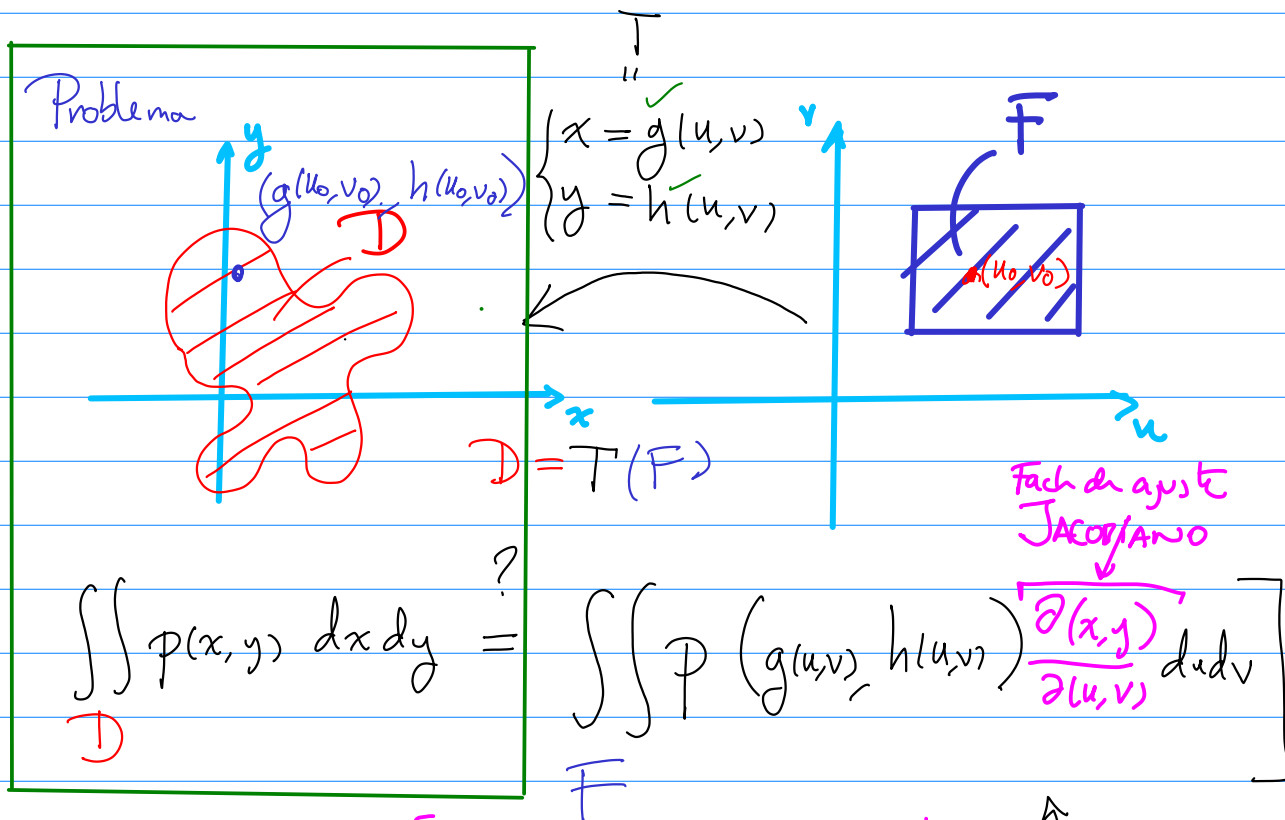


Hoy: (1) Teorema del cambio de variable

(2) "elección de coordenadas"



$$\frac{\partial(x, y)}{\partial(u, v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

Teorema: [del cambio de variable]

Si $T: \mathbb{R}^2_{(u, v)} \rightarrow \mathbb{R}^2_{(x, y)}$

y $T(F) = D$, entonces

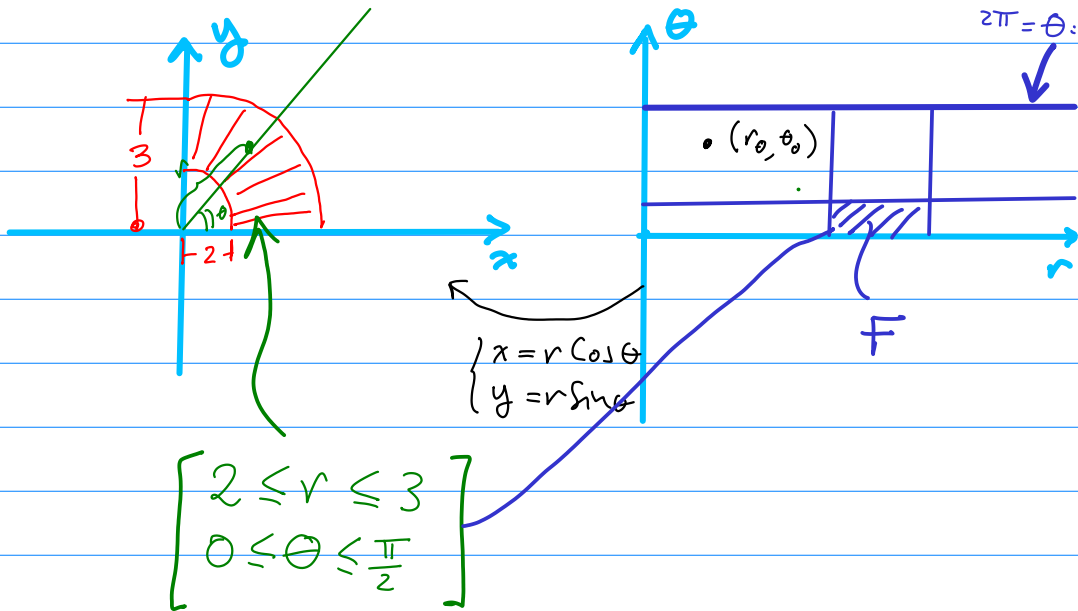
es diferenciable y 1-1

(1)

es válida.

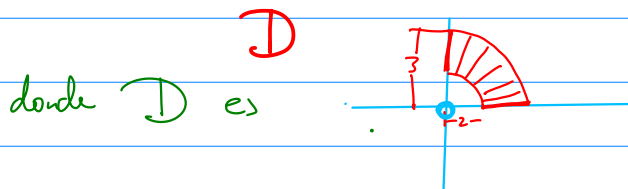
Ejemplo de transformación: $\mathbb{R}^2_{(x,y)} \xleftarrow{T} \mathbb{R}^2_{(\theta,r)}$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad \text{"Coordenadas polares"}$$



Ejercicio:

Calcule $\iint_D x+y \, dA =$



$$\frac{\partial(x,y)}{\partial(r,\theta)} = \det \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix}$$

Jacobiano
cambio a
polares

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad \det \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix} =$$

$$|r \cos^2 \theta - (-r \sin^2 \theta)| =$$

$$|r [\cos^2 \theta + \sin^2 \theta]| = |r| = r$$

$$\iint_D x+y \, dA = \int_0^{\frac{\pi}{2}} \int_2^3 (r \cos \theta + r \sin \theta) \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_2^3 r^2 [\cos \theta + \sin \theta] \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left([\cos \theta + \sin \theta] \int_2^3 r^2 \, dr \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} [\cos \theta + \sin \theta] \left[\frac{3^3}{3} - \frac{2^3}{3} \right] d\theta$$

$$= \left[\frac{3^3}{3} - \frac{2^3}{3} \right] \cdot (\sin \theta - \cos \theta) \Big|_{\theta=0}^{\theta=\frac{\pi}{2}}$$

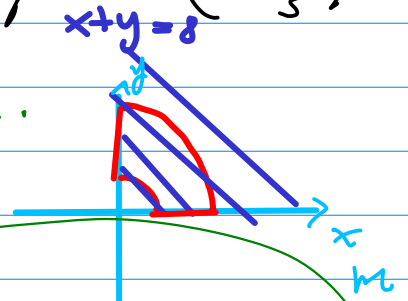
$$= \left[\frac{3^3}{3} - \frac{2^3}{3} \right] (1 - (-1)) =$$

$$= \left(\frac{3^3}{3} - \frac{2^3}{3} \right) \cdot 2 = \left(3^2 - \frac{2^3}{3} \right) \cdot 2 = \left(9 - \frac{8}{3} \right) \cdot 2$$

$$= \left[18 - \frac{16}{3} \right]$$

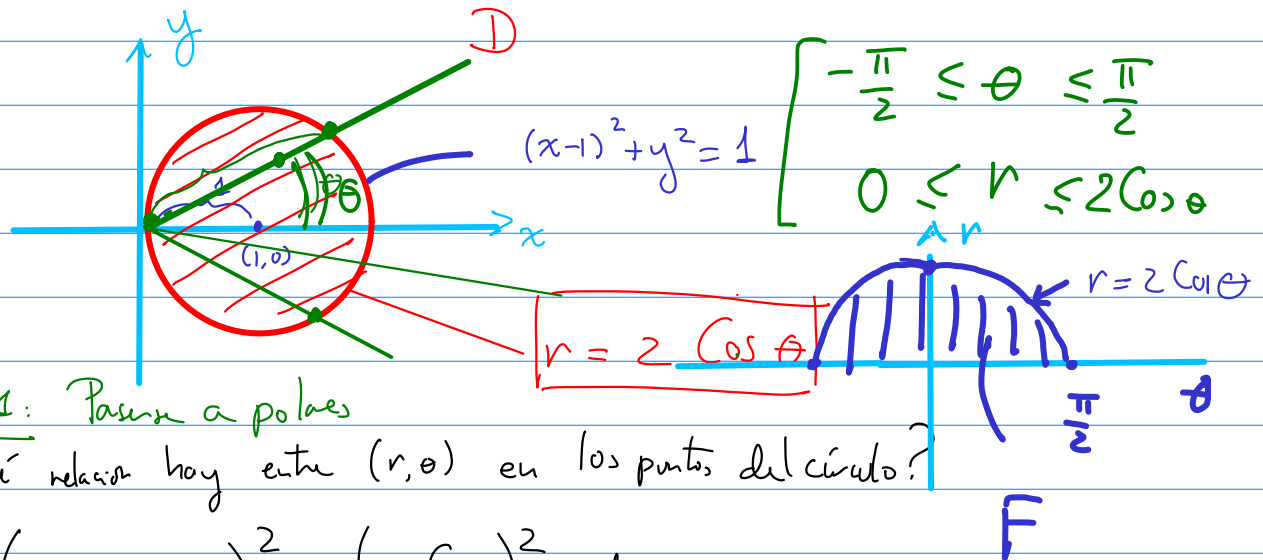
Respecto Kg.

$$\iint_D x+y \, dA$$



"Un plato tiene la forma de la región D y la densidad en Kg/m^2 está dada por $f(x,y) = x+y$. Calcule la masa total?"

Ejercicio: Calcular $\iint_D x^2 + y^2 dA = ?$



Sol 1: Pasen a polares

¿Qué relación hay entre (r, θ) en los puntos del círculo?

$$(r \cos \theta - 1)^2 + (r \sin \theta)^2 = 1$$

$$r^2 [\cos^2 \theta + \sin^2 \theta] - 2r \cos \theta + 1 = 1$$

$$r^2 - 2r \cos \theta = 0$$

$$[r(r - 2 \cos \theta) = 0]$$

$$\iint_D x^2 + y^2 dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} r^2 \cdot r dr d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{r^4}{4} \Big|_{r=0}^{r=2 \cos \theta} \right) d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^4}{4} \cos^4 \theta d\theta =$$

$$= 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta \, d\theta =$$

$$\begin{cases} 1 = \cos^2 \theta + \sin^2 \theta \\ \cos(2\theta) = \cos^2 \theta - \sin^2 \theta \end{cases}$$

$$\left[\frac{1 + \cos(2\theta)}{2} = \cos^2 \theta \right]$$

$$= 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1 + \cos(2\theta)}{2} \right)^2 d\theta$$

$$= 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + 2\cos(2\theta) + \cos^2(2\theta)}{2} d\theta$$

$$= \pi + 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(2\theta) d\theta + \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(2\theta) d\theta}_{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + \cos(4\theta)}{2} d\theta}$$

$$= \pi + \left. \sin(4\theta) \right|_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} + ()$$

$$= \pi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{z} + \frac{\cos(4\theta)}{2} \right) d\theta$$

$$= \pi + \frac{\pi}{2} + \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(4\theta) d\theta$$

error
enclose

$$- \left. \frac{\sin(4\theta)}{4} \right|_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}$$

$$= \boxed{\frac{3\pi}{2}}$$

Sol 2:

$$\begin{cases} x = r \cos \theta + 1 \\ y = r \sin \theta \end{cases}$$

$$\begin{cases} 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \end{cases}$$