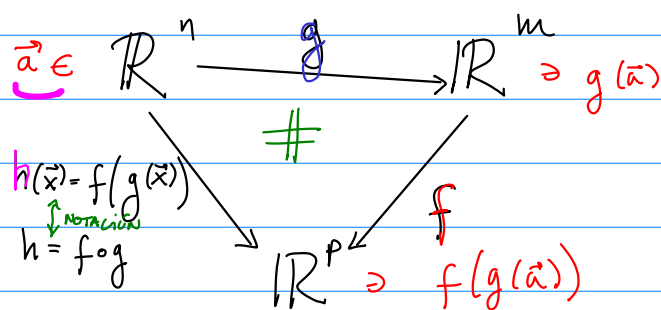


Hoy: Regla de la cadena = Método para calcular derivadas de composiciones.



Cómo se calcula $Dh(\vec{a})$ en términos de $Df(\vec{m})$ y $Dg(\vec{a})$?

Clase anterior:

Si $f: \mathbb{R}^a \rightarrow \mathbb{R}^b$ es diferenciable en $\vec{u} \in \mathbb{R}^a$ su derivada es una matriz

$$Df(\vec{u}) = \begin{bmatrix} f_1 & \dots & x_1 & x_2 & \dots & x_a \\ \vdots & & \vdots & \vdots & & \vdots \\ f_b & & \vdots & \vdots & & \vdots \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\vec{u}) & \dots & \frac{\partial f_1}{\partial x_a}(\vec{u}) \\ \vdots & & \vdots \\ \frac{\partial f_b}{\partial x_1}(\vec{u}) & \dots & \frac{\partial f_b}{\partial x_a}(\vec{u}) \end{bmatrix}$$

$b \times a$

La derivada de f EVALUADA en \vec{u} .

Teorema [Regla de la cadena] Suponga que

(i) g es diferenciable en \vec{a}

(ii) f es diferenciable en $g(\vec{a})$

entonces la composición $h(x) = f(g(x))$ cumple:

(i) h es diferenciable en \vec{a}

* (ii) Su derivada en \vec{a} es:

$$Dh(\vec{a}) = Df(g(\vec{a})) \cdot Dg(\vec{a})$$

$p \times m$ $m \times n$

$p \times n$

Obs: Útil porque permite calcular $Dh(\vec{a})$ sin CONOCER $h(x)$ (e incluso sabiendo sólo las derivadas de f y de g en los lugares apropiados)

Obs: Ya conocen esta fórmula en el caso de una sola variable

Si $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

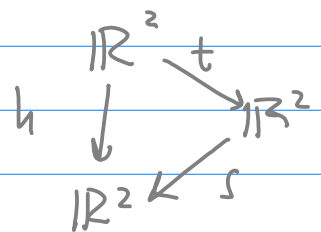
$h'(a) = f'(g(a)) \cdot g'(a)$

derivar "hacia" es la regla de la cadena

Ejercicio: $[t(x, y) = (x^2 + y^2, x^2 - y^2)]$
 $s(A, B) = (B^2, A - B)$

Defina $[r(u, v) := s(t(u, v))]$

Calcule $Dr(1, 2) \doteq$



Sol: $Dr(1, 2) = \underbrace{Ds(t(1, 2))}_{t: \mathbb{R}^n \rightarrow \mathbb{R}^m} \cdot \underbrace{Dt(1, 2)}$

$Dt(x, y) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$

$Dt(1, 2) = \begin{bmatrix} 2 & 4 \\ 2 & -4 \end{bmatrix}$

$s(A, B) = (B^2, A - B)$

$$Ds(A, B) = \begin{matrix} & A & B \\ \begin{matrix} B^2 \\ A-B \end{matrix} & \begin{pmatrix} 0 & 2B \\ 1 & -1 \end{pmatrix} \end{matrix}$$

$$t(1, 2) = (1^2 + 2^2, 1^2 - 2^2) = (\overset{A}{\underset{B}{5}}, \overset{A}{\underset{B}{-3}})$$

$$Ds(t(1, 2)) = \begin{pmatrix} 0 & -6 \\ 1 & -1 \end{pmatrix}.$$

$$Dr(1, 2) = \begin{pmatrix} 0 & -6 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 \\ 2 & -4 \end{pmatrix}$$

$$Dr(1, 2) = \begin{pmatrix} -12 & 24 \\ 0 & 8 \end{pmatrix}$$

$\begin{matrix} \downarrow u & \downarrow v \\ r_1 \rightarrow & r_2 \rightarrow \end{matrix}$

$$r(u, v) = (r_1(u, v), r_2(u, v))$$

$$\frac{\partial r_2}{\partial v}(1, 2) = 8$$

Ejercicio: Sea $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ una función escalar

$$[h(r, \theta) = f(r \cos \theta, r \sin \theta)]$$

Calcule $\frac{\partial h(r, \theta)}{\partial r} \overset{?}{=} \frac{\partial f}{\partial x}$ y $\frac{\partial h(r, \theta)}{\partial \theta} \overset{?}{=} \frac{\partial f}{\partial y}$ ($Dh(r, \theta) \overset{?}{=}$)
 es función de $\frac{\partial f}{\partial x}$ y $\frac{\partial f}{\partial y}$ ✓

Sugerencia: $g: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
 $(r, \theta) \longmapsto (r \cos \theta, r \sin \theta)$

$$[h(r, \theta) = f(g(r, \theta))]$$

Por regla de la cadena

$$Dh(r, \theta) = \underbrace{Df(g(r, \theta))}_{\text{?} \checkmark} \cdot \underbrace{Dg(r, \theta)}_{\text{?}}$$

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$\begin{matrix} x & y \\ x, y \end{matrix}$
 $f(x, y) \checkmark$

$$Df(x, y) = f \left[\underbrace{\frac{\partial f}{\partial x}(x, y)}_{\checkmark} \quad \underbrace{\frac{\partial f}{\partial y}(x, y)}_{\checkmark} \right]$$

$$Df(g(r, \theta)) = \left[\frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) \quad \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta) \right]$$

$$g(r, \theta) = (r \cos \theta, r \sin \theta)$$

$$Dg(r, \theta) = \begin{matrix} r & \theta \\ r \cos \theta & -r \sin \theta \\ r \sin \theta & r \cos \theta \end{matrix}$$

$$Dh(r, \theta) = \left[\frac{\partial f}{\partial x}(\cdot) \quad \frac{\partial f}{\partial y}(\cdot) \right] \cdot \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix}$$

$$\left[\frac{\partial h}{\partial r}(r, \theta), \frac{\partial h}{\partial \theta}(r, \theta) \right]$$

$$Dh(r, \theta) = \left[\cos \theta \left(\frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) \right) + \sin \theta \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta) \right]$$

$$-r \sin \theta \cdot \frac{\partial f}{\partial x}(\cdot) + r \cos \theta \cdot \frac{\partial f}{\partial y}(\cdot)$$

Resumiendo:

$$\begin{cases} \frac{\partial h}{\partial r}(r, \theta) = \frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) \cdot \cos \theta + \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta) \sin \theta \\ \frac{\partial h}{\partial \theta}(r, \theta) = \frac{\partial f}{\partial x}(\cdot) \cdot (-r \sin \theta) + \frac{\partial f}{\partial y}(\cdot) [r \cos \theta] \end{cases}$$