1. A

$$\begin{aligned} f(\beta) &= (y - X\beta)^T W (y - X\beta) \\ &= \beta^T X^T W X \beta - 2y^T W X \beta + \cdots \\ &= \beta^T X^T W X \beta - 2y^T W^T X^T (X^T W X)^{-1} X^T W X \beta + \cdots \\ &= (\beta - (X^T W X)^{-1} X^T W y)^T X^T W X (\beta - (X^T W X)^{-1} X^T W y) + \cdots \end{aligned} \tag{1}$$

2. A

$$\begin{aligned} l(\beta) &= -\log(\prod_i \omega_i^{y_i} (1 - \omega_i)^{1-y_i}) \\ &= -(\sum_i y_i \log \omega_i + (1 - y_i) \log(1 - \omega_i)) \end{aligned} \tag{2}$$

$$\frac{\partial \omega_i}{\partial \beta_j} = \frac{x_{ij} \exp(-x_i \beta)}{(1 + \exp(-x_i \beta))^2} = x_{ij} \omega_i (1 - \omega_i)$$

$$\frac{\partial l}{\partial \beta_j} = \sum_i (\frac{1 - y_i}{1 - \omega_i} - \frac{y_i}{\omega_i}) x_{ij} \omega_i (1 - \omega_i) = \sum_i (\omega_i - y_i) x_{ij}$$

B See code.

C Write down the taylor expansion of l:

$$l(\beta) = l(\beta_0) + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \frac{1}{2} \sum_{jk} \frac{\partial^2 l(\beta_0)}{\partial \beta_k \partial \beta_j} \beta_j \beta_k + \cdots$$

Compute second order derivative:

$$\begin{aligned} & \frac{\partial^2 l}{\partial \beta_k \partial \beta_j} \\ = & \sum_i \omega_i (1 - \omega_i) x_{ij} x_{ik} \end{aligned} \tag{3}$$

l can be written as a quadratic form of diagonal matrix W whose diagonal entries are $\omega_i(1 - \omega_i)$

$$\begin{aligned} l(\beta) &= l(\beta_0) + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \frac{1}{2} \sum_{jk} x_{ij} x_{ik} f_i(\beta_0) \beta_j \beta_k + \cdots \\ &= \frac{1}{2} \beta^T X^T W X \beta + \sum_j \frac{\partial l(\beta_0)}{\partial \beta_j} \beta_j + \cdots \\ &= \frac{1}{2} (z - X\beta)^T W (z - X\beta) + \cdots \end{aligned} \tag{4}$$

Determine z:

$$\begin{aligned} z^T W X &= -\frac{\partial l(\beta_0)}{\partial \beta_i} \\ \sum_i z_i \omega_i (1-\omega_i) x_{ij} &= -\sum_i (\omega_i - y_i) x_{ij} \\ z_i &= \frac{y_i - \omega_i}{\omega_i (1-\omega_i)} \qquad (5) \end{aligned}$$