X complex conjugate pairs

$\mathbb{R}[X]$

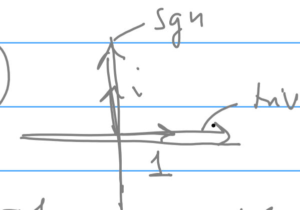
$\ell : \mathbb{R}[X] \longrightarrow \mathbb{R}$

$\left[\ell(q^2) \geq 0 \quad \forall q \text{ to } q \in A \subseteq \mathbb{R}[x]\right]$

Q corresponds to then have $1+$, $1-$

$$\left[\ell(f) = \sum_i \left[\underbrace{\left[y_i f(z_i) + \overline{y_i} f(\overline{z_i})\right]}\right]_p\right]$$

$\mathbb{C}$



$$\frac{\mathbb{R}[x]}{(x^2+1)} \rightsquigarrow \frac{\mathbb{C}[x]}{(x^2+1)} \cong \mathbb{C} \oplus \mathbb{C}$$

$\overline{1} = 1$

$\overline{i} = -i$

$K \supseteq \mathbb{Q}$ galois

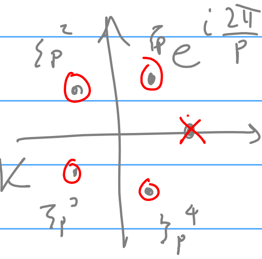
X depend on $\mathbb{Q}$, because distinct points in $K \supseteq \mathbb{Q}$

$G := Gal(K/\mathbb{Q})$, Assume X is a single orbit.

<u>Example:</u> p-prime $\frac{\mathbb{Q}[x]}{(1+x+x^2+\dots+x^{p-1})}$

S_n!

because distinct in $\mathbb{Q}(\zeta_p) = K$



$$Gal(\mathbb{Q}(\zeta_p), \mathbb{Q}) = U(\mathbb{Z}/p\mathbb{Z})$$

$\underbrace{0, 1, \dots, p-1}_{j}$

$[\sigma_j(\zeta_p) = \zeta_p^j]$

σ_j $\mathbb{Q}$-hom.

$\ell : \mathbb{Q}[X] \longrightarrow \mathbb{Q}$

$\alpha_i \in K$

$$\ell = \sum_i \left[\underbrace{\sum_{g \in G} \left[g \circ \alpha_i f(g^{-1}(z_i))\right]}_{\text{blocks ?}}\right]$$

$\alpha_i \in \mathbb{Q}$

$|Gal|$ choices

<u>Example:</u> $p = 7$

$$\left[\sum f(g^{-1}(z_i))\right] \approx \text{Rational pt } (ev_{z_i})$$

$$
\begin{array}{c|c}
 & 1 \quad \zeta \quad \cdots \quad \zeta^5 \\
\hline
\begin{array}{c} 1 \\ \zeta \\ \vdots \\ \zeta^5 \end{array} &
\begin{bmatrix}
1 & \zeta & \cdots & \zeta^5 \\
 & \zeta^2 & \cdots & \zeta^6 \\
 & & \ddots & \\
 & & &
\end{bmatrix}
\end{array}
\xrightarrow{ev_\zeta} \sum_{g \in G_{al}} g\left(\quad \right) =
\left[\begin{array}{cc|cccc}
5 & -1 & -1 & \cdots & & -1 \\
-1 & -1 & -1 & \cdots & & -1 \\
\hline
-1 & -1 & -1 & -1 & 5 & \\
-1 & -1 & -1 & -1 & 5 & \\
\vdots & \vdots & & & \ddots & \\
 & & 5 & & -1 & -1
\end{array}\right]
$$

$$
\begin{bmatrix} \underbrace{2+} & \underbrace{2-} \end{bmatrix}
$$