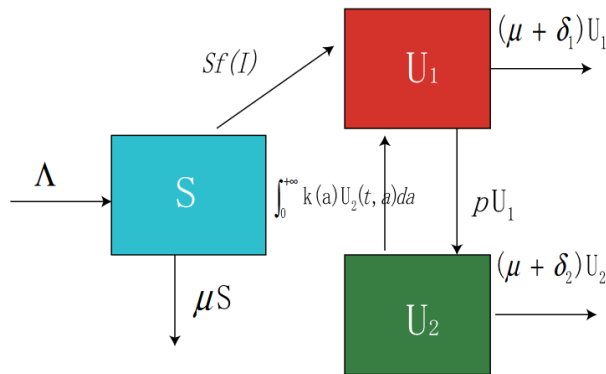
The revised parameters for Model-6 are as follow:

Parameter	Description
p	Recruitment rate from general population into susceptible population (S).
k_1	Rate at which susceptible population (S) become light drug users (L) without the effects of drug barons (D).
k_2	Rate at which light users (L) escalates to heavy drug use (H).
k_3	Proportion of heavy drug users (H) exposed to police search.
k_4	Proportion of light drug users (L) exposed to police search.
k_5	Intensity of policing / police search.
k_6	Rate of relapse from remission (Re) to light drug use (L).
k_7	Rate of relapse from remission (Re) to heavy drug use (H).
k_8	Rate of susceptible population (S) become heavy drug users (H).
k_9	Availability of drugs in the system.
b	Proportion of drug users caught for in-patient treatment (T_i). Therefore, the proportion of drug users caught for out-patient treatment (T_o) is $(1-b)$.
g_1	Rate at which light users (L) quit and become susceptible (S) again.
g_2	Rate at which heavy users (H) become light users (L), which includes amelioration.
g_3	Rate at which in-patient treatment (T_i) reverted to heavy drug use (H).
g_4	Rate at which in-patient treatment (T_i) reverted to light drug use (L).
g_5	Rate at which in-patient treatment (T_i) enter remission (Re).
g_6	Proportion of light drug users (L) entering remission (Re) on their own accord.
g_7	Rate at which out-patient treatment (T_o) reverted to heavy drug use (H).
g_8	Rate at which out-patient treatment (T_o) reverted to light drug use (L).

g9	Rate at which out-patient treatment (T_o) enter remission (Re).
a1	Effective contact rate between drug barons (D) and susceptible population (S).
a2	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
r1	Per capita mortality rate of population.
r2	Removal rate of heavy users (H) due to events related to drug usage.
r3	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	Rate at which in-patient treatment (T_i) permanently quit (Q).
r5	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	Rate of susceptible (S) maturing into non-susceptible (M)
r7	Rate of light users (L) quitting drug use permanently (Q)
r8	Rate at which out-patient treatment (T_o) permanently quit (Q).
c1	Rate of out-patient treatment (T_o) entering in-patient treatment (T_i)
c2	Rate of in-patient treatment (T_i) entering out-patient treatment (T_o)

S20. Description of Yang et al.'s Model

Yang et al. [14] model examine susceptible (S) to drug or alcohol usage (U_1), and from drug or alcohol usage (U_1) to treatment (U_2).

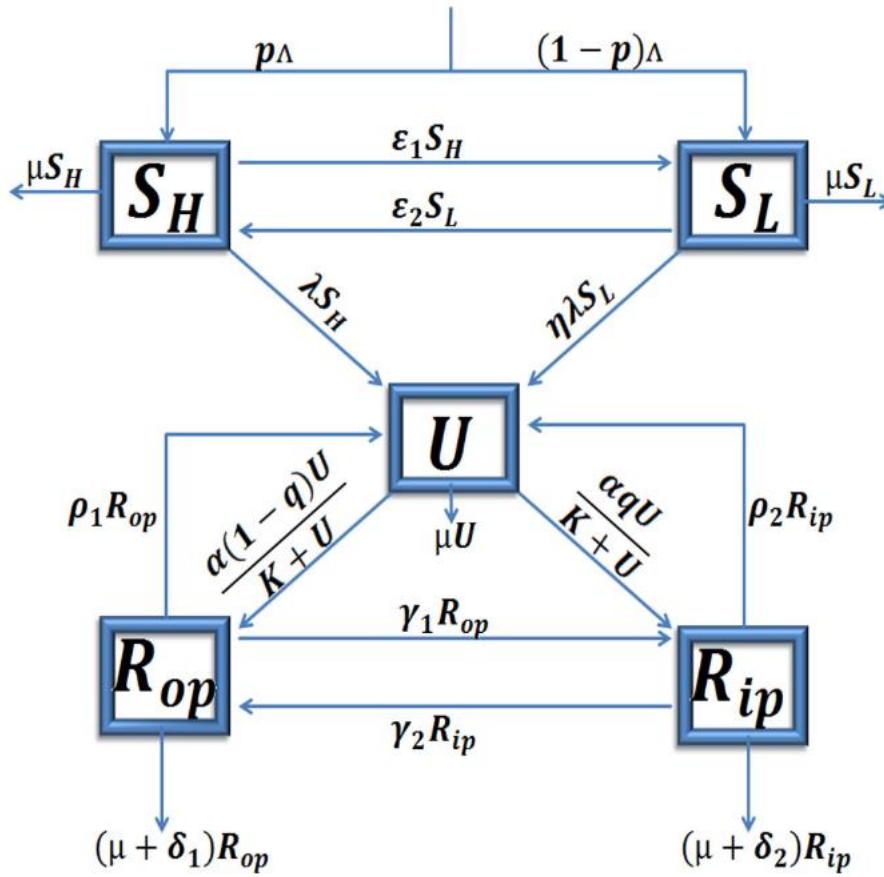


S21. Stage 12: Model-6 incorporates Yang et al.'s Model

Susceptible (S), drug usage (U_1), and treatment (U_2) in Yang et al. [14] are addressed in Model-6.

S22. Description of Mushanyu et al. 2016's Model

Mushanyu et al. [15] extend Mushanyu et al. [13] model by splitting susceptible (S) into high (S_H) and low (S_L) susceptibility to drug use (U).

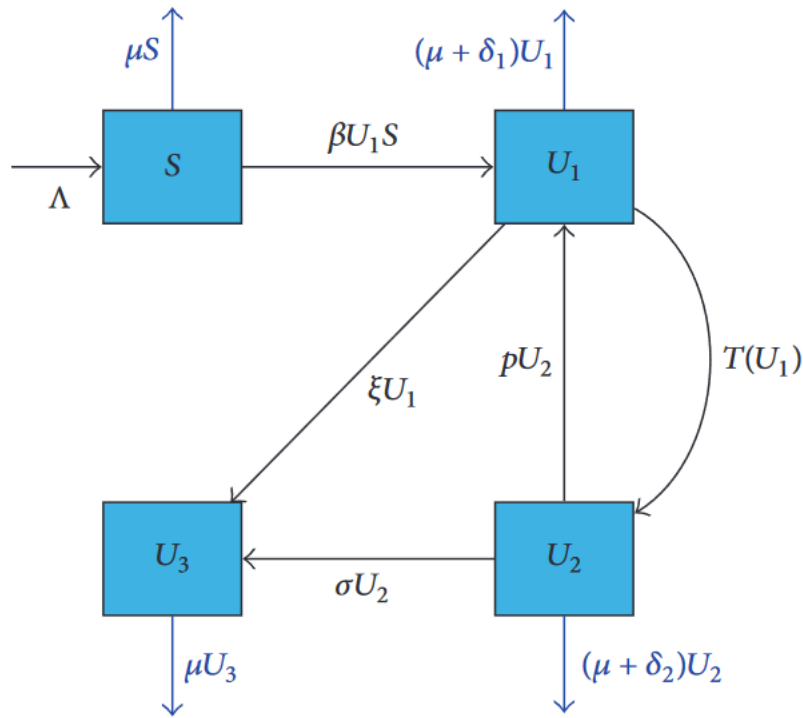


S23. Stage 13: Model-6 partially incorporates Mushanyu et al. 2016's Model

Model-6 addresses in-patient and out-patient treatment, which are deemed to be more visible than low and high susceptibility. In addition, it is generally difficult to accurately categorize a susceptible person as high or low susceptibility as compared to in-patient and out-patient treatment, which are usually documented. Hence, splitting of susceptibility is difficult. Therefore, Model-5 partially incorporates Mushanyu et al. [15] model.

S24. Description of Wangari and Stone's Model

Wangari and Stone's [16] model defines S as susceptible, U_1 as drug users, U_2 as drug users under treatment, and U_3 as individuals successfully treated.

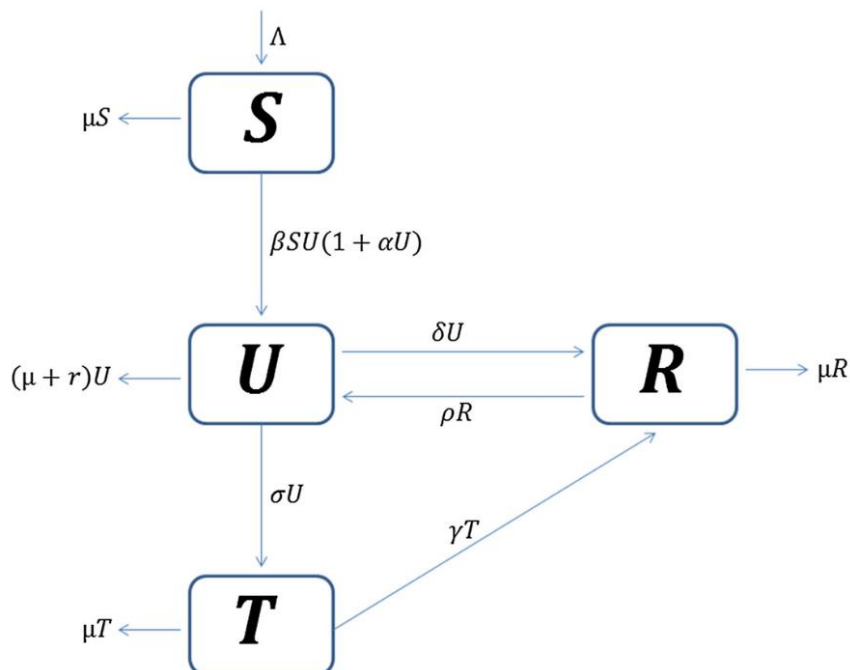


S25. Stage 14: Model-6 incorporates Wangari and Stone's Model

Model-6 addresses Wangari and Stone's [16] model as light (L) and heavy drug (H) users are collectively denoted as U_1 , users under treatment (T) corresponds to U_2 , and users in remission (Re) corresponds to U_3 .

S26. Description of Mushanyu et al. 2017's Model

Mushanyu et al.'s [17] model defines S as susceptible, U as drug users for both current and relapsed users, T as drug users under treatment, and R as individuals successfully treated or recovered.

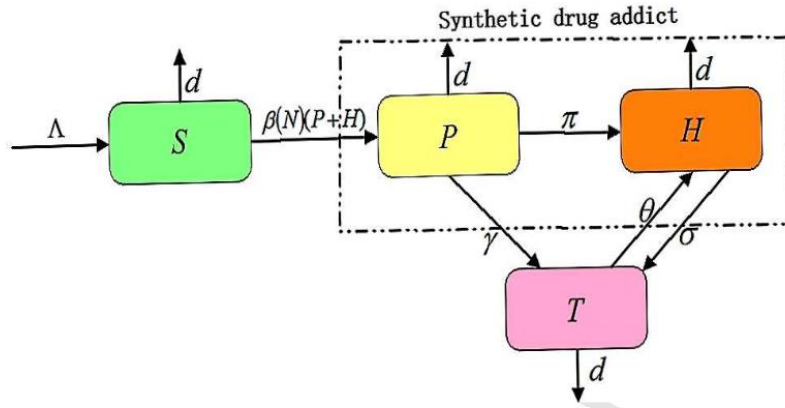


S27. Stage 15: Model-6 incorporates Mushanyu et al. 2017's Model

Model-6 addresses Mushanyu et al.'s [17] model as light (L) and heavy drug (H) users are collectively denoted as U, users under treatment (T) has the same definition in both models, and users in remission (Re) corresponds to R (recovered users with potential relapse).

S28. Description of Ma et al.'s Model

Ma et al.'s [18] model defines S as susceptible, P as psychological addicts, H as physiological addicts, and T as users under treatment.

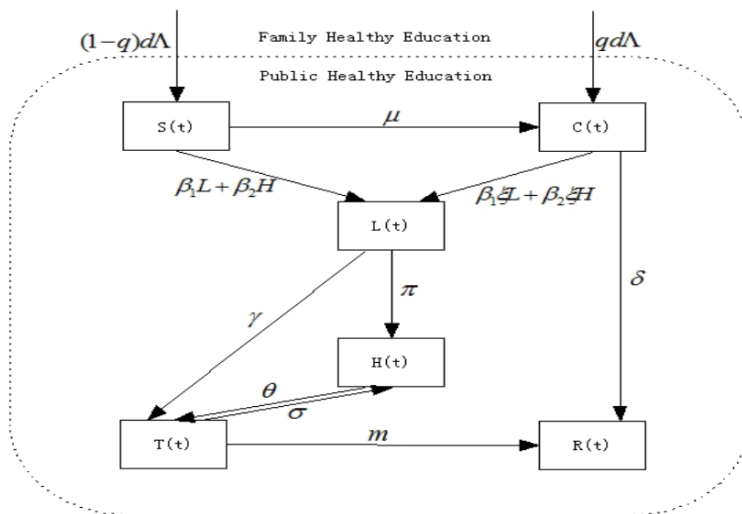


S29. Stage 16: Model-6 incorporates Ma et al.'s Model

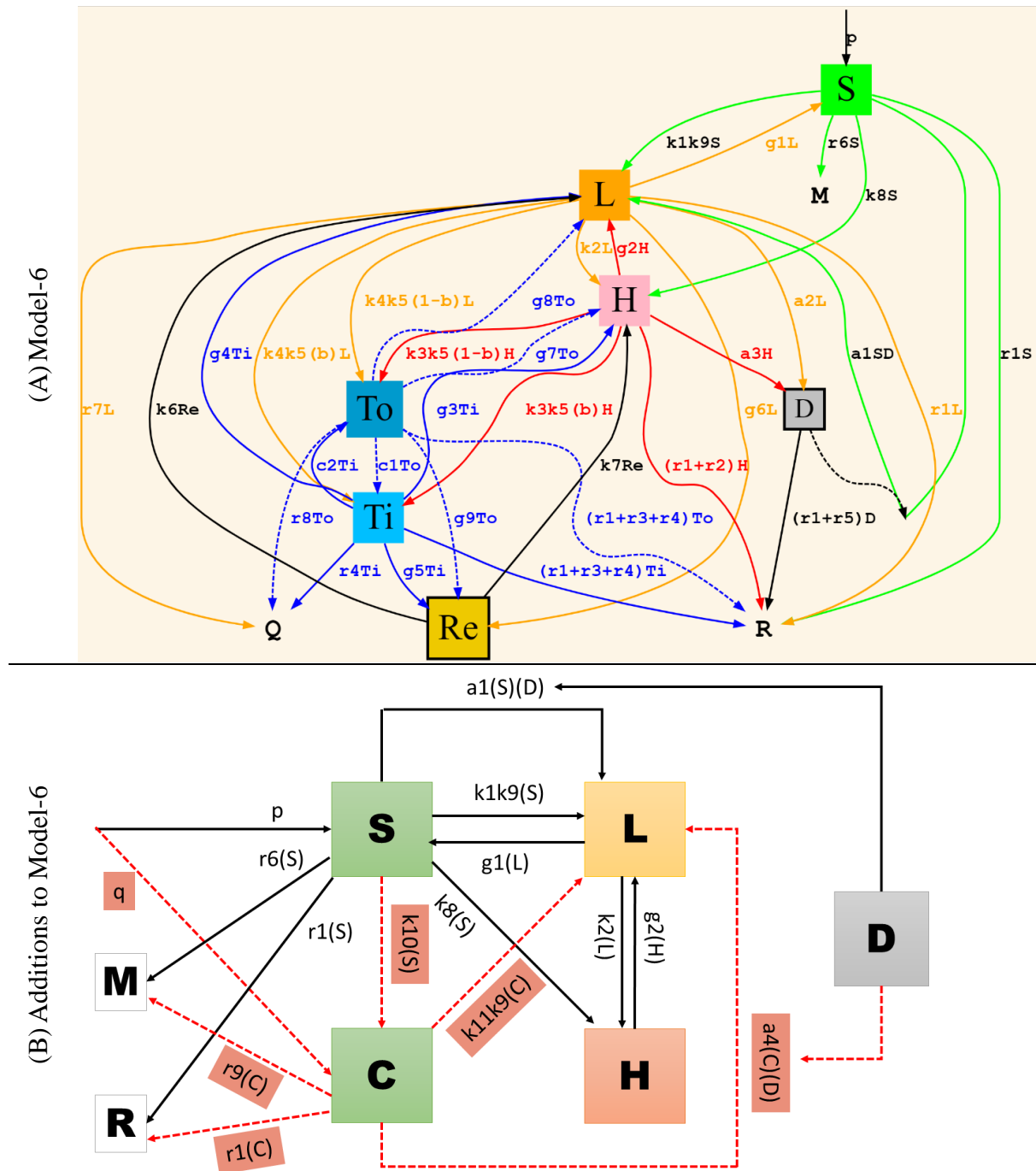
Model-6 addresses Ma et al.'s [18] model as psychological addicts (P) and physiological addicts (H) can correspond to light (L) and heavy (H) users, respectively.

S30. Description of Li and Ma's Model

Li and Ma [19] defines susceptibility into 2 classes – those that refused health education (S) and those accepted health education (C). However, both S and C may still end up as light drug users (L) but at different rates. Yet, there may be a small group of C vowed never to do drugs (deemed as removed or R).



S31. Stage 17: Assemble Model-6 and Li and Ma's Model into Model-7
 Li and Ma [19] model differentiated susceptible in Model-6 (S) into 2 classes – those that refused health education (S) and those accepted health education (C). Hence, S in Model-6 is split into S and C. This resulted in the addition of the following processes – (1) recruitment from general population to susceptible accepting health education (C), (2) maturing and removal from C, (3) individuals moving from without health education (S) to accepting health education (C), (4) persuasion of C to light drug use (L) by drug sellers (D), and (4) susceptible accepting health education (C) moving into light drug use (L) without drug sellers (D).



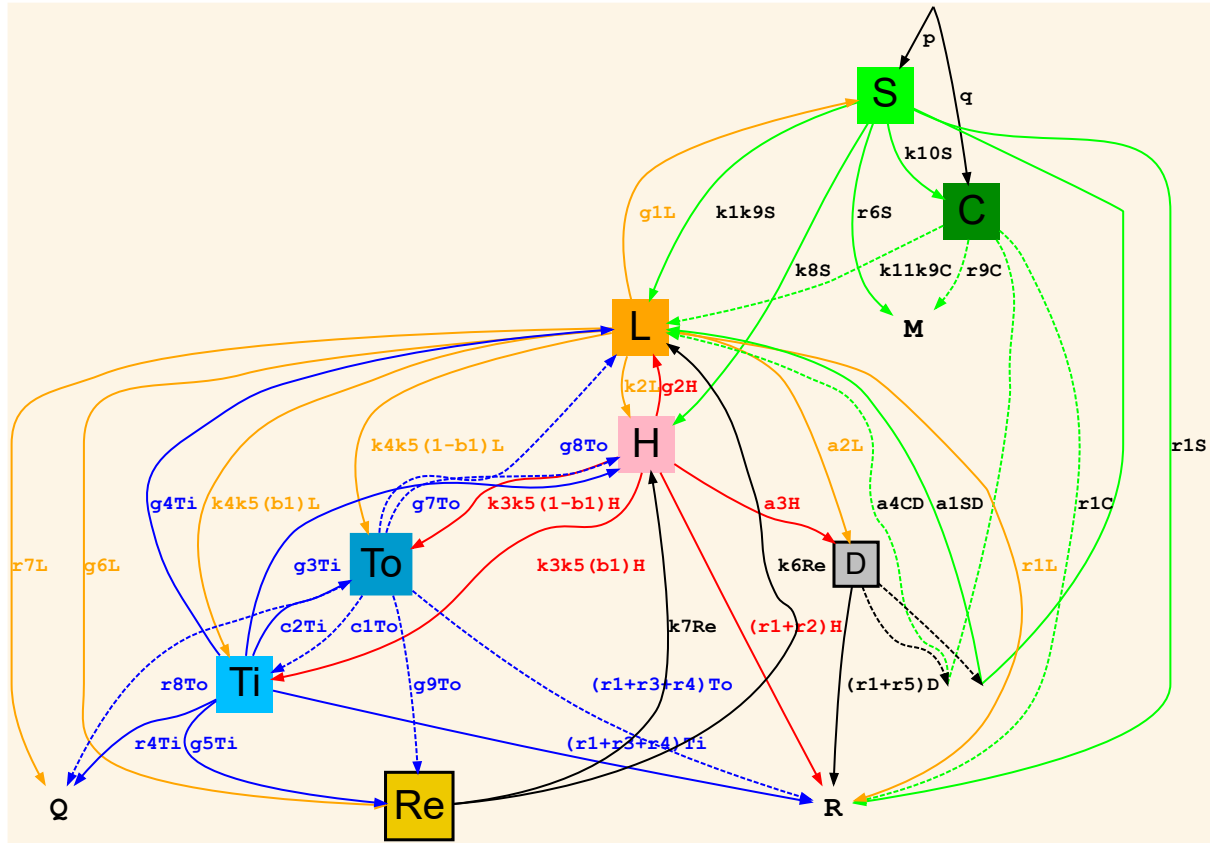


Figure S16. Model-7.

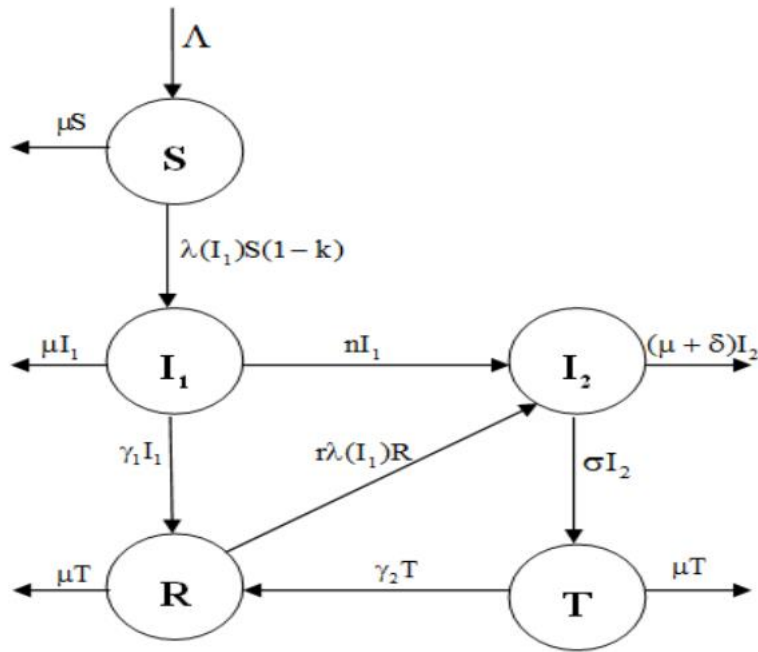
The revised parameters for Model-7 are as follow:

Parameter	Description
p	Recruitment rate from general population into susceptible population without health education (S).
q	Recruitment rate from general population into susceptible population with health education (C).
k1	Rate at which susceptible population without health education (S) become light drug users (L) without the effects of drug barons (D).
k2	Rate at which light users (L) escalates to heavy drug use (H).
k3	Proportion of heavy drug users (H) exposed to police search.
k4	Proportion of light drug users (L) exposed to police search.
k5	Intensity of policing / police search.
k6	Rate of relapse from remission (Re) to light drug use (L).
k7	Rate of relapse from remission (Re) to heavy drug use (H).
k8	Rate of susceptible population without health education (S) become heavy drug users (H).
k9	Availability of drugs in the system.
k10	Rate at which susceptible population without health education (S) accepts health education (C)
k11	Rate at which susceptible population with health education (C) become light drug users (L) without the effects of drug barons (D).
b	Proportion of drug users caught for in-patient treatment (Ti). Therefore, the proportion of drug users caught for out-patient treatment (To) is (1-b).

g1	Rate at which light users (L) quit and become susceptible without health education (S) again.
g2	Rate at which heavy users (H) become light users (L), which includes amelioration.
g3	Rate at which in-patient treatment (T_i) reverted to heavy drug use (H).
g4	Rate at which in-patient treatment (T_i) reverted to light drug use (L).
g5	Rate at which in-patient treatment (T_i) enter remission (Re).
g6	Proportion of light drug users (L) entering remission (Re) on their own accord.
g7	Rate at which out-patient treatment (T_o) reverted to heavy drug use (H).
g8	Rate at which out-patient treatment (T_o) reverted to light drug use (L).
g9	Rate at which out-patient treatment (T_o) enter remission (Re).
a1	Effective contact rate between drug barons (D) and susceptible population without health education (S).
a2	Rate at which light users (L) convert from consumer to seller / promoter (D).
a3	Rate at which heavy users (H) convert from consumer to seller / promoter (D).
a4	Effective contact rate between drug barons (D) and susceptible population with health education (S).
r1	Per capita mortality rate of population.
r2	Removal rate of heavy users (H) due to events related to drug usage.
r3	Removal rate of rehabilitated users (T) due to events related to drug usage.
r4	Rate at which in-patient treatment (T_i) permanently quit (Q).
r5	Removal rate of drug barons (D), which constitutes mainly to law enforcement.
r6	Rate of susceptible without health education (S) maturing into non-susceptible (M)
r7	Rate of light users (L) quitting drug use permanently (Q)
r8	Rate at which out-patient treatment (T_o) permanently quit (Q).
r9	Rate of susceptible with health education (C) maturing into non-susceptible (M)
c1	Rate of out-patient treatment (T_o) entering in-patient treatment (T_i)
c2	Rate of in-patient treatment (T_i) entering out-patient treatment (T_o)

S32. Description of Naowarat and Kumat's Model

Naowarat and Kumat [20] define 5 compartments – S as susceptible, I_1 as light drug users, I_2 as heavy drug users, T as drug users in treatment, and R as recovered drug users.

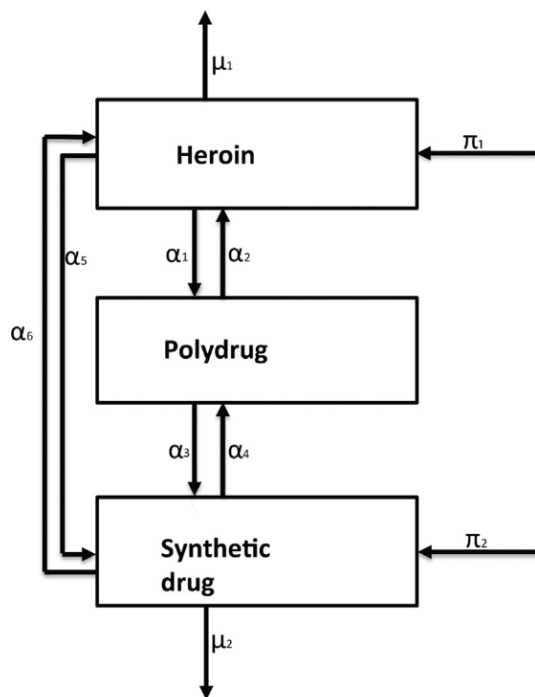


S33. Stage 18: Model-7 incorporates Naowarat and Kumat's Model

Model-7 addresses the 5 compartments in Naowarat and Kumat's model [20] – S as susceptible (corresponding to S and C), I_1 as light drug users (corresponding to L), I_2 as heavy drug users (corresponding to H), T as drug users in treatment (corresponding to T_o and T_i), and R as recovered drug users (corresponding to R_e).

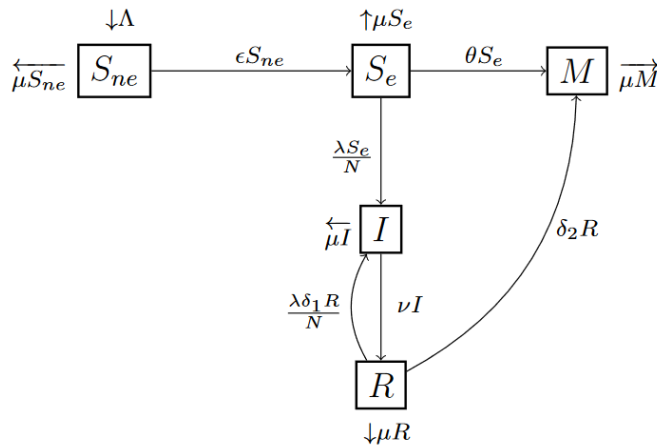
S34. Description of Su et al.'s Model

Su et al. [21] examines the shuttling between single drug use (heroin), multidrug use, and synthetic drugs.



S35. Description of Memarbashi and Pourhossieni's Model

Memarbashi and Pourhossieni [22] use a 5-compartment model – (i) non-educated susceptible (S_{ne}), (ii) educated susceptible (S_e), (iii) mature and will not use drugs forever (M), (iv) infected which are drug users (I), and (v) users under treatment or rehabilitation (R).

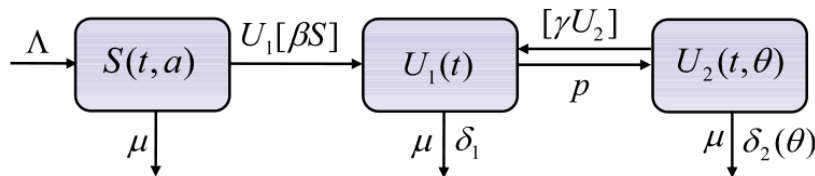


S36. Stage 19: Model-7 incorporates Memarbashi and Pourhossieni's Model

Model-7 addresses the 5 compartments in Memarbashi and Pourhossieni [22] – (i) non-educated susceptible (S_{ne}) can be addressed by susceptible without or refusing health education (S), (ii) educated susceptible (S_e) can be addressed by susceptible with or accepted health education (C), (iii) mature and will not use drugs forever (M) can be addressed by removed (R), (iv) infected (I) can be addressed by light (L) and heavy (H) drug users, and (v) users under treatment or rehabilitation (R) can be addressed by in-patient (T_i) and out-patient (T_o) treatment.

S37. Description of Liu and Liu's Model

Liu and Liu [23] model defines S as susceptible, U_1 as drug users not under treatment, and U_2 as drug users under treatment.



S38. Stage 20: Model-7 incorporates Liu and Liu's Model

Model-7 addresses the 3 compartments in Liu and Liu [23] – (i) susceptible (S) is addressed by susceptible not accepting health education (S) and susceptible accepted health education (C), (ii) U_1 can be addressed by light (L) and heavy (H) drug users, and (v) U_2 can be addressed by in-patient (T_i) and out-patient (T_o) treatment.

S39. Description of Saha and Samanta's Model

Saha and Samanta's [24] model is similar to Ma et al.'s [18] model, which define 4 compartments as (i) susceptible (S), (ii) psychological addicts (P_1), (iii) physiological addicts (P_2), and (iv) addicts under treatment (T).

S40. Stage 21: Model-7 incorporates Saha and Samanta's Model

Model-7 addresses Saha and Samanta's [24] model as psychological addicts (P_1) and physiological addicts (H) can correspond to light (P_2) and heavy (H) users, respectively.

S41. Description of Duan et al.'s Model

Duan et al. [25] define a 5-compartment model for heroin and HIV co-infection as (a) susceptible (S), (ii) heroin drug users (U), (iii) HIV-infected individuals (V), (iv) heroin/HIV coinfecting individuals (I), and (v) individuals with AIDS (A).

S42. SubstanceUseModel in Graphviz DOT Language

```
digraph epidemiology {
    bgcolor=oldlace

    start -> S [label="p", penwidth=2, fontsize=20, fontname="Courier-Bold"]
    start -> C [label="q", penwidth=2, fontsize=20, fontname="Courier-Bold"]

    S -> R [label="r1S", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    S -> M [label="r6S", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    S -> C [label="k10S", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    S -> L [label="k1k9S", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    S -> H [label="k8k9S", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    S -> D_1 [arrowhead = none, penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]
    D_1 -> L [label="a1SD", penwidth=2, color=green, fontcolor=olive,
    fontsize=20, fontname="Courier-Bold"]

    C -> M [label="r1C", penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]
    C -> R [label="r9C", penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]
    C -> L [label="k11k9C", penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]
    C -> H [label="k12k9C", penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]
    C -> D_2 [arrowhead = none, penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]
    D_2 -> L [label="a4CD", penwidth=2, color=green, style=dashed,
    fontcolor=olive, fontsize=20, fontname="Courier-Bold"]

    D -> D_1 [penwidth=2, style=dashed]
    D -> D_2 [penwidth=2, style=dashed]
    D -> R [label="(r1+r5)D", penwidth=2, fontsize=20, fontname="Courier-
    Bold"]

    L -> S [label="g1L", penwidth=2, color=orange, fontcolor=orange1,
    fontsize=20, fontname="Courier-Bold"]
}
```

```

L -> H [label="k2L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]
L -> Ti [label="k4k5(b1)L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]
L -> To [label="k4k5(1-b1)L", penwidth=2, color=orange,
fontcolor=orange1, fontsize=20, fontname="Courier-Bold"]
L -> R [label="r1L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]
L -> D [label="a2L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]
L -> Re [label="g6L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]
L -> Q [label="r7L", penwidth=2, color=orange, fontcolor=orange1,
fontsize=20, fontname="Courier-Bold"]

H -> D [label="a3H", penwidth=2, color=red, fontcolor=red, fontsize=20,
fontname="Courier-Bold"]
H -> L [label="g2H", penwidth=2, color=red, fontcolor=red, fontsize=20,
fontname="Courier-Bold"]
H -> R [label="(r1+r2)H", penwidth=2, color=red, fontcolor=red,
fontsize=20, fontname="Courier-Bold"]
H -> Ti [label="k3k5(b2)H", penwidth=2, color=red, fontcolor=red,
fontsize=20, fontname="Courier-Bold"]
H -> To [label="k3k5(1-b2)H", penwidth=2, color=red, fontcolor=red,
fontsize=20, fontname="Courier-Bold"]

Ti -> L [label="g4Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]
Ti -> H [label="g3Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]
Ti -> To [label="c2Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]
Ti -> Re [label="g5Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]
Ti -> Q [label="r4Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]
Ti -> R [label="(r1+r3+r4)Ti", penwidth=2, color=blue, fontcolor=blue,
fontsize=20, fontname="Courier-Bold"]

To -> L [label="g8To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]
To -> H [label="g7To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]
To -> Ti [label="c1To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]
To -> Re [label="g9To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]
To -> Q [label="r8To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]
To -> R [label="(r1+r3+r4)To", penwidth=2, color=blue, style=dashed,
fontcolor=blue, fontsize=20, fontname="Courier-Bold"]

Re -> L [label="k6Re", penwidth=2, fontsize=20, fontname="Courier-Bold"]
Re -> H [label="k7Re", penwidth=2, fontsize=20, fontname="Courier-Bold"]
Re -> R [label="r1Re", penwidth=2, fontsize=20, fontname="Courier-Bold"]

start [shape=point, width=0]
D_1 [shape=point, width=0]
D_2 [shape=point, width=0]
S [shape=square, fillcolor=green1, style=filled, penwidth=0, fontsize=40,
fontname="Arial-Bold"]

```

```

C [shape=square, fillcolor=green4, style=filled, penwidth=0, fontsize=40,
fontname="Arial-Bold"]
L [shape=square, fillcolor=orange1, style=filled, penwidth=0,
fontsize=40, fontname="Arial-Bold"]
H [shape=square, fillcolor=pink1, style=filled, penwidth=0, fontsize=40,
fontname="Arial-Bold"]
Ti [shape=square, fillcolor=deepskyblue1, style=filled, penwidth=0,
fontsize=40, fontname="Arial-Bold"]
To [shape=square, fillcolor=deepskyblue3, style=filled, penwidth=0,
fontsize=40, fontname="Arial-Bold"]
Re [shape=square, fillcolor=gold2, style=filled, fontsize=40, penwidth=3,
fontname="Arial-Bold"]
D [shape=square, fillcolor=grey, style=filled, penwidth=3, fontsize=30,
fontname="Arial-Bold"]
R [shape=square, penwidth=0, fontsize=30, fontname="Courier-Bold"]
M [shape=square, penwidth=0, fontsize=30, fontname="Courier-Bold"]
Q [shape=square, fontsize=30, penwidth=0, fontname="Courier-Bold"]
}

```

S43. Python Implementation of SubstanceUseModel (SubstanceUseModel.py)

```

"""
Substance use epidemiological model written in Python
by
Yap et al. (2024) Assembly of Substance Use or Abuse Epidemiological Models.
"""
import argparse

# Step 1: Setup command-line parser
parser = argparse.ArgumentParser(prog="python SubstanceUseModel.py",
description="Substance use epidemiological model (written in Python) by Yap
et al. (2024) Assembly of Substance Use Epidemiological Models.")
# Step 1.1: Compartment initial conditions (in percentage)
parser.add_argument("-S", type=float, default=99.9957, help="Percentage of
population that is susceptible without health education (S).
Default=49.9939")
parser.add_argument("-C", type=float, default=99.9957, help="Percentage of
population that is susceptible with health education (S). Default=50.000")
parser.add_argument("-L", type=float, default=0.005, help="Percentage of
population that is light drug user (L). Default=0.005")
parser.add_argument("-H", type=float, default=0.0005, help="Percentage of
population that is heavy drug user (H). Default=0.0005")
parser.add_argument("-Ti", type=float, default=0.0002, help="Percentage of
population that is under in-patient treatment (Ti). Default=0.0002")
parser.add_argument("-To", type=float, default=0.0003, help="Percentage of
population that is under out-patient treatment (To). Default=0.0003")
parser.add_argument("-D", type=float, default=0.0001, help="Percentage of
population that is drug sellers (D). Default=0.0001")
parser.add_argument("-Re", type=float, default=0.0005, help="Percentage of
population that is in remission (Re). Default=0.0005")
parser.add_argument("-R", type=float, default=0.0, help="Percentage of
population that is dead or removed (R). Default=0.0")
parser.add_argument("-Q", type=float, default=0.0, help="Percentage of
population that permanently quitted drug use (Q). Default=0.0")
parser.add_argument("-M", type=float, default=0.0, help="Percentage of
population that matured from susceptible (M). Default=0.0")
# Step 1.2: Model parameters

```

```

parser.add_argument("-p", type=float, default=0.05, help="Recruitment rate
from general population into susceptible population without health education
(S). Default=0.05")
parser.add_argument("-q", type=float, default=0.15, help="Recruitment rate
from general population into susceptible population with health education
(C). Default=0.15")
parser.add_argument("-k1", type=float, default=0.2, help="Rate at which
susceptible population without health education (S) become light drug users
(L) without the effects of drug barons (D). Default=0.2")
parser.add_argument("-k2", type=float, default=0.5, help="Rate at which
light users (L) escalates to heavy drug use (H). Default=0.5")
parser.add_argument("-k3", type=float, default=0.4, help="Proportion of
heavy drug users (H) exposed to police search. Default=0.4")
parser.add_argument("-k4", type=float, default=0.2, help="Proportion of
light drug users (L) exposed to police search. Default=0.2")
parser.add_argument("-k5", type=float, default=1.00, help="Intensity of
policing / police search. Default=1.00")
parser.add_argument("-k6", type=float, default=0.05, help="Rate of relapse
from remission (Re) to light drug use (L). Default=0.05")
parser.add_argument("-k7", type=float, default=0.01, help="Rate of relapse
from remission (Re) to heavy drug use (H). Default=0.01")
parser.add_argument("-k8", type=float, default=0.01, help="Rate of
susceptible population without health education (S) become heavy drug users
(H) without the effects of drug barons (D). Default=0.01")
parser.add_argument("-k9", type=float, default=1.00, help="Availability of
drugs in the system. Default=1.00")
parser.add_argument("-k10", type=float, default=0.3, help="Rate at which
susceptible population without health education (S) accepts health education
(C). Default=0.3")
parser.add_argument("-k11", type=float, default=0.1, help="Rate at which
susceptible population with health education (C) become light drug users (L)
without the effects of drug barons (D). Default=0.1")
parser.add_argument("-k12", type=float, default=0.001, help="Rate of
susceptible population with health education (C) become heavy drug users (H)
without the effects of drug barons (D). Default=0.001")
parser.add_argument("-b1", type=float, default=0.2, help="Proportion of
light drug users (L) caught for in-patient treatment (Ti). Therefore, the
proportion of light drug users caught for out-patient treatment (To) is (1-
b1). Default=0.2")
parser.add_argument("-b2", type=float, default=0.8, help="Proportion of
heavy drug users (H) caught for in-patient treatment (Ti). Therefore, the
proportion of heavy drug users caught for out-patient treatment (To) is (1-
b2). Default=0.8")
parser.add_argument("-g1", type=float, default=0.2, help="Rate at which
light users (L) quit and become susceptible without health education (S)
again. Default=0.2")
parser.add_argument("-g2", type=float, default=0.4, help="Rate at which
heavy users (H) become light users (L), which includes amelioration.
Default=0.4")
parser.add_argument("-g3", type=float, default=0.01, help="Rate at which in-
patient treatment (Ti) reverted to heavy drug use (H). Default=0.01")
parser.add_argument("-g4", type=float, default=0.02, help="Rate at which in-
patient treatment (Ti) reverted to light drug use (L). Default=0.02")
parser.add_argument("-g5", type=float, default=0.2, help="Rate at which in-
patient treatment (Ti) enter remission (Re). Default=0.2")
parser.add_argument("-g6", type=float, default=0.015, help="Proportion of
light drug users (L) entering remission (Re) on their own accord.
Default=0.015")
parser.add_argument("-g7", type=float, default=0.015, help="Rate at which
out-patient treatment (To) reverted to heavy drug use (H). Default=0.015")

```

```

parser.add_argument("-g8", type=float, default=0.025, help="Rate at which
out-patient treatment (To) reverted to light drug use (L). Default=0.025")
parser.add_argument("-g9", type=float, default=0.2, help="Rate at which out-
patient treatment (To) enter remission (Re). Default=0.2")
parser.add_argument("-a1", type=float, default=0.4, help="Effective contact
rate between drug barons (D) and susceptible population without health
education (S). Default=0.4")
parser.add_argument("-a2", type=float, default=0.04, help="Rate at which
light users (L) convert from consumer to seller / promoter (D).
Default=0.04")
parser.add_argument("-a3", type=float, default=0.08, help="Rate at which
heavy users (H) convert from consumer to seller / promoter (D).
Default=0.08")
parser.add_argument("-a4", type=float, default=0.2, help="Effective contact
rate between drug barons (D) and susceptible population with health education
(S). Default=0.2")
parser.add_argument("-r1", type=float, default=0.2, help="Per capita
mortality rate of population. Default=0.2")
parser.add_argument("-r2", type=float, default=0.001, help="Removal rate of
heavy users (H) due to events related to drug usage. Default=0.001")
parser.add_argument("-r3", type=float, default=0.003, help="Removal rate of
rehabilitated users (T) due to events related to drug usage. Default=0.003")
parser.add_argument("-r4", type=float, default=0.1, help="Rate at which in-
patient treatment (Ti) permanently quit (Q). Default=0.1")
parser.add_argument("-r5", type=float, default=0.02, help="Removal rate of
drug barons (D), which constitutes mainly to law enforcement. Default=0.02")
parser.add_argument("-r6", type=float, default=0.005, help="Rate of
susceptible without health education (S) maturing into non-susceptible (M).
Default=0.005")
parser.add_argument("-r7", type=float, default=0.01, help="Rate of light
users (L) quitting drug use permanently (Q). Default=0.01")
parser.add_argument("-r8", type=float, default=0.1, help="Rate at which out-
patient treatment (To) permanently quit (Q). Default=0.1")
parser.add_argument("-r9", type=float, default=0.01, help="Rate of
susceptible with health education (C) maturing into non-susceptible (M).
Default=0.01")
parser.add_argument("-c1", type=float, default=0.001, help="Rate of out-
patient treatment (To) entering in-patient treatment (Ti). Default=0.001")
parser.add_argument("-c2", type=float, default=0.01, help="Rate of in-
patient treatment (Ti) entering out-patient treatment (To). Default=0.01")
# Step 1.3: Simulation parameters
parser.add_argument("-step", type=float, default=0.00274, help="Simulation
time step. Default=0.00274")
parser.add_argument("-end", type=float, default=10.0, help="Simulation end
time. Default=10.0")
# Step 1.4: Get command-line options
args = parser.parse_args()

# Step 2: Initial conditions in percentage
y = list(range(11))
y[0] = args.S
y[1] = args.C
y[2] = args.L
y[3] = args.H
y[4] = args.Ti
y[5] = args.To
y[6] = args.D
y[7] = args.Re
y[8] = args.R
y[9] = args.Q
y[10] = args.M

```

```

print("--- Compartment Initial Conditions ---")
compartment_names = ["S", "C", "L", "H", "Ti", "To", "D", "Re", "R", "Q",
"M"]
for i in range(len(compartment_names)):
    print("%s = %f" % (compartment_names[i], y[i]))

# Step 3: Model parameters
print("--- Model Parameters ---")
p = args.p
print("p = %f" % p)
q = args.q
print("q = %f" % q)
k1 = args.k1
print("k1 = %f" % k1)
k2 = args.k2
print("k2 = %f" % k2)
k3 = args.k3
print("k3 = %f" % k3)
k4 = args.k4
print("k4 = %f" % k4)
k5 = args.k5
print("k5 = %f" % k5)
k6 = args.k6
print("k6 = %f" % k6)
k7 = args.k7
print("k7 = %f" % k7)
k8 = args.k8
print("k8 = %f" % k8)
k9 = args.k9
print("k9 = %f" % k9)
k10 = args.k10
print("k10 = %f" % k10)
k11 = args.k11
print("k11 = %f" % k11)
k12 = args.k12
print("k12 = %f" % k12)
b1 = args.b1
print("b1 = %f" % b1)
b2 = args.b2
print("b2 = %f" % b2)
g1 = args.g1
print("g1 = %f" % g1)
g2 = args.g2
print("g2 = %f" % g2)
g3 = args.g3
print("g3 = %f" % g3)
g4 = args.g4
print("g4 = %f" % g4)
g5 = args.g5
print("g5 = %f" % g5)
g6 = args.g6
print("g6 = %f" % g6)
g7 = args.g7
print("g7 = %f" % g7)
g8 = args.g8
print("g8 = %f" % g8)
g9 = args.g9
print("g9 = %f" % g9)
a1 = args.a1
print("a1 = %f" % a1)
a2 = args.a2

```

```

print("a2 = %f" % a2)
a3 = args.a3
print("a3 = %f" % a3)
a4 = args.a4
print("a4 = %f" % a4)
r1 = args.r1
print("r1 = %f" % r1)
r2 = args.r2
print("r2 = %f" % r2)
r3 = args.r3
print("r3 = %f" % r3)
r4 = args.r4
print("r4 = %f" % r4)
r5 = args.r5
print("r5 = %f" % r5)
r6 = args.r6
print("r6 = %f" % r6)
r7 = args.r7
print("r7 = %f" % r7)
r8 = args.r8
print("r8 = %f" % r8)
r9 = args.r9
print("r9 = %f" % r9)
c1 = args.c1
print("c1 = %f" % c1)
c2 = args.c2
print("c2 = %f" % c2)

# Step 4: Set up ODEs
def S(t, y):
    # Susceptible without health education (S), y[0] = args.S
    # S = [p + g1L] - [r1S + r6S + k1k9S + k8k9S + a1SD + k10S]
    incoming = p + g1*y[2]
    outgoing = r1*y[0] + r6*y[0] + k1*k9*y[0] + k8*k9*y[0] + a1*y[0]*y[6] +
k10*y[0]
    return incoming - outgoing
def C(t, y):
    # Susceptible with health education (C), y[1] = args.C
    # C = [q + k10S] - [r9C + r1C + k9k11C + k9k12C + a4CD]
    incoming = q + k10*y[0]
    outgoing = r9*y[1] + r1*y[1] + k9*k11*y[1] + k9*k12*y[1] + a4*y[1]*y[6]
    return incoming - outgoing
def L(t, y):
    # Light drug users (L), y[2] = args.L
    # L = [k1k9S + a1SD + k9k11C + a4CD + g2H + g4Ti + g8To + k6Re] - [g1L
+ k2L + k4k5(b1)L + k4k5(1-b1)L + r1L + a2L + g6L + r7L]
    incoming = k1*k9*y[0] + a1*y[0]*y[6] + k9*k11*y[1] + a4*y[1]*y[6] +
g2*y[3] + g4*y[4] + g8*y[4] + k6*y[7]
    outgoing = g1*y[2] + k2*y[2] + k4*k5*(b1)*y[2] + k4*k5*(1-b1)*y[2] +
r1*y[2] + a2*y[2] + g6*y[2] + r7*y[2]
    return incoming - outgoing
def H(t, y):
    # Heavy drug users (H), y[3] = args.H
    # H = [k8k9S + k9k12C + k2L + g3Ti + g7To + k7Re] - [a3H + g2H + (r1+r2)H
+ k3k5(b2)H + k3k5(1-b2)H]
    incoming = k8*k9*y[0] + k9*k12*y[1] + k2*y[2] + g3*y[4] + g7*y[4] +
k7*y[7]
    outgoing = a3*y[3] + g2*y[3] + (r1+r2)*y[3] + k3*k5*(b2)*y[3] + k3*k5*(1-
b2)*y[3]
    return incoming - outgoing
def Ti(t, y):

```



```

    # In-patient treatment (Ti), y[4] = args.Ti
    # Ti = [k4k5(b1)L + k3k5(b2)H + c1To] - [g4Ti + g3Ti + c2Ti + g5Ti +
r4Ti + (r1+r2+r3)Ti]
    incoming = k4*k5*(b1)*y[2] + k3*k5*(b2)*y[3] + c1*y[5]
    outgoing = g4*y[4] + g3*y[4] + c2*y[4] + g5*y[4] + r4*y[4] +
(r1+r2+r3)*y[4]
    return incoming - outgoing
def To(t, y):
    # Out-patient treatment (To), y[5] = args.To
    # To = [k4k5(1-b1)L + k3k5(1-b2)H + c2Ti] - [g8To + g7To + c1To + g9To
+ r8To + (r1+r3+r4)To]
    incoming = k4*k5*(1-b1)*y[2] + k3*k5*(1-b2)*y[3] + c2*y[4]
    outgoing = g8*y[5] + g7*y[5] + c1*y[5] + g9*y[5] + r8*y[5] +
(r1+r3+r4)*y[5]
    return incoming - outgoing
def D(t, y):
    # Drug sellers (D), y[6] = args.D
    # D = [a2L + a3H] - [(r1+r5)D]
    incoming = a2*y[2] + a3*y[3]
    outgoing = (r1+r5)*y[6]
    return incoming - outgoing
def Re(t, y):
    # Remission (Re), y[7] = args.Re
    # Re = [g6L + g5Ti + g9To] - [k6Re + k7Re + r1Re]
    incoming = g6*y[2] + g5*y[5] + g9*y[5]
    outgoing = k6*y[7] + k7*y[7] + r1*y[7]
    return incoming - outgoing
def R(t, y):
    # Removed (R), y[8] = args.R
    # R = r1S + r1C + (r1+r5)D + r1L + (r1+r2)H + (r1+r2+r3)Ti + (r1+r3+r4)To
+ r1Re
    incoming = r1*y[0] + r1*y[1] + (r1+r5)*y[6] + r1*y[2] + (r1+r2)*y[3] +
(r1+r2+r3)*y[4] + (r1+r3+r4)*y[5] + r1*y[7]
    return incoming
def Q(t, y):
    # Quitted (Q), y[9] = args.Q
    # Q = r7L + r4Ti + r8To
    incoming = r7*y[2] + r4*y[4] + r8*y[5]
    return incoming
def M(t, y):
    # Matured (M), y[10] = args.M
    # M = [r6S + r9C]
    incoming = r6*y[0] + r9*y[2]
    return incoming

# Step 5: Model setup
f = list(range(11))
f[0] = S
f[1] = C
f[2] = L
f[3] = H
f[4] = Ti
f[5] = To
f[6] = D
f[7] = Re
f[8] = R
f[9] = Q
f[10] = M

# Step 6: ODE solver
def DP5(funcs, x0, y0, step, xmax,

```

```

        overflow=1e100, zerodivision=1e100):
yield [x0] + y0
def solver(funcs, x0, y0, step):
    n = len(funcs)
    f1, f2, f3 = [0]*n, [0]*n, [0]*n
    f4, f5, f6, f7 = [0]*n, [0]*n, [0]*n, [0]*n
    y1 = [0]*n
    for i in range(n):
        try: f1[i] = funcs[i](x0, y0)
        except TypeError: pass
        except ZeroDivisionError: f1[i] = zerodivision
        except OverflowError: f1[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (0.2*step*f1[j])
    for i in range(n):
        try: f2[i] = funcs[i]((x0+(0.2*step)), y1)
        except TypeError: pass
        except ZeroDivisionError: f2[i] = zerodivision
        except OverflowError: f2[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (3*step*f1[j]/40.0) + (9*step*f2[j]/40.0)
    for i in range(n):
        try: f3[i] = funcs[i]((x0+(0.3*step)), y1)
        except TypeError: pass
        except ZeroDivisionError: f3[i] = zerodivision
        except OverflowError: f3[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (44*step*f1[j]/45.0) + (-56*step*f2[j]/15.0) + \
            (32*step*f3[j]/9.0)
    for i in range(n):
        try: f4[i] = funcs[i]((x0+(0.8*step)), y1)
        except TypeError: pass
        except ZeroDivisionError: f4[i] = zerodivision
        except OverflowError: f4[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (19372*step*f1[j]/6561.0) + \
            (-25360*step*f2[j]/2187.0) + \
            (64448*step*f3[j]/6561.0) + \
            (-212*step*f4[j]/729.0)
    for i in range(n):
        try: f5[i] = funcs[i](x0+(8*step/9.0), y1)
        except TypeError: pass
        except ZeroDivisionError: f5[i] = zerodivision
        except OverflowError: f5[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (9017*step*f1[j]/3168.0) + \
            (-355*step*f2[j]/33.0) + (46732*step*f3[j]/5247.0) + \
            (49*step*f4[j]/176.0) + (-5103*step*f5[j]/18656.0)
    for i in range(n):
        try: f6[i] = funcs[i](x0+step, y1)
        except TypeError: pass
        except ZeroDivisionError: f6[i] = zerodivision
        except OverflowError: f6[i] = overflow
    for j in range(n):
        y1[j] = y0[j] + (35*step*f1[j]/384.0) + \
            (500*step*f3[j]/1113.0) + (125*step*f4[j]/192.0) + \
            (-2187*step*f5[j]/6784.0) + (11*step*f6[j]/84.0)
    for i in range(n):
        try: f7[i] = funcs[i](x0+step, y1)
        except TypeError: pass

```

```

        except ZeroDivisionError: f7[i] = zerodivision
        except OverflowError: f7[i] = overflow
    for i in range(n):
        try: y1[i] = y0[i] + (step * \
            ((35*f1[i]/384.0) + (500*f3[i]/1113.0) + \
            (125*f4[i]/192.0) + (-2187*f5[i]/6784.0) + \
            (11*f6[i]/84.0)))
        except TypeError: pass
        except ZeroDivisionError: y1[i] = zerodivision
        except OverflowError: y1[i] = overflow
    return y1
while x0 < xmax:
    y1 = solver(funcs, x0, y0, step)
    y0 = y1
    x0 = x0 + step
    yield [x0] + y0

# Step 7: Simulation parameters
time_step = args.step
end_time = args.end

# Step 9: Simulate model
print("--- Simulation Results ---")
print(",".join(["Time", "S", "C", "L", "H", "Ti", "To", "D", "Re", "R", "Q",
"M"]))
for i in [x for x in DP5(f, 0.0, y, time_step, end_time)]:
    print(",".join([str(z) for z in i]))

```

S44. Commandline Usage of SubstanceUseModel.py

```

usage: python SubstanceUseModel.py [-h] [-S S] [-C C] [-L L] [-H H] [-Ti
TI] [-To TO] [-D D] [-Re RE] [-R R] [-Q Q] [-M M] [-p P] [-q Q] [-k1
K1] [-k2 K2] [-k3 K3] [-k4 K4] [-k5 K5] [-k6 K6] [-k7 K7] [-k8 K8]
[-k9 K9] [-k10 K10] [-k11 K11] [-k12 K12] [-b1 B1] [-b2 B2] [-g1 G1]
[-g2 G2] [-g3 G3] [-g4 G4] [-g5 G5] [-g6 G6] [-g7 G7] [-g8 G8] [-g9
G9] [-a1 A1] [-a2 A2] [-a3 A3] [-a4 A4] [-r1 R1] [-r2 R2] [-r3 R3]
[-r4 R4] [-r5 R5] [-r6 R6] [-r7 R7] [-r8 R8] [-r9 R9] [-c1 C1] [-c2
C2] [-step STEP] [-end END]

```

Substance use epidemiological model (written in Python) by Yap et al.
(2024) Assembly of Substance Use Epidemiological
Models.

optional arguments:

```

-h, --help      show this help message and exit
-S S            Percentage of population that is susceptible without health
education (S). Default=49.9939
-C C            Percentage of population that is susceptible with health
education (S). Default=50.000
-L L            Percentage of population that is light drug user (L).
Default=0.005
-H H            Percentage of population that is heavy drug user (H).
Default=0.0005
-Ti TI          Percentage of population that is under in-patient treatment
(Ti). Default=0.0002
-To TO          Percentage of population that is under out-patient treatment
(To). Default=0.0003
-D D            Percentage of population that is drug sellers (D).
Default=0.0001
-Re RE          Percentage of population that is in remission (Re).
Default=0.0005

```

-R R Percentage of population that is dead or removed (R).
Default=0.0

-Q Q Percentage of population that permanently quitted drug use
(Q). Default=0.0

-M M Percentage of population that matured from susceptible (M).
Default=0.0

-p P Recruitment rate from general population into susceptible
population without health education (S).
Default=0.05

-q Q Recruitment rate from general population into susceptible
population with health education (C).
Default=0.15

-k1 K1 Rate at which susceptible population without health education
(S) become light drug users (L) without
the effects of drug barons (D). Default=0.2

-k2 K2 Rate at which light users (L) escalates to heavy drug use
(H). Default=0.5

-k3 K3 Proportion of heavy drug users (H) exposed to police search.
Default=0.4

-k4 K4 Proportion of light drug users (L) exposed to police search.
Default=0.2

-k5 K5 Intensity of policing / police search. Default=1.00

-k6 K6 Rate of relapse from remission (Re) to light drug use (L).
Default=0.05

-k7 K7 Rate of relapse from remission (Re) to heavy drug use (H).
Default=0.01

-k8 K8 Rate of susceptible population without health education (S)
become heavy drug users (H) without the
effects of drug barons (D). Default=0.01

-k9 K9 Availability of drugs in the system. Default=1.00

-k10 K10 Rate at which susceptible population without health education
(S) accepts health education (C).
Default=0.3

-k11 K11 Rate at which susceptible population with health education
(C) become light drug users (L) without the
effects of drug barons (D). Default=0.1

-k12 K12 Rate of susceptible population with health education (C)
become heavy drug users (H) without the effects
of drug barons (D). Default=0.001

-b1 B1 Proportion of light drug users (L) caught for in-patient
treatment (Ti). Therefore, the proportion of
light drug users caught for out-patient treatment (To) is (1-
b1). Default=0.2

-b2 B2 Proportion of heavy drug users (H) caught for in-patient
treatment (Ti). Therefore, the proportion of
heavy drug users caught for out-patient treatment (To) is (1-
b2). Default=0.8

-g1 G1 Rate at which light users (L) quit and become susceptible
without health education (S) again.
Default=0.2

-g2 G2 Rate at which heavy users (H) become light users (L), which
includes amelioration. Default=0.4

-g3 G3 Rate at which in-patient treatment (Ti) reverted to heavy
drug use (H). Default=0.01

-g4 G4 Rate at which in-patient treatment (Ti) reverted to light
drug use (L). Default=0.02

-g5 G5 Rate at which in-patient treatment (Ti) enter remission (Re).
Default=0.2

-g6 G6 Proportion of light drug users (L) entering remission (Re) on
their own accord. Default=0.015

-g7 G7 Rate at which out-patient treatment (To) reverted to heavy drug use (H). Default=0.015
 -g8 G8 Rate at which out-patient treatment (To) reverted to light drug use (L). Default=0.025
 -g9 G9 Rate at which out-patient treatment (To) enter remission (Re). Default=0.2
 -a1 A1 Effective contact rate between drug barons (D) and susceptible population without health education (S). Default=0.4
 -a2 A2 Rate at which light users (L) convert from consumer to seller / promoter (D). Default=0.04
 -a3 A3 Rate at which heavy users (H) convert from consumer to seller / promoter (D). Default=0.08
 -a4 A4 Effective contact rate between drug barons (D) and susceptible population with health education (S). Default=0.2
 -r1 R1 Per capita mortality rate of population. Default=0.2
 -r2 R2 Removal rate of heavy users (H) due to events related to drug usage. Default=0.001
 -r3 R3 Removal rate of rehabilitated users (T) due to events related to drug usage. Default=0.003
 -r4 R4 Rate at which in-patient treatment (Ti) permanently quit (Q). Default=0.1
 -r5 R5 Removal rate of drug barons (D), which constitutes mainly to law enforcement. Default=0.02
 -r6 R6 Rate of susceptible without health education (S) maturing into non-susceptible (M). Default=0.005
 -r7 R7 Rate of light users (L) quitting drug use permanently (Q). Default=0.01
 -r8 R8 Rate at which out-patient treatment (To) permanently quit (Q). Default=0.1
 -r9 R9 Rate of susceptible with health education (C) maturing into non-susceptible (M). Default=0.01
 -c1 C1 Rate of out-patient treatment (To) entering in-patient treatment (Ti). Default=0.001
 -c2 C2 Rate of in-patient treatment (Ti) entering out-patient treatment (To). Default=0.01
 -step STEP Simulation time step. Default=0.00274
 -end END Simulation end time. Default=10.0

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