Modeling Investor Optimism with Fuzzy Connectives

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Abstract— *Optimism or pessimism of investors is one of the important characteristics that determine the investment behavior in financial markets. In this paper, we propose a model of investor optimism based on a fuzzy connective. The advantage of the proposed approach is that the influence of different levels of optimism can be studied by varying a single parameter. We implement our model in an artificial financial market based on the LLS model. We find that more optimistic investors create more pronounced booms and crashes in the market, when compared to the unbiased efficient market believers of the original model. In the case of extreme optimism, the optimistic investors end up dominating the market, while in the case of extreme pessimism, the market reduces to the benchmark model of rational informed investors.*

Keywords— artificial financial market, agent-based modeling, behavioral finance, fuzzy aggregation, investor sentiment, optimism index.

1 Introduction

Artificial financial markets are models for studying the link between individual investor behavior and financial market dynamics. They are often computational models of financial markets, and are usually comprised of a number of heterogeneous and boundedly rational agents, which interact through some trading mechanism, while possibly learning and evolving. These models are built for the purpose of studying agents' behavior, price discovery mechanisms, the influence of market microstructure or the reproduction of the stylized facts of real-world financial time-series (e.g. fat tails of return distributions and volatility clustering).

A similar bottom-up approach has been utilized in agent-based computational economics (ACE) - the computational study of economies modeled as evolving systems of autonomous interacting agents [22]. A methodology analogous to agent-based modeling also comes from the physical sciences as the Macroscopic Simulation - a tool for studying complex systems by simulating many interacting microscopic elements [16]. A number of reviews of studies with artificial financial markets are available, e.g. [15] and [10].

Since agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics [4], such simulations are particularly suitable to test and generate various behavioral hypothesis. The idea of individual investors who are prone to biases in judgment, who are frame-dependent and use various heuristics, which might lead to anomalies on the market level, has been explored within the field of behavioral finance. Behavioral finance is the branch of finance which applies knowledge from psychology and sociology to discover and explain phenomena inconsistent with the paradigm of the expected utility of wealth and narrowly defined rational behavior [7]. A number of surveys and books on behavioral finance and behavioral economics topics can be found, for example [20], [8], and [2]. This complementarity of behavioral finance research and the agent-based methodology has been recognized in the literature (e.g. [15], [5]). Some of the early studies that pursue the idea of explicit accounting for behavioral theories in agent-based financial market simulations are [21] and [9]. In [21] the focus is on overconfidence and loss aversion, while [9] study the social interaction between investors.

One of the key characteristics that govern investor behavior is the optimism or pessimism of the investors. The link between asset valuation and investor sentiment has been the subject of considerable debate in the finance, and has been studied in the context of mispricing (departures from the fundamentals) [3], the limits of arbitrage [17], as well as the underreaction and overreaction of stock prices [1]. Two methodological approaches can be found in the finance literature. One is concerned with finding adequate proxies for the aggregate investor sentiment, and using them in statistical analysis to explain the variation of stock prices and the occurrences of mispricing, such as bubbles and crashes. The other one is a bottom-up approach that aims at modeling individual investor optimism and pessimism by using the insights from psychological theories. For these theories, it is important to have a flexible framework that can be adapted to capture the complexity of human decision making behavior.

In fuzzy decision theory, a wide range of connectives (aggregation operators) has been proposed and studied in order to model the flexibility of human decision making. In this sense, the use of fuzzy connectives for modeling elements of behavioral finance is promising, since the wide range of behaviors documented in the behavioral finance literature necessitates the use of a flexible framework for aggregating information. In this paper, we make a step in this direction by proposing a model of investor optimism based on fuzzy aggregation.

In probabilistic decision theory, such as the Prospect Theory [12, 23] and Rank-Dependent Utility Theory [19], optimism and pessimism are modeled using the probability weighting function. If, for example, the decision under risk is considered, a decision problem is presented using risky prospects, i.e. a set of possible outcomes and their probabilities. Because of the probability weighting, the decision weights asso-

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ciated with the outcomes are not equal to their probabilities (as would be in the case of Expected Value Theory or Expected Utility Theory). To model optimism we would need to specify and parameterize such a probability weighting function that gives more decision weight to good outcomes and less decision weight to bad outcomes. However, an empirically observed probability weighting function is usually S-shaped, which means that when dealing with such prospects, people are at the same time optimistic about the best outcomes, pessimistic about the worst outcomes, and insensitive to middle outcomes [25].

A decision maker's optimism or pessimism has also been studied within a fuzzy decision making setting. Various fuzzy connectives studied in this context have parameters that denote explicitly the optimism or pessimism degree of a decision maker. Apart from the well-known Hurwicz operator [11], the grade of compensation in Zimmermann–Zysno operator [26] can also be interpreted as an index of optimism. All these operators view the decision as a mixture of conjunctive and disjunctive behavior, and the degree of optimism determines which aggregation type dominates and to which degree. Another optimism–pessimism index was proposed in [24], where the parameter of the generalized averaging operator [6] is interpreted as the decision maker's characteristic degree of optimism. This is an intuitive way of modeling degree of optimism, since optimism is now modeled as the disposition of the decision maker to believe or give importance to positive events compared to his/her disposition to consider negative events [14]. An application of this operator in the risk management of power networks has been considered in [13].

In this paper, we propose a model of investor optimism based on the generalized averaging operator. The advantage of the proposed approach is that the influence of different levels of optimism can be studied by varying a single parameter. We study the effects of investor optimism in an artificial financial market based on the Levy, Levy, Solomon (LLS) model [16]. In previous publications, investor psychological biases such as overconfidence have also been studied by using this model [18].

The outline of the paper is as follows. Section 2 explains the basics of the LLS model in which we study the investor optimism and pessimism. Section 3 describes the setup of the experiments we have conducted. Section 4 presents the results of the simulations. Section 5 concludes the paper and discusses possible extensions for the future research.

2 Model Description

The proposed model of investor optimism is based on the LLS microscopic simulation model [16] with a small homogeneous subpopulation of efficient market believers (EMBs) as described in [16]. LLS model is a well-known and early econophysics model, rooted in a utility maximization framework. Variants of the model have been published in a number of articles and a book, and the model has also been critically evaluated in [27].

2.1 Asset Classes

As in the original LLS model, there are two investments alternatives: a risky stock (or market index) and a risk-free asset (bond). This is in line with many of the agent-based artificial financial markets, which typically do not deal with portfolio selection in multi-asset environments. The risky asset pays at the beginning of each period a dividend which follows a multiplicative random walk according to

$$\tilde{D}_{t+1} = D_t(1 + \tilde{z}), \tag{1}$$

where $\tilde{z}$ is a random variable distributed uniformly in the interval $[z_1, z_2]$. The bond pays interest with a rate of r_f.

2.2 Agent Behavior

Many early agent-based artificial financial markets were based on a small number of relatively simple strategies. Such markets have been labeled as *few-type models* [15]. Typically, strategies (or agents who employ them) could be divided into two groups: *fundamental* (based on a perceived fundamental value) and *technical* (based on the past prices, e.g. some form of trend extrapolation). *Zero-intelligence* framework in which agents trade randomly, might be useful for to studying the influence of market microstructure, and sometimes a small number of such agents is included into a few type model to provide liquidity for other agents.

LLS model follows a standard framework where preferences (and risk attitude) are captured by an agent's utility function, and the objective is the maximization of expected utility. But even in such a framework there are many possibilities for the functional form of the utility, which differ in descriptive validity and analytical tractability. When empirical support is taken into account, most evidence suggests DARA (Decreasing Absolute Risk Aversion) and CRRA (Constant Relative Risk Aversion), which motivates the choice of power (myopic) utility function in [16]

$$U(W) = \frac{W^{1-\alpha}}{1-\alpha}. \tag{2}$$

LLS model contains two types of investors: (1) Rational Informed Investors (RII) and (2) Efficient Market Believers (EMB).

2.2.1 RII investors

RII investors know the dividend process, and therefore can estimate fundamental value as the discounted stream of future dividend, according to the Gordon model

$$P^f_{t+1} = \frac{D_t(1+\tilde{z})(1+g)}{k-g}, \tag{3}$$

where k is the discount factor of the expected rate of return demanded by the market for the stock, and g is the expected growth rate of the dividend. RII investors assume that the price will converge to the fundamental value in the next period. In each period RII investor i chooses the proportion of wealth to invest in stocks and bonds so that he or she maximizes the expected utility of wealth in the next period, given by the following equation from [16]:

$$\begin{aligned} EU(\tilde{W}^i_{t+1}) &= \frac{(W^i_h)^{1-\alpha}}{(1-\alpha)(2-\alpha)} \frac{1}{(z_2-z_1)} \left(\frac{k-g}{k+1}\right) \frac{P_h}{xD_t} \\ &\times \left\{ \left[(1-x)(1+r_f) + \frac{x}{P_h}\left(\frac{k+1}{k-g}\right) D_t(1+z_2)\right]^{(2-\alpha)} \right. \\ &- \left. \left[(1-x)(1+r_f) + \frac{x}{P_h}\left(\frac{k+1}{k-g}\right) D_t(1+z_1)\right]^{(2-\alpha)} \right\}. \end{aligned} \tag{4}$$

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Based on the optimal proportion, they determine the number of stocks demanded by multiplying this optimal proportion with their wealth. Since all RII investors are assumed to have the same degree or risk aversion (parameter α), they will all have the same optimal proportion x. The actual number of demanded shares might differ only if investors differ in their wealth. However, as in the experiments of [16] we assume that they all start with the same initial wealth.

2.2.2 EMB investors

EMB investors believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use *ex post* distribution of stock returns to estimate the *ex ante* distribution. EMB investor i uses a rolling window of size m^i, and is in the original model [16] said to be *unbiased* if, in absence of additional information, he or she assigns the same probability to each of the past m^i return observations [16]. Hence, the original, unbiased EMBs assume that returns come from a discrete uniform distribution

$$\mathrm{Pr}^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{1}{m^i}, \text{ for } j = 1, ..., m^i. \tag{5}$$

The expected utility of EMB investor i is given by [16]

$$EU(\tilde{W}^i_{t+1}) = \frac{(W^i_h)^{1-\alpha}}{(1-\alpha)} \sum_{j=1}^{m^i} \mathrm{Pr}^i(\tilde{R}_{t+1} = R_{t-j}) \times \left[(1-x)(1+r_f) + xR_{t-j}\right]^{(1-\alpha)}. \tag{6}$$

In accordance with the LLS model [16], for all EMB investors an investor specific noise is added to the optimal investment proportion x^* (that maximizes the expected utility) in order to account for various departures from rational optimal behavior ($\tilde{\varepsilon}^i$ is truncated so that $0 \leq x^i \leq 1$, imposing the constraint of no borrowing and no short-selling), i.e.

$$x^i = x^{*i} + \tilde{\varepsilon}^i. \tag{7}$$

2.2.3 Sentiment EMBs

In this paper we create a new EMB type, called the *sentiment EMBs* by using a fuzzy set connective. *Sentiment EMBs* use generalized aggregation operator to estimate future returns, using the rolling window of size m^i. The prediction of the next period return for each investor i is given by

$$\tilde{R}_{t+1} = \left(\frac{1}{m^i} \sum_{j=1}^{m^i} (R_{t-j})^s \right)^{1/s}. \tag{8}$$

The higher the parameter s, the higher the estimate of the return (more closer to the maximum value from the sample), and vice versa. In such a way, we use parameter s to capture the phenomena of investor optimism and pessimism.

In our experiments we consider a several special cases of the generalized mean:

- $s \to -\infty$, the minimum of the sample;
- $s = -1$, the harmonic mean;
- $s \to 0$, the geometric mean;
- $s = 1$, the arithmetic mean;
- $s = 2$, the quadratic mean;
- $s \to \infty$, the maximum of the sample.

Since there is only one value for the expected return, instead of a probability distribution, the expected utility of sentiment EMB investor i is given by

$$EU(\tilde{W}^i_{t+1}) = \frac{(W^i_h)^{1-\alpha}}{(1-\alpha)} \left[(1-x)(1+r_f) + x\tilde{R}_{t+1}\right]^{(1-\alpha)}. \tag{9}$$

The investors will maximize this expected utility if in each period they invest all their wealth either in the stock or in the bond, depending on the actual comparison between the expected return on the stock $\tilde{R}_{t+1}$ and the return on the riskless bond $(1 + r_f)$.

2.3 Market Mechanism

LeBaron in [15] describes four types of market mechanisms used in agent-based artificial financial markets. In this paper, as in the original LLS model, we use clearing by *temporary market equilibrium*. RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the future price, which is in the current period unknown. Investors therefore need to determine optimal proportions, and respective demands for shares, for various hypothetical prices. The equilibrium price P_t is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares, according to

$$\sum_i N^i_h(P_t) = \sum_i \frac{x^i_h(P_t) W^i_h(P_t)}{P_t} = N. \tag{10}$$

Table 1: Parametrization

Symbol	Value	Explanation
M	950	Number of RII investors
M_2	50	Number of EMB investors
m	10	Memory length of EMB investors
α	1.5	Risk aversion parameter
N	10000	Number of shares
r_f	0.01	Riskless interest rate
k	0.04	Required rate of return on stock
z_1	-0.07	Maximal one-period dividend decrease
z_2	0.10	Maximal one-period dividend growth
g	0.015	Average dividend growth rate

3 Experiments with investor optimism

In the benchmark model where only RII investors are present in the market, there is no trade, the log prices follow random walk, and there is no excess volatility of the market price [16]. In the experiment with a small fraction of homogeneous (with respect to memory length) and unbiased EMB investors (of the original model), the market dynamics show semi-predictable (unrealistic) booms and crashes, with substantial trading in the

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market and excess volatility [16]. This experimental setup of [16] is also the basis for the experiments in this paper.

In our new model we conduct six experiments for six different levels of optimism of EMB investors, that correspond to the special cases of the parameter s. In each experiment the market consists of 95% RII investors and 5% EMB investors, with the parametrization given in Table 1. We run 100 independent 1000-period-long simulations, with different initial seeds of the random number generators. The results in the Table 2 are averaged over these 100 simulations.

Table 2: Results

	$s=-\infty$	$s=-1$	$s=0$
$\sigma(P)$	6.0249	12.8370	17.8668
$\sigma(P^f)$	5.7159	5.7159	5.7159
excess volatility %	5.41	124.59	212.58
mean volume p.p. %	0.48	9.04	6.40
	$s=1$	$s=2$	$s=\infty$
$\sigma(P)$	27.4739	28.8751	25.0327
$\sigma(P^f)$	5.7159	5.7159	5.7159
excess volatility %	380.66	405.18	337.95
mean volume p.p. %	2.82	1.18	0.12

4 Results

Fig. 1 shows a typical price dynamics from the first experiment with pessimistic EMB investors. The market price closely follows the fundamental price which is driven by the random dividend process. Hence, this experiment resembles the benchmark model in which there are only RII investors in the market. Pessimistic investors predict next period return with the minimum return in the sample of past returns. The minimum return is almost always bellow the risk-less return, so the optimal investment for pessimistic EMB investors is to invest everything in bond. The actual investment proportion will slightly vary due to the error term in (7). Only in rare occasions when there is a series of returns higher than the risk-less return, the EMB investors will invest in the risky asset. The results in Table 2 show that for this experiment the volatility of the market price is similar to the volatility of the fundamental price, which means that there is a low excess volatility. The relative mean volume per period shows that there is very little trading in the market, i.e. from period to period the investors do not change much their portfolio holdings.

Fig. 2 shows the price development for the second experiment with slightly more optimistic investors that predict future return using the harmonic mean. The results of this experiment qualitatively and quantitatively resemble the results of the original model with a small fraction of unbiased EMB investors (which predict future returns using a uniform discrete distribution over the observed returns). The market exhibits cyclical booms and crashes to the fundamental value. According to Table 2, the market is more volatile, and there is also more trading. This exchange of risky assets between RII and EMB investors occurs mostly when the booms begin and when they crash.

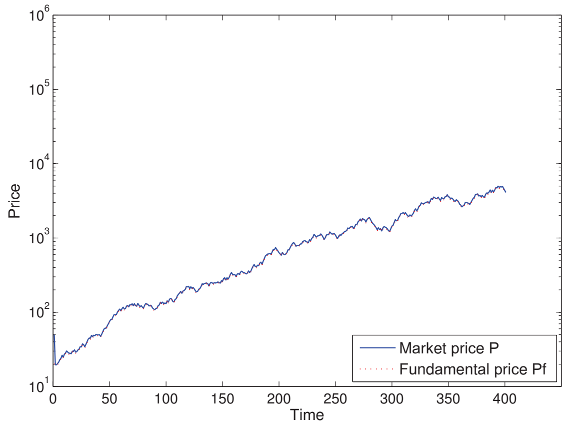

Figure 1: Price dynamics with 95% RII and 5% minimum sentiment EMB.

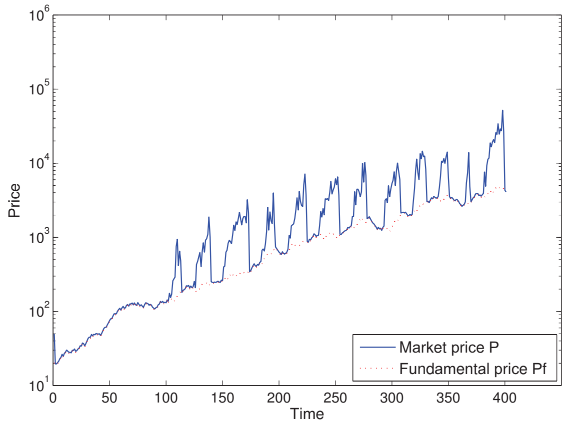

Figure 2: Price dynamics with 95% RII and 5% harmonic sentiment EMB.

Fig. 3, Fig. 4, and Fig. 5 depict the market dynamics when EMB investors are even more optimistic. As the index of optimism increases the market shows more extreme (longer lasting) booms, followed by very sharp crashes. During these bubbles the EMB investors aggressively invest in the risky asset, while the RII investor divest expecting that the overvalued asset would fall to its fundamental value. The crash occurs when there is a series of low returns, due to low dividend realizations, so the EMB investors suddenly shift toward a riskless asset. However, as soon as a better return is realized, EMB investors invest in the risky asset and a new boom starts. From Table 2 it is also evident that the more optimistic EMB investors are, the more volatile market price is. However, the trading is reduced because the booms are longer lasting, i.e. the cycles of booms and crashes appear less frequently.

In the case of full optimism, there is an ongoing market bubble, as shown in Fig. 6. The market does not crash because the maximum return in the rolling window of past returns is always above the risk-less return, so the EMB investors are always highly invested in the risky asset. The trading in this experiment is even more reduced, but the volatility of the market price is also somewhat reduced. The reason for the latter is that the crash does not occur within the experiments.

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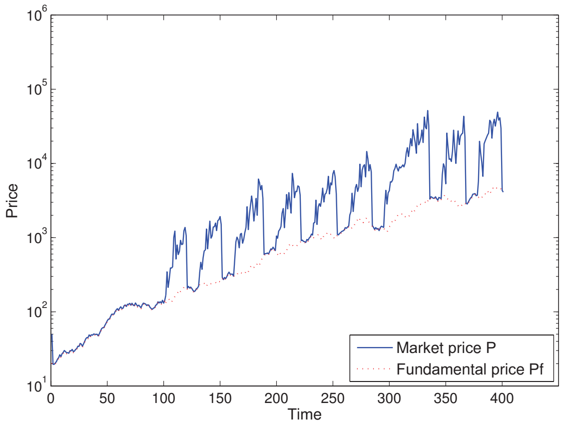

Figure 3: Price dynamics with 95% RII and 5% geometric sentiment EMB.

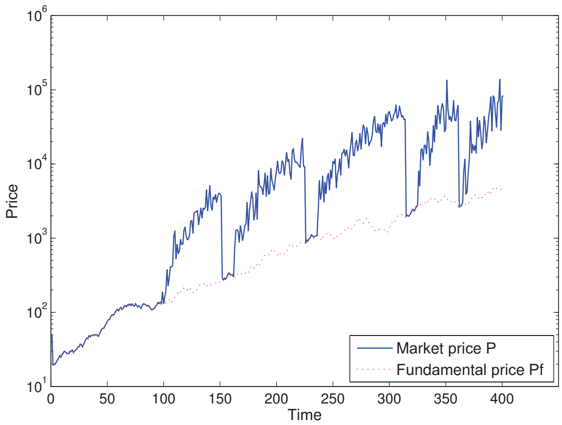

Figure 4: Price dynamics with 95% RII and 5% arithmetic sentiment EMB.

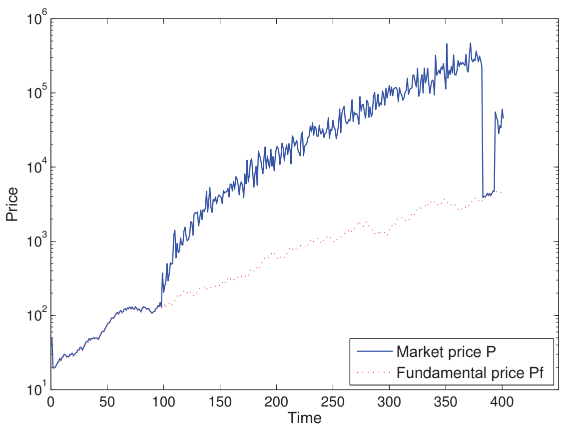

Figure 5: Price dynamics with 95% RII and 5% quadratic sentiment EMB.

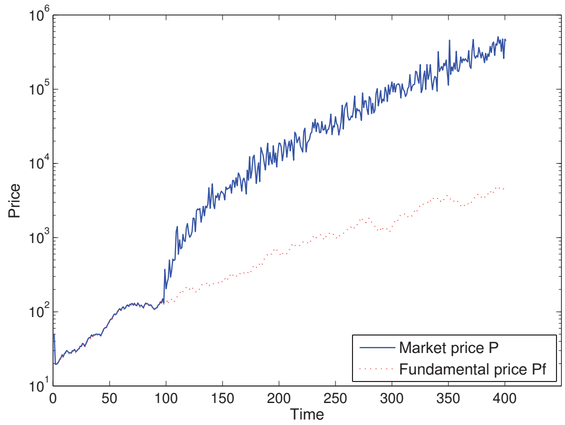

Figure 6: Price dynamics with 95% RII and 5% maximum sentiment EMB.

Fig. 7 shows the development of the relative wealth of RII investors over time. At the beginning, RII investors posses 95% of all the wealth in the market. In the case of extremely pessimistic EMB investors, RII investors end up asymptotically dominating the market. This is because the LLS market is a growing market, and only RII investors are investing in the risky asset and exploiting that growth. Conversely, in the case of extreme optimism, EMB investors are highly invested in the stock, and eventually dominate the market. In non-extreme cases of optimism, both types of investor coexist in the market.

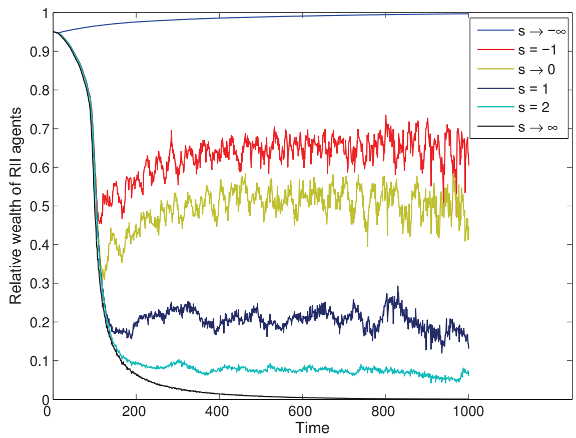

Figure 7: Relative wealth dynamics of RII against sentiment EMBs with various levels of optimism.

5 Discussion and conclusions

In this paper we have used a fuzzy connective to study investor optimism in the modified LLS model of the stock market [16]. We show how changes in the formation of expectations by EMB investors can have a marked impact on the price dynamics. The levels of investor optimism are related to the occurrences of market booms and crashes, as well as measures of excess volatility and trading volume.

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Since our current experiments focus only on the case of homogeneous EMB investors, we would like to conduct our next experiments in the case of heterogeneous EMB investors with various memory lengths. We expect that the choice of memory length has a great impact on the occurrence of booms and crashes, particularly in extreme cases of optimism, because the larger that window of past returns is, the less likely is that all the returns are bellow or above the risk-less return. In future research, we will extend our analysis to the interplay of investors with various degrees of optimism within the same market. Furthermore, we would like to implement an updating mechanism by which the level of investor optimism changes based on the past performance.

As we have used the same model to study investor overconfidence [18], a distinct although related behavioral phenomenon, it would also be interesting to study both phenomena at the same time. The overconfidence in the model [18] refers to the peakedness of the return distribution around the mean of return observations, while optimism in this model determines how that mean is chosen (ranging from the minimum observation to the maximum observation in the sample of past returns).

This paper demonstrates the advantage of using a fuzzy connective for modeling investor optimism, as we were able to control investor optimism by varying only a single parameter. The results of our experiments show that this parameter was a valid choice for an index of optimism in the context of financial markets. In future research other fuzzy set connectives could be investigated for agent decision making.

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