Sólido de Revolución

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Contorno de la botella con Desmos

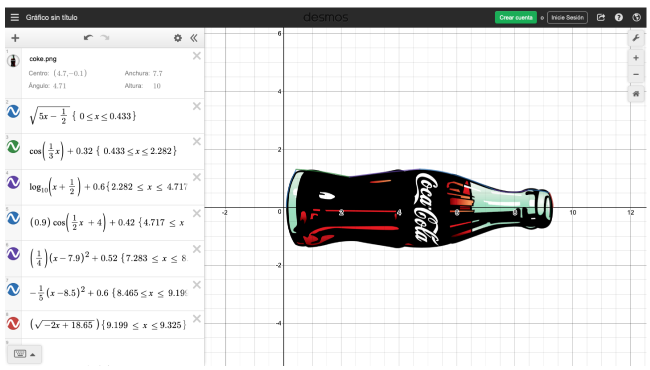

Figura 1: Funciones con la imagen

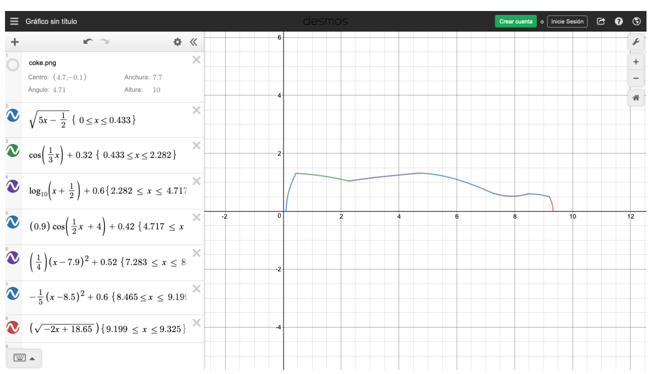

Figura 2: Contorno de la botella

Sólido de revolución en Mathematica

Código fuente en Mathematica

```
coke = Piecewise[
  {
    {Sqrt[5x-(1/2)],   0 <= x <= 0.433},
    {Cos[1/3x]+0.32, 0.433 <= x <= 2.282},
    {Log10[x+1/2]+0.6, 2.282 <= x <= 4.717},
    {(0.9)Cos[1/2x + 4] + 0.42, 4.717 <= x <= 7.283},
    {(1)/(4) (x-7.9)^(2)+0.52, 7.283 <= x <= 8.465},
    {(-1)/(5) (x-8.5)^(2)+0.6, 8.465 <= x <= 9.199},
    {Sqrt[-2x+18.65], 9.199 <= x <= 9.325}
  }
];

RevolutionPlot3D[
    coke,
    {x, 0, 9.325},
    RevolutionAxis->"X"
]
```

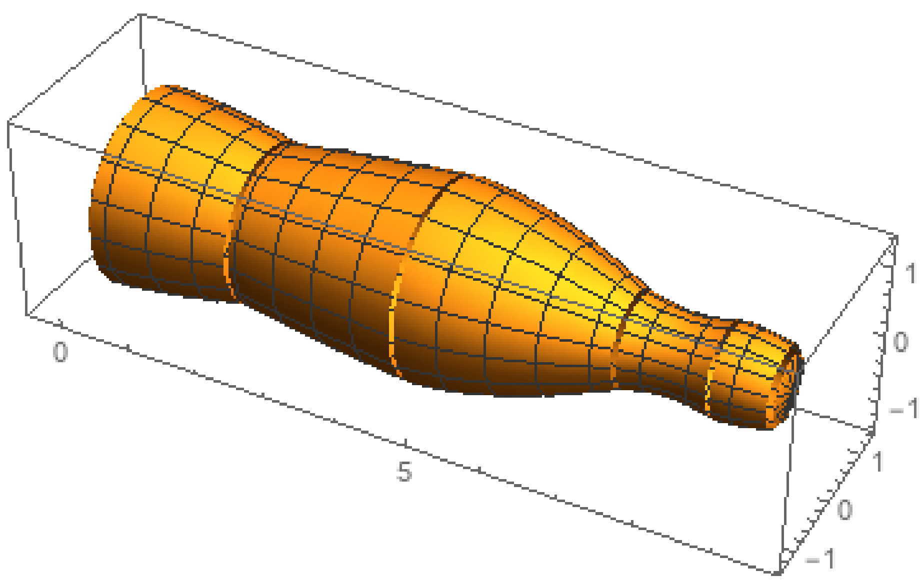

Figura 3: Funciones con la imagen

Calculando los volúmenes de cada función por el método del disco

1. $\sqrt{5x - \frac{1}{2}}$

$$v = \pi \int_{0}^{0.433} (\sqrt{5x - \frac{1}{2}})^2 dx$$

Simplificando:

$$= \pi \int 5x - \frac{1}{2} dx$$

$$= \pi (\int 5x dx - \int \frac{1}{2} dx)$$

$$= \pi (5 \int x dx - \frac{1}{2} \int 1 dx)$$

$$= 5\pi \int x dx - \frac{\pi}{2} \int 1 dx$$

Integrando:

$$= 5\pi \frac{x^2}{2} - \frac{\pi}{2} x$$

Simplificando:

$$= \frac{5\pi x^2 - \pi x}{2}$$

$$= \frac{\pi x(5x - 1)}{2}$$

Sustituyendo intervalos:

$$= \frac{\pi(0.433)(5(0.433) - 1)}{2} - \frac{\pi 0(5(0) - 1)}{2}$$

$$= \frac{\pi(0.433)(5(0.433) - 1)}{2}$$

$$= \frac{100889\pi}{400000}$$

Por lo tanto, el resultado es:

$$\frac{100889\pi}{400000} u^3$$

O aproximadamente $0.7923u^3$

2. $\cos\left(\frac{x}{3}\right) + 0.32$

$$v = \pi \int_{0.433}^{2.282} (\cos\left(\frac{x}{3}\right) + 0.32)^2 dx$$

Simplificando:

$$= \pi \int \cos^2\left(\frac{x}{3}\right) + 0.64\cos\left(\frac{x}{3}\right) + 0.1024dx$$

$$= \pi(\int \cos^2\left(\frac{x}{3}\right) dx + \int 0.64\cos\left(\frac{x}{3}\right) dx + \int 0.1024dx)$$

Integrando $\int \cos^2\left(\frac{x}{3}\right) dx$:

$$= 3\int \cos^2(u)du$$

$$= 3(\frac{2-1}{2}\int \cos^0(u)du + \frac{\cos(u)\sin(u)}{2})$$

Simplificando e integrando:

$$= 3(\frac{1}{2}\int 1du + \frac{\cos(u)\sin(u)}{2})$$

$$= 3(\frac{u}{2} + \frac{\cos(u)\sin(u)}{2})$$

$$= \frac{3u}{2} + \frac{3\cos(u)\sin(u)}{2}$$

$$= \frac{3u + 3\cos(u)\sin(u)}{2}$$

Sustituyendo u:

$$= \frac{3(\frac{x}{3}) + 3\cos(\frac{x}{3})\sin(\frac{x}{3})}{2}$$

Simplificando:

$$= \frac{x}{2} + \frac{3}{4}\sin\left(\frac{2x}{3}\right)$$

Ahora integrando $\int 0.64\cos\left(\frac{x}{3}\right) dx$:

$$= \frac{48}{25}\int \cos(u)du$$

$$= \frac{48}{25}\sin(u)$$

Sustituyendo u:

$$= \frac{48}{25}\sin(\frac{x}{3})$$

Por último integrando $\int 0.1024dx$:

Simplificando:

$$= \int \frac{64}{625}dx$$

Integrando:

$$= \frac{64}{625}x$$

Sustituyendo las integrales resueltas:

$$= \pi(\frac{x}{2} + \frac{3}{4}\sin\left(\frac{2x}{3}\right) + \frac{48}{25}\sin(\frac{x}{3}) + \frac{64}{625}x)$$

Sustituyendo intervalos:

$$= \pi(\frac{2.282}{2} + \frac{3}{4}\sin\left(\frac{2(2.282)}{3}\right) + \frac{48}{25}\sin(\frac{2.282}{3}) + \frac{64}{625}(2.282))$$

$$-(\pi(\frac{0.433}{2} + \frac{3}{4}\sin\left(\frac{2(0.433)}{3}\right) + \frac{48}{25}\sin(\frac{0.433}{3}) + \frac{64}{625}(0.433)))$$

Por lo tanto, el resultado es:

$$8.4726u^3$$

3. $\frac{\ln\left(x+\frac{1}{2}\right)}{\ln(10)}+0.6$

$$v=\pi\int_{4.717}^{2.282}\left(\frac{\ln\left(x+\frac{1}{2}\right)}{\ln(10)}+0.6\right)^2dx$$

Tomemos $u=x+\frac{1}{2}$ y $du=dx$, los limites cambian a $u=\frac{1}{2}+2.282=2.782$ y $u=\frac{1}{2}+4.717=5.217$

$$\int_{2.782}^{5.217}\left(\frac{\log u}{log10}+\frac{3}{5}\right)^2du=\int_{2.782}^{5.217}\left(\frac{\log^2 u}{\log^2 10}+\frac{6\log u}{5\log 10}+\frac{9}{25}\right)du=...$$

$$...=\frac{6}{5\log 10}\int_{2.782}^{5.217}(\log u)du+\frac{1}{\log^2 10}\int_{2.782}^{5.217}(\log^2 u)du+\frac{9}{25}\int_{2.782}^{5.217}(1)du=...$$

$$...=\left.\frac{6u\log u}{5\log 10}\right|_{2.782}^{5.217}+\left(\frac{9}{25}-\frac{6}{5\log 10}\right)\int_{2.782}^{5.217}(1)du+\frac{1}{\log^2 10}\int_{2.782}^{5.217}(\log^2 u)du=...$$

$$...=3.0079+\left.\left(u\left(\frac{9}{25}-\frac{6}{5\log 10}\right)\right)\right|_{2.782}^{5.217}+\frac{1}{\log^2 10}\int_{2.782}^{5.217}(\log^2 u)du=...$$

$$...=2.61549+\left.\frac{u\log^2 u}{\log^2 10}\right|_{2.782}^{5.217}-\frac{2}{\log^2 10}\int_{2.782}^{5.217}(\log u)du=...$$

$$...=4.75133+\left.\left(-\frac{2u\log u}{\log^2 10}\right)\right|_{2.782}^{5.217}+\frac{2}{\log^2 10}\int_{2.782}^{5.217}(1)du=...$$

$$...=2.57414+\left.\frac{2u}{\log^2 10}\right|_{2.782}^{5.217}=3.49268$$

Ahora multiplicamos el resultado por la constante:

$3.49268\cdot\pi=10.972577829339999$

Por lo tanto, el volumen es aproximadamente $10.9725u^3$

4. $0.9\cos\left(\frac{x}{2}+4\right)+0.42$

$$v=\pi\int_{7.283}^{4.717}(0.9\cos\left(\frac{x}{2}+4\right)+0.42)^2dx$$

Simplificando:

$$=\pi\int(\frac{9\cos\left(\frac{x}{2}+4\right)}{10}+\frac{21}{50})^2dx$$

$$=\frac{9}{2500}\pi\int(15\cos(\frac{x}{2}+4)+7)^2$$

Integrando $\int(15\cos(\frac{x}{2}+4)+7)^2$:

$$=2\int(15\cos(u)+7)^2$$

Resolviendo $\int(15\cos(u)+7)^2$:

$$=\int 225\cos^2(u)+210\cos(u)+49dx$$

$$=225\int\cos^2(u)dx+210\int\cos(u)dx+49\int 1dx$$

Integrando $225\int\cos^2(u)dx$:

$$=\frac{\cos(u)\sin(u)}{2}+\frac{1}{2}\int 1du$$

$$=\frac{\cos(u)\sin(u)}{2}+\frac{1}{2}u$$

Ahora integramos $\int\cos(u)du$:

$$=\sin(u)$$

Reemplazando las integrales resueltas de $\int(15\cos(u)+7)^2$:

$$=225(\frac{\cos(u)\sin(u)}{2}+\frac{1}{2}u)+210\sin(u)+49u$$

$$=225\frac{\cos(u)\sin(u)}{2}+\frac{225}{2}u+210\sin(u)+49u$$

$$=225\frac{\cos(u)\sin(u)}{2}+210\sin(u)+\frac{323}{2}u$$

Reemplazamos las integrales resueltas en $2\int(15\cos(u)+7)^2$

$$=2(225\frac{\cos(u)\sin(u)}{2}+210\sin(u)+\frac{323}{2}u)$$

$$= 225\cos(u)\sin(u) + 420\sin(u) + 323u)$$

Deshaciendo la sustitución:

$$= 225\cos(\frac{x}{2}+4)\sin(\frac{x}{2}+4) + 420\sin(\frac{x}{2}+4) + 323(\frac{x}{2}+4))$$

Reemplazando la integral resuelta en $\frac{9}{2500}\pi\int(15\cos(\frac{x}{2}+4)+7)^2$

$$= \frac{9}{2500}\pi(225\cos(\frac{x}{2}+4)\sin(\frac{x}{2}+4) + 420\sin(\frac{x}{2}+4) + 323(\frac{x}{2}+4)))$$

Sustituyendo los intervalos:

$$= \frac{9}{2500}\pi(225\cos(\frac{7.283}{2}+4)\sin(\frac{7.283}{2}+4)+420\sin(\frac{7.283}{2}+4)+323(\frac{7.283}{2}+4))$$
$$-(\frac{9}{2500}\pi(225\cos(\frac{4.717}{2}+4)\sin(\frac{4.717}{2}+4)+420\sin(\frac{4.717}{2}+4)+323(\frac{4.717}{2}+4)))$$

Por lo tanto, el volumen es igual a:

$$\frac{370287}{125000}\pi u^3$$

O aproximadamente:

$$9.3063u^3$$

5. $\frac{1}{4}(x-7.9)^2+0.52$

$$v=\pi\int_{4.717}^{7.283}(\frac{1}{4}(x-7.9)^2+0.52)^2dx$$

Expandiendo:

$$=\pi(\int\frac{x^4}{16}-\frac{79x^3}{40}+\frac{18923x^2}{800}-\frac{508839x}{4000}+\frac{41486481}{160000}dx)$$

Simplificando:

$$=\pi(\frac{1}{16}\int x^4dx-\frac{79}{40}\int x^3dx+\frac{18923}{800}\int x^2dx-\frac{508839}{4000}\int xdx+\frac{41486481}{160000}\int 1dx)$$

Resolviendo las integrales:

$$=\pi(\frac{1}{16}(\frac{x^5}{5})-\frac{79}{40}(\frac{x^4}{4})+\frac{18923}{800}(\frac{x^3}{3})-\frac{508839}{4000}(\frac{x^2}{2})+\frac{41486481}{160000}x)$$

$$=\pi(\frac{x^5}{80}-\frac{79x^4}{160}+\frac{18923x^3}{2400}-\frac{508839x^2}{8000}+\frac{41486481}{160000}x)$$

Sustituyendo los intervalos:

$$=\pi(\frac{(8.465)^5}{80}-\frac{79(8.465)^4}{160}+\frac{18923(8.465)^3}{2400}-\frac{508839(8.465)^2}{8000}+\frac{41486481}{160000}(8.465))$$

$$-(\pi(\frac{(7.283)^5}{80}-\frac{79(7.283)^4}{160}+\frac{18923(7.283)^3}{2400}-\frac{508839(7.283)^2}{8000}+\frac{41486481}{160000}(7.283)))$$

Por lo tanto, el resultado es:

$$\frac{13277654599067991}{40000000000000000}\pi u^3$$

O aproximadamente:

$$1.0428u^3$$

6. $0.6-\frac{1}{5}(x-8.5)^2$

$$v=\pi\int_{8.465}^{9.199}(0.6\,-\frac{1}{5}(x-8.5)^2)^2dx$$

Simplificando:

$$=\pi\int\left(\frac{3}{5}-\frac{\left(x-\frac{17}{2}\right)^2}{5}\right)^2\mathrm{d}x$$

$$=\frac{1}{400}\pi\int((2x-17)^2-12)^2dx$$

Sustituyendo por u:

$$=\frac{1}{400}\frac{1}{2}\pi\int(u^2-12)^2du$$

$$=\frac{1}{800}\pi\int(u^2-12)^2du$$

Integrando $\int(u^2-12)^2du$

Expandiendo:

$$=\int\left(u^4-24u^2+144\right)\mathrm{d}u$$

Simplificando:

$$=\int u^4du-24\int u^2du+144\int 1du$$

Resolviendo las integrales:

$$=\frac{u^5}{5}-24(\frac{u^3}{3})+144u$$

$$=\frac{u^5}{5}-8u^3+144u$$

Reemplazando las integrales resueltas en $\frac{1}{800}\pi\int(u^2-12)^2du$

Deshaciendo la sustitución:

$$=\frac{1}{800}\pi(\frac{(2x-17)^5}{5}-8(2x-17)^3+144(2x-17))$$

Simplificando:

$$=\frac{(2x-17)^5\pi}{4000}-\frac{8\pi(2x-17)^3}{800}+\frac{144\pi(2x-17)}{800}$$

Sustituyendo los intervalos:

$$= \frac{(2(9.199)-17)^5\pi}{4000} - \frac{8\pi(2(9.199)-17)^3}{800} + \frac{144\pi(2(9.199)-17)}{800}$$

$$-(\frac{(2(8.465)-17)^5\pi}{4000} - \frac{8\pi(2(8.465)-17)^3}{800} + \frac{144\pi(2(8.465)-17)}{800})$$

Por lo tanto, el resultado es:

$$\frac{14890561618812687\pi}{62500000000000000}u^3$$

O aproximadamente $0.7484u^3$

7. $\sqrt{18.65 - 2x}$

$$v = \pi \int_{9.199}^{9.325} (\sqrt{18.65 - 2x})^2 dx$$

Simplificando:

$$= \pi \int 18.65 - 2x dx$$

$$= \pi (18.65 \int 1 dx - 2 \int x dx)$$

Resolviendo las integrales:

$$= \pi (18.65x - 2\frac{x^2}{2} dx)$$

Simplificando:

$$= \pi (18.65x - x^2)$$

$$= 18.65\pi x - \pi x^2$$

$$= \pi x(18.65 - x)$$

Sustituyendo los intervalos:

$$= 9.325\pi(18.65 - 9.325) - (9.199\pi(18.65 - 9.199))$$

Por lo tanto, el resultado es:

$$\frac{3969\pi}{250000}$$

Lo que es aproximadamente: $0.0498u^3$

Volumen total de la botella

Para calcular el volumen total, solo basta con sumar los individuales:

$$0.7923 + 8.4726 + 10.9725 + 9.3063 + 1.0428 + 0.7484 + 0.0498$$

Lo que nos da como resultado 31.3847, por lo que el volumen total de la botella es $31.3847u^3$