1. A

$$
\begin{aligned}
f(\beta) &= (y - X\beta)^T W (y - X\beta) \\
&= \beta^T X^T W X \beta - 2y^T W X \beta + \cdots \\
&= \beta^T X^T W X \beta - 2y^T W^T X^T (X^T W X)^{-1} X^T W X \beta + \cdots \\
&= (\beta - (X^T W X)^{-1} X^T W y)^T X^T W X (\beta - (X^T W X)^{-1} X^T W y) + \cdots
\end{aligned} \tag{1}
$$

2. A

$$
\begin{aligned}
l(\beta) &= -\log(\prod_i (y_i \omega_i(\beta) + (1 - y_i)(1 - \omega_i(\beta)))) \\
&= -\sum_i \log((2y_i - 1)\omega_i(\beta) + (1 - y_i))
\end{aligned} \tag{2}
$$

$$
\frac{\partial \omega_i}{\partial \beta_j} = \frac{x_{ij} \exp(-x_i \beta)}{(1 + \exp(-x_i \beta))^2} = x_{ij} \omega_i (1 - \omega_i)
$$

$$
\frac{\partial l}{\partial \beta_j} = \sum_i \frac{x_{ij} \omega_i (1 - \omega_i)(2y_i - 1)}{(2y_i - 1)\omega_i + (1 - y_i)}
$$

B See code.

C Write down the taylor expansion of l:

$$
l(\beta) = l(\beta_0) + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \frac{1}{2} \sum_{jk} \frac{\partial^2 l(\beta_0)}{\partial \beta_k \partial \beta_j} \beta_j \beta_k + \cdots
$$

Compute second order derivative:

$$
\begin{aligned}
&= \frac{\partial^2 l}{\partial \beta_k \partial \beta_j} \\
&= \sum_i \frac{x_{ij}(1 - 2\omega_i) x_{ik} \omega_i (1 - \omega_i)((2y_i - 1)\omega_i + (1 - y_i))(2y_i - 1) - (2y_i - 1)^2 x_{ik} x_{ij} \omega_i^2 (1 - \omega_i)^2}{((2y_i - 1)\omega_i + (1 - y_i))^2} \\
&= x_{ij} x_{ik} w_{ii}
\end{aligned} \tag{3}
$$

$$
w_{ii} = \omega_i (1 - \omega_i)(2y_i - 1) \frac{(1 - 2\omega_i)((2y_i - 1)\omega_i + 1 - y_i) - (2y_i - 1)\omega_i (1 - \omega_i)}{((2y_i - 1)\omega_i + (1 - y_i))^2}
$$

l can be written as a quadratic form of diagonal matrix W

$$
\begin{aligned}
l(\beta) &= l(\beta_0) + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \frac{1}{2} \sum_{jk} x_{ij} x_{ik} f_i(\beta_0) \beta_j \beta_k + \cdots \\
&= \frac{1}{2} \beta^T X^T W X \beta + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \cdots \\
&= (z - X\beta)^T W (z - X\beta) + \cdots
\end{aligned} \tag{4}
$$

Determine z:

$$
\begin{aligned}
z^T W X &= -\frac{\partial l(\beta_0)}{\partial \beta_i} \\
\sum_i z_i w_{ii} x_{ij} &= -\sum_i \frac{x_{ij}\omega_i(\beta_0)(1-\omega_i(\beta_0))(2y_i-1)}{(2y_i-1)\omega_i(\beta_0)+(1-y_i)} \\
z_i &= -\frac{\omega_i(\beta_0)(1-\omega_i(\beta_0))(2y_i-1)}{((2y_i-1)\omega_i(\beta_0)+(1-y_i))w_{ii}} \\
&= -\frac{((2y_i-1)\omega_i(\beta_0)+(1-y_i))(2y_i-1)}{(1-2\omega_i(\beta_0))((2y_i-1)\omega_i(\beta_0)+1-y_i)-(2y_i-1)^2\omega_i(\beta_0)(1-\omega_i(\beta_0))}
\end{aligned}
\tag{5}
$$