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REGULAR ARTICLE

Heterogeneity in economics

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Abstract This paper argues that heterogeneity of agents' characteristics plays a fundamental role in the economy and should do so in economic models. Three aspects are considered. Firstly the notion that assuming heterogeneity was a solution to the problem posed by the results of Sonnenschein Mantel and Debreu is considered and it is suggested that the more pragmatic approach adopted by Hildenbrand is likely to be more productive. Next the role of adaptation and evolution which necessarily involve variety or heterogeneity is examined. It is suggested that heterogeneity will persist since agents will only slowly learn to adapt and that in the meantime the environment will change. Lastly the role of heterogeneity in financial markets is examined. It is suggested that heterogeneous and varying expectations may account for many of the stylised facts which do not seem to be consistent with the standard financial markets model.

Keywords Heterogeneity · Uniqueness and stability of equilibrium · Adaptation · Learning · Varying expectations

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"Variety is the spice of life"

1 Introduction

It is odd that heterogeneity does not play a greater role in economic models. In so many other disciplines it is fundamental. In the theory of evolution,

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variation, that is, heterogeneity even of a limited sort is crucial. In sociology it is the differences between individuals and the role and consequences of those differences that occupy centre stage. Yet, in economics, heterogeneity plays an ambiguous role. On the one hand, it is obvious that trade, for example, will not take place between identical individuals, and again, if all agents hold accurate views about the evolution of uncertain financial assets no trade will take place. But macroeconomic models reduce the economy to an individual who, evidently, cannot trade with herself. Indeed, the standard model of financial markets is one in which no trade takes place. No arbitrage is possible. Yet, in reality, many individuals make their living from arbitrage. Indeed, trade and exchange is an essential feature of the economy. Someone who was brought up in the General equilibrium tradition will immediately object to the suggestion that heterogeneity is ignored. She will argue that, in that model, there is room for as much heterogeneity as one wants and the essence of the simple exchange models which are used to characterise that model is that they are always based on individuals with different tastes or endowments trading with each other. Why then resort to representative agent models?

2 The theoretical problem

To see this, consider the basic Arrow–Debreu framework. In that model, each agent in an economy maximises some concave or quasi-concave objective function over a convex set defined by parameters over which he has no control. The natural way to analyse such a system is to look for an equilibrium state, that is, values of the parameters determining the constraints of the individuals and choices of those individuals such that none of them has any incentive to modify his action. The passage from micro to macro behaviour is simply additive. One seeks a vector of central signals which will induce the individuals to demand and supply just the total amounts required to clear the market, that is, to be in equilibrium. Furthermore, no explanation is given as to how the equilibrium comes about. This is the subject of stability analysis which has usually been reduced to examining the convergence of artificial price adjustment processes. Yet, as Morishima (1964) has pointed out

> "If economists successfully devise a correct general equilibrium model, even if it can be proved to possess an equilibrium solution, should it lack the institutional backing to realise an equilibrium solution, then the equilibrium solution will amount to no more than a utopian state of affairs which bear not relation whatsoever to the real economy."

It is here that the well known Sonnenschein (1972), Debreu (1974), and Mantel (1974) results reveal their full force. They show that the standard and restrictive assumptions on the preferences of individuals do not guarantee stability. Without this the intrinsic interest of economic analysis based on the General Equilibrium model is extremely limited. It is worth just explaining here in what sense uniqueness and stability cannot be guaranteed. The reason is quite simple.

Let me just write down the basic problem for the simplest case. The argument goes through for economies with standard production sectors. Consider an economy with l goods, in which each consumer is characterised by fixed resources and an excess demand function f which maps the prices $p \in \Re^l_{++}$ into a demand, that is a commodity bundle $f(p) \in \Re^l_{+}$. The excess demand functions in the set $\Im$ that we consider are endowed with all the properties of those that are derived from preferences satisfying all the standard conditions, (see Hildenbrand and Kirman 1988). In the most general terms the consumption sector of this economy can be described as a measure ν on $\Im$. In this exchange economy which is often considered as a canonical example, there is of course, no other sector. Now what we are interested in is the aggregate, (or if we are considering a large or infinite population, the mean), excess demand. This can be written simply as

$$F(p) = \int\limits_{\Im} f(p)\mathrm{d}\nu. \tag{1}$$

If we want to consider the finite case we just replace the integral with a sum. Under the standard assumptions on individual preferences and resultant demands there will be an equilibrium price vector i.e. a vector p^* such that:

$$F\left(p^*\right) = 0. \tag{2}$$

Now what we would like is to know what properties on the individual demand functions, or more precisely on the preferences that generate them, will guarantee *uniqueness* and *stability* of equilibrium of this sector or economy This, it should be repeated, is a minimal requirement for this sort of model to have any intrinsic interest.

It is here that the unfortunate role of the SMD results can be seen. Those results say that the only conditions imposed by the standard conditions on individual preferences, on the functions f in F are the following:

1. *Continuity* on the set of positive prices, that is for all $p \in \Re^l_{++}$
2. *Homogeneity of degree 0*. $f(\lambda p) = f(p)$ for all p and $\lambda > 0$
3. *Walras law*, $p \cdot f(p) = 0$ for all $p \in \Re^l_{++}$
4. *Boundary condition*, if $p_t \to \bar{p}$ and for some $i, \bar{p}_i = 0$ then $\|f(p_t)\| \to \infty$.

Different versions of this result have been proposed and by bounding prices away from zero one can leave out condition 4. Nevertheless the result says that these are the *only* conditions imposed by the standard assumptions on aggregate demand functions. This is bad news for a simple reason. It is easy to write down functions which satisfy the above conditions but which exhibit multiple and unstable equilibria. Another way of expressing the SMD result is to say that to every function satisfying the four conditions above, corresponds a well behaved economy, i.e. one which has the function in question as the excess demand function and, furthermore one can write down preferences satisfying the standard conditions which generate precisely that function. Debreu showed that one only needed l demand functions to do this.

But this means that the only hope of obtaining uniqueness and stability of equilibria, is to impose some other conditions. For example one might require that all the agents in the economy have Cobb–Douglas preferences. In terms of our original formulation this amounts to concentrating the measure ν on a tiny set. But this would be manifestly too restrictive. So, if one is not satisfied with assuming a very special case the situation looks grim.

Hildenbrand (1994) expressed his strong reaction to this development by saying,

> "Up to that time I had the naive illusion that the microeconomic foundations of the General Equilibrium model which I admired so much, does not only allow us to prove that the model and the concept of equilibrium are logically consistent (existence of equilibrium) but also allows us to show that the equilibrium is well determined. This illusion, or should I say this hope, was destroyed, once and for all, ..."

Thus a number of leading theorists realised that the SMD results showed that the basic theoretical model of the economics discipline was empty of empirical content. This is clearly therefore an important topic. But what has all of this got to do with heterogeneity? Precisely because an appeal to heterogeneity might, it has been suggested, solve the problem. Recall that we could assume that the measure ν was concentrated on a very special set. This would not be acceptable but it does however, point the way to another solution, also based on assumptions on the measure ν. Thus instead of trying to modify the assumptions on individuals we might want to make assumptions on the *distribution* of their characteristics. Indeed, it has been argued that, this could be a way to overcome the difficulties with the General Equilibrium model and, by so doing to generate the structure necessary to obtain uniqueness and stability of equilibria. This stems from ideas advanced already by Cournot. The idea is the following. If the economy consists of a large number of sufficiently heterogeneous agents, properties like uniqueness and stability of equilibrium may be restored (see Grandmont 1987, 1992; Hildenbrand 1983, 1994). Thus structure may be introduced into aggregate behaviour by the presence of enough differences between the characteristics of the agents. Put another way, if the measure ν is sufficiently spread out we may obtain uniqueness and stability of equilibrium. That is, heterogeneity can replace assumptions strictly restricted to those on individuals.

The first step in this direction was taken by Cournot who argued that individual demand was likely to be highly discontinuous, but, that by adding up the demands of a large number of individuals, we should get smooth demand curves. This intuition was confirmed by Hildenbrand and Sondermann and the key condition was that the slope of all consumer's indifference curves or surfaces, should all be different at any point in the consumption space. This is surely a description of heterogeneity What this idea suggested was that dispersion, or heterogeneity might have a general role to play in providing structure for aggregate relations.

A further step was taken by Hildenbrand (1983) where he showed that heterogeneity of characteristics, (in this case income) can also impose structure. He showed that "the Law of Demand", for aggregate behaviour will hold if the distribution of income is monotonically decreasing. But if the "Law of Demand" holds then the economy has an unique and stable equilibrium. Now, admittedly this is a special form of heterogeneity and certainly cannot apply to any economy that we know. What it requires is that starting from zero income, there will be more individuals in the lowest percentile of income than in the next percentile and so forth all the way to the highest income bracket. Chiappori (1985) made this more palatable by showing that for very low incomes the assumption need not apply. Once again there was a glimmer of hope that heterogeneity might be the answer to SMD. The next step was made by Grandmont who developed a class of preferences by using a method derived from the "household equivalent" approach. Essentially what he did was to use a base of preferences and then to modify them to produce a whole class of preferences. It is important to note that this is a very special way of generating heterogeneity, it consists of "stretching" in a particular way, the basic preferences with which one starts. The family of preferences generated in this way in turn generates a family $\Im$ of excess demand functions. However, it is important to note that this approach consists of parameterising the preferences one generates and that the space of parameters is finite dimensional.

How exactly does Grandmont proceed? As I have said, he, like Quah (1997) works with a finite dimensional space of demand functions and he achieves this by parametrising the excess demand functions by this space which can be considered as a subset C of $\Re^n$. This is done by defining a map T from C to $\Im$. Under this map the image measure μ of a distribution ν on C is a consumption sector as in our original definition. What Grandmont does is to examine what happens when the distribution or measure ν on C becomes increasingly flat. (increasingly flat can be considered as saying that sets of "similar size" in the basic sense have "similar weight". The distribution does not have most of its weight concentrated on a "small set". Here is the danger, for an increasingly dispersed distribution on C, in this sense, does not imply as Quah (1997) has pointed out, "heterogeneity in any meaningful sense), of the distribution of demand functions".

This is a real and important problem. One should not confuse diffused parameters with diffused excess demands. In what sense has Grandmont really introduced heterogeneity? If we are to say that a set of preferences exhibits heterogeneity then we would surely require that the behaviour, or choices, that they generate should also be heterogeneous. A closer look at Grandmont's result shows the real problem. This is just the point that has been raised by Billette de Villemeur (1999) and which was reinforced by Hildenbrand and John (2003). Indeed, most of the preferences generated by the Grandmont method from a fixed arbitrary set of preferences, are, in fact, Cobb Douglas. Thus what seemed like a way of generating a variety of preferences, generates, in reality, a very small family concentrated on a very special set of excess demand functions. What happens, as a result, is that the aggregate excess demand becomes essentially like one generated by a Cobb Douglas utility function. In other words the shares of expenditure on each good are price invariant. So, without apparently having to make the restrictive assumption, at the outset, that all individuals have such utility functions, the aggregate turns out to behave as if its excess demand was generated by such a function. This, of course guarantees uniqueness and stability of equilibrium. Thus, one has initially the impression that heterogeneity produces "well behaved excess demand". As we have seen this is an illusion because the distribution on the family $\Im$ is very concentrated and the alternative that seems unacceptable at the outset, assuming that all consumers are Cobb Douglas, is very close to what is being done here. Thus heterogeneity, in one sense, is restriction in another.

This is, at least until now, a sad story since heterogeneity could have played a key role in justifying the general equilibrium model but in reality has failed to do so. Is there then nothing more to be said about the underlying intuition? Not quite, since Hildenbrand (1994) suggested a much more radical departure from traditional theory and proposed that we start with individual demands without deriving them from preferences and that we then see if we could find some condition on the dispersal of consumption choices that would give us back uniqueness and stability. What he showed in his book was that if, with increasing income, the consumption choices of individuals become more dispersed in a very precise sense, then the aggregate demand will satisfy the aggregate "Law of Demand" . He works with demand functions, but this is not important here, and the condition which says that *F(p)*, the aggregate demand satisfies the "Law of Demand" is that, for any two price vectors p and q,:

$$(p - q)F(p) \leq (p - q)F(q). \tag{3}$$

This property guarantees the uniqueness and stability of equilibrium in a simple economy. It is worth noting that what Hildenbrand does is far from the traditional approach. He starts with a condition on observable choices which he does not assume are derived from any utility maximisation and then deduces the "Law of Demand". Thus the method is completely rigorous but does not depend on assumptions about the source of the demand behaviour. What he requires is that the choices of individuals should be sufficiently heterogeneous and that this heterogeneity should increase with income. This pragmatic approach, which starts with Hardle et al. (1991) and Hildenbrand and Kneip (1993) and which has been pursued in Hildenbrand and Kneip (2005), does not encounter the difficulties seen in the earlier discussion and it is surprising that it has not had a larger echo.

Everything that I have said so far has turned on one special role that theoreticians has hoped to attribute to heterogeneity and the bottom line is that it has not yet showed any results if one sticks to the basic general equilibrium model and its assumptions on individual preferences. More empirically based results seem to be more promising.

Yet for many economists this will have been a rather arcane discussion and they might ask whether heterogeneity has not been considered as important in other economic settings. My first assertion was that it does, indeed play an essential role in evolutionary biology for example. Variety and selection are the two keywords there. There is a long tradition, even though not a dominant one, in economics, of using evolution at least as a metaphor. If this metaphor is to be meaningful, it must be the case that variety or heterogeneity plays an important role. A context in which such a debate has taken place is that concerning the nature and origin of optimising behaviour, that most fundamental assumption of economics.

3 Adaptation and evolution or optimisation?

At least since Marshall there has been an effort to replace mechanical optimisation by more biological notions of learning and adaptation. The use of arguments based on natural selection has been present in economic analysis over a long period although such arguments have, in general, not been regarded as central. Hayek,[1] for example, systematically used evolutionary arguments to justify the emergence of social and economic order. He maintained that the emergence of the market system was a result of evolutionary forces operating not only on individuals' behaviour but also on rules and social structure. His view of the result as optimal would, of course, be contestable from a strictly evolutionary standpoint, but his view of economic organisation itself as evolving endogenously was far from being widely accepted. Where does heterogeneity play a role here? If selection is to happen then there must be differences among the objects, organisms or types of behaviour amongst which the selection is being made. If convergence to an "optimum" was the true story evolution would cease. Yet in nature it continues because there is constant variation and the constant introduction of novelty. Thus heterogeneity is an essential part of the evolutionary story. In the period after Darwin the emphasis was on selection and not much attention was paid to the nature and origin of variation. More recently attention has increasingly been turned to the mechanisms that generate variation. One of the oldest questions for evolutionary theory is how long would the random mutations take to develop a complex organism? Too long, has been the answer of the "intelligent design" clan. Thomas Payley, when developing his argument about the watch he found on the heath argued that it was much too complex to have arrived there as the result of a process of trial and error. This led him to the necessity for a "creator" who had designed and assembled the watch. This is the argument that is directly attacked in Dawkins' well-known book, "The Blind Watchmaker".

Yet a question remains, how can we be sure that the mutations necessary to produce the variation on which selection operates do, indeed, happen fast enough? Recently, a way out of this dilemma has been proposed with the theory of "facilitated variation" (see Kirschner and Gerhart 2005). This argues that there are properties of the development and physiology of organisms that

[1] See von Hayek (1989) for example.

facilitate certain types of variation and that guide the direction that evolution takes. This suggests that the apparently totally improbable chain of change necessary for the evolution of organisms is not so unlikely after all. This completes the picture and shows how the important role of variation can be played. Whilst all of this may be intriguing, the reader may well ask what has this to do with economics? The answer is rather a lot, since evolutionary arguments, even if only metaphors, have come to be widely used. It is useful to see these arguments in the context of modern evolutionary theory to get an understanding of their nature.

Natural selection, it would be fair to say, has been the central theme, of the evolutionary approach, but often in a very negative and simplistic way.

Indeed, for the most part, evolutionary arguments have been used by economists in the mainstream to suggest that optimising behaviour can been justified in economics as the result of adaptive behaviour which is often taken to be synonomous with natural selection. Friedman's (1953) remarks are so well known that it is not worth repeating them here. Taking profitablity as a criterion for fitness, he argued that non-optimising firms cannot be around because they must have been selected against. Though the logical objections to his position are clear, they reflect a widely held view. Alchian developed this reasoning and argued that natural selection would engender optimising behaviour. If this is the case, many economists have argued that one might as well take the short-cut of assuming complete rationality and studying the equilibria that would arise. Thus, Johnson (1968, p. 5) states without qualification:

> "it has been shown… that whether firms consciously seek to maximize profits and minimize costs or not, competition will eliminate the inefficient firms; and that whether consumer behaviour is rational or purely random, the demand curves for a product will tend to slope downwards as in the Marshallian analysis. In consequence, it is possible for economists to treat the economy as an interdependent system responding to change according to certain general principles of a rational kind, with considerably more confidence than appeared justifiable thirty years ago."

What Johnson is saying is that the system has properties which are not dependent on the specific behaviour of individuals. He is arguing that the aggregate structure that Hildenbrand derives from the dispersion of behaviour may result from natural selection. The basic argument is not formal but it leads, as does Hildenbrand's work, to an important observation. Johnson is arguing that the system as a whole has a certain rationality. This rationality emerges at the aggregate level and is of the same order as that which economists *assume* for individuals. As a result the system as a whole behaves like a large optimising individual.

This is far from the idea of self organisation used in biological or physical systems which is rather that the behaviour of the system may be qualitatively different from the behaviour of an individual. In that view, without imposing any specific rationality on individuals some sort of collective rationality may emerge. Indeed, this is one way of interpreting Hildenbrand's (1994) work on Market Demand.

Johnson's argument is extremely simple and is close to the position held by Friedman since he simply argues that the role of selection eliminates the inefficient or unfit individuals. Thus for Johnson selection takes place at the level of the individual and order emerges as a result of this. This is related to the position adopted by Lucas (1988) when he says,

> "In general we view or model an individual as a collection of decision rules (rules that dictate the action to be taken in given situations) and a set of preferences used to evaluate the outcomes arising from particular situation-action combinations. These decision rules are continuously under review and revision: new decisions are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. I use the term "adaptive" to refer to this trial-and-error process through which our modes of behaviour are determined."

However, Lucas then goes on to argue that we can safely ignore the dynamics of this process since,

> "Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates."

Thus the difference between the approach of many economists who rely on standard theory and those who argue for a biological style of analysis, is one concerning the stability of the adjustment to equilibrium and the speed of that adjustment. The basic tenet of those who concentrate on equilibrium is that the individuals in an economy learn, or are selected, relatively rapidly to behave optimally and that the economic environment changes sufficiently slowly so that in the resulting situation they have no need to continue to learn. Such a view does little to explain the variety of behaviour, and of what economists are pleased to call "anomalies" that are observed. People other than the optimisers would quickly disappear from the system. The mistaken few who generate heterogeneity would be transitory and can therefore, the thinking goes, be eliminated from consideration.

This contrasts with the contention of authors who, like myself, hold the view that the economy is a complex adaptive system and that the very process of learning and adaptation and the feedback from the consequences of that adaptation, generate highly complicated dynamics which may well not converge to any standard economic equilibrium. However what is clear is that adaptation and selection have not been absent from economic theory except in the most rigorous General Equilibrium models. They were used as a means of justifying the assumption of complex optimisation by economic agents rather than as an alternative to that optimisation.

It is also true that such a position was not left unchallenged even in its earlier stages. Winter (1962) took issue with Friedman and said:

> "There is then a basic difficulty in the existing statements of the selection argument, a difficulty which is rooted in the fact that the relative deviations from profit maximization of different firms may change through time. Since there has been no careful treatment of the dynamic process by which some patterns of behaviour are shown to be viable and others nonviable, it has escaped notice that firms cannot in general be unambiguously ranked in terms of their closeness to maximising behaviour. Such a ranking must, in general, presume a particular state of the environment, but the environment is changed by the dynamic process itself."

This brings us face to face with the most difficult problem with the evolutionary approach. Fitness, or utility, or the capacity to survive and reproduce are measured with respect to the environment. Yet, the environment consists, in economics, of the behaviour of the other agents and the organisational set-up. If one agent is adapting or learning, then so probably are the others. Thus, in simple game theoretic terms we cannot assume that a change is profitable just because it would be so if other agents did not modify their behaviour. Agents who adopt strategies which were better in the past may make them less beneficial in the future. This, in turn, will lead them to change again and there is no guarantee that this process will converge. Thus, one cannot hope, in general, to use adaptive behaviour to rationalise the equilibria that would have been achieved by fully optimising agents. This does not mean that one should abandon the assumption of adaptive behaviour but rather casts some doubt on the value of the equilibrium notion and suggests that we should be more interested in the dynamics of disequilibrium.

Once again, there should be very little variation in the population of economic agents if we adopt the simplistic view of evolution. Since all agents are converging on optimal behaviour, it must take some exogenous shock or change in the system to knock them off course. Why they should be affected differently by such a change is, in general, not explained. If the only source of differences between agents are those due to their asymmetric reactions to shocks, then we have to go one step further back and ask why this asymmetry should exist. An alternative explanation is the existence of "idiosyncratic shocks". This says that the players are, in fact, different, but that these differences can be captured by the fact that they receive individual shocks. Once again the source of the differences, i.e. of the heterogeneity of the players, is left unexplained.

A better explanation of the nature and source of variation is that given by Nelson and Winter (1982). They suggest that firms, under pressure because of low profitability, experiment with new routines to try to improve their situation. These mutations can improve or worsen the situation and the positive ones are selected for. To model this sort of idea formally, one normally defines a list of possible routines or strategies a priori and then argues that those strategies which are most successful will expand and those that are less successful will contract. The usual way to model this is to use "replicator dynamics". This approach does not however show how, or why, individuals choose, or learn to choose the better strategies. One way to derive this is to allow the firms or players to choose from among the available strategies with probabilities which are related to the firm's performance when using them. This sort of "reinforcement learning" can then be applied and one can see whether the players converge to some vector of strategies and if that vector can be considered as an equilibrium, for example in the sense of Nash. One extreme cases of this approach would be ones in which individuals systematically chose the "best response" to the current choices of the others. Another would be that in which agents always choose with equal probabilities between the alternatives. There are, of course, many intermediate cases.

A typical example of the reinforcement sort of learning is that given by the following function:

$$P_t^{ik} = \frac{\mathrm{e}^{\beta W_t^{ik}}}{\sum_j \mathrm{e}^{\beta W_t^{ij}}}, \tag{4}$$

where β is a constant that reflects the importance that individual i attaches to previous experience and W_t^{ij} is the, possibly discounted, total gain that individual got from using rule or strategy j up to time t. In other words, at each time period an agent decides which strategy to use but does so stochastically, with probabilities that are an increasing function of success. Note that with such rules there will always be those who change their rule, even if, over time, the probability of this diminishes. Thus heterogeneity will persist over time.

The particular rule shown here for the probability for the transition from one rule to another will be familiar from a number of economic applications (see McFadden 1974; Anderson et al. 1993; Brock 2003; Brock and Durlauf 2001a,b). It has been widely used in the psychological literature on reinforcement learning, and in game theory, where it is referred to as the "quantal response rule". One way of deriving it is to consider the problem of the trade-off between obtaining information about sources of profit and exploiting those which have proved profitable in the past. This "exploration" versus "exploitation" arbitrage can be analysed by maximising a linear combination of the gain to be had from trying new alternatives and the expected gain given the experience in the past with the different rules, (see Brock 2003; Weisbuch et al. 1998).

By using this sort of tool we can explain the persistence of differences in the population. However, in this sort of situation the basic notion is one of competition and not one of coordination and cooperation. In many situations there is a voluntary creation of heterogeneity as a result of a collective desire to exploit increasing returns, for example. The division of labour is a remarkable and fundamental phenomenon which can be found in a wide number of circumstances particularly in social insects, where different tasks are taken on by different members of the hive or nest. The nature of increasing returns or "non-convexities of the production set", is one that has been widely discussed in economics ever since Adam Smith gave the example of the pin factory. Whilst the precise modelling of this phenomenon has been very limited its nature is intuitive. If animals or humans specialise on specific tasks then they require much less cognitive capacity and thus overall productivity is increased since the tasks can be done faster. Specialisation means that one can no longer describe overall activity as the result of one typical agent but the phenomenon is so prevalent that we can surely not afford to ignore it.

A fascinating example is given by honeybees who specialise in their tasks. Not only do we find heterogeneity of roles but these roles change. The kind of work performed by the worker depends largely upon her age. The first three weeks of her adult life, during which she is referred to as a house bee, are devoted to activities within the hive, while the remainder are devoted to field work, so that she is called a field bee, (see Adjare 1990). Thus there is important heterogeneity even within individuals;

The tasks for house bees are varied but temperature control is one of the important duties. When the temperature is low, bees cluster to generate heat for themselves, but when it is high, *some of them* fan their wings to circulate air throughout the hive. The general hive temperature required is between 33 and 36°C, while the brood chamber requires a constant heat of 35°. Honey has to be cured in order to ripen, and this also requires the help of circulating air. According to Crane (1999), 12 fanning bees positioned across a hive entrance 25 cm wide can produce an air flow amounting to 50–60 litres per minute. This fanning can go on day and night during the honey-flow season. Honey bees' wings beat 11,400 times per minute, thus making their distinctive buzz.

What is the lesson here for us? The typical economist's response to this phenomenon would be to consider a representative bee and then study how its behaviour responds to the ambient temperature. This would be a smooth function of temperature, wing beats going up with the temperature. Yet this is not what happens at all. Bees have different threshold temperatures and they are either on (beating at 11,400 beats per minute) or off. As the temperature rises more bees join in. Thus collectively with very simple 1, 0 rules the bees produce a smooth response. This sort of coordination, with each agent doing something simple, can only be explained by having a distribution of temperature thresholds across bees. Aggregation of individuals with specific local and differentiated behaviour produces smooth and sophisticated aggregate behaviour.

When there are non-convexities and specialisation is advantageous, it can emerge. The same is true in markets where information is dispersed and agents are not in contact with all the others. Each agent can learn to do something specific and a situation can be obtained in which different individuals are behaving differently. The division of labour is such a longstanding and prevalent feature of our society that the names that people are given often reflect professions. The profession, or specialised activity, one undertakes conditions one's tastes, and one's place, in society. Thus this voluntary diversification is, to some extent, self perpetuating.

This is one type of heterogeneity, but another involves people in identical situations, in which there is, apparently, a rational way to behave, behaving differently. Take, for example, simple experiments on contributions to public goods games. In the most basic version of these games the Nash equilibrium involves nobody contributing whereas in the social optimum everyone would give everything. The results of these experiments reveal that some cooperation emerges though, in general, the social optimum is not obtained. (see Ledyard 1997). The standard explanation is that the players are "learning to play Nash" and that, if the game went on long enough, all the players would be free-riding and all heterogeneity of behaviour would have disappeared. Yet, careful examination of the individuals' behaviour (see Hichri and Kirman 2005) shows that heterogeneity persists and that some players play in a rather altruistic way, some are non-cooperative all the time and others seem to signal their intentions to other players. This mixture of behaviour does not disappear. The explanation for this phenomenon is far from clear and the puzzle remains. The question becomes whether people have more or less altruistic preferences and what is the origin of this. One suggestion is that altruism is selected for but this selection cannot again be uniform. A large literature on the emergence and persistence of fairness and altruism has sprung up (see Fehr and Gächter 2000) and the debate is far from closed.

To conclude this section it is worth remarking that almost all of the formal evolutionary models in economics consider individuals choosing from a fixed set of alternatives. This is not compatible with a biological view because no new possibilities emerge in such models. The variations produced by mutations are absent so the result of the individual choices must be either convergence to a point or some complicated dynamics within the set. If we were to pursue the evolutionary analogy we would allow for the arrival of new strategies. A very heavily exploited example is that of the well known prisoner's dilemma game. A great deal has been made of the possibility that cooperative behaviour will emerge from a tournament. Lindgren (1991) is one of the rare authors to allows for the possible introduction of new strategies as a result of mutation and he did it in this context. At each period players play one of two actions "cooperate" or "defect" and the pay-offs are given for each of the one shot games. However a strategy for the repeated game is more complicated since it says that conditional on the history observed the action to be taken will be this. Clearly any finite memory strategy in the repeated prisoner's dilemma framework can be written as a binary sequence which specifies the history of the actions taken by both the players and the action to be taken. This "genetic code" can then be modified by point mutations, which change one of the elements in the sequence, by split mutations which divide the sequence in half and retain one or the other half and by gene duplication which duplicates the sequence. Thus although the system starts out with strategies with a fixed memory length other strategies with different memory lengths enter the population. As time goes by strategies with increasing memory length become possible.

Lindgren's simulations show that the evolution of the system is complicated, with periods in which strategies of increasing memory length dominate and then periods of extinction. There are also periods in which the average pay-off is low as a new mutant exploits the existing species. Such an open ended system can exhibit a number of interesting features but it is worth noting that the evolutionarily stable strategy still plays an important role and the vulnerability to invasion by a new strategy is an important determinant of the evolution of the system. In this sort of situation we see the population of strategies changing, sometimes there is herding on one strategy, at others there is a whole ecology of strategies. Thus heterogeneity plays an important role in the evolution of even such a limited system. Yet an important thing to emphasise is that new strategies are constantly becoming available. This is a world which is open and changing.

The sort of phenomena that I have just described with heterogeneity varying over time in the population is one that is often evoked in discussions of financial markets. This is the last area which I would like to discuss. This may seem rather distant from the aspects that I have discussed up to now but, in fact, the parallels are striking.

4 Heterogeneity in financial markets

Financial markets are populated by agents who may, in reality, differ in many ways, (for an excellent and detailed account of the problem of heterogeneity in financial markets see Hommes 2006). In the most basic financial markets model agents do not differ form each other. In particular, a crucial feature of these markets is the nature and formation of expectations. In a standard "rational expectations" framework there is no room for individuals who have different expectations. Again, in terms of the financial market literature, when markets are efficient all information is contained in prices and nobody can make a profitable trade or arbitrage. The only thing that can explain a change in the prices of assets is a change, expected by nobody, in some underlying variable.

The underlying model for this is the classic one from finance where the price of an asset has to follow, for the reasons just given, a random walk with possibly a drift. This is referred to as *geometric Brownian motion*. Thus the value of the asset S_t at time t can be expressed as

$$\mathrm{d}S_t = S_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_t), \tag{5}$$

where the term W_t is a random walk and can be interpreted as corresponding to the underlying fundamentals. Thus the frequently observed phenomenon of a shift in an asset price with no corresponding change in the related "fundamentals" is inexplicable.

As a first example consider the Dow Jones industrial average at the end of the 1980s. The largest 1-day percentage drop in the last 50 years occurred on "Black Monday" in 1987 when the average fell by 22.6% (Fig. 1). Yet there was no specific news to justify this. There has been a great deal of discussion about the technical explanations in terms of the use of "portfolio insurance" etc but suffice it to say there is no reason to attribute this abrupt change to some underlying fundamental change. Quite the opposite, there seem to be good reasons to believe that herding behaviour took over and people came to the view that they had to act on the rapidly developing decline of the market.

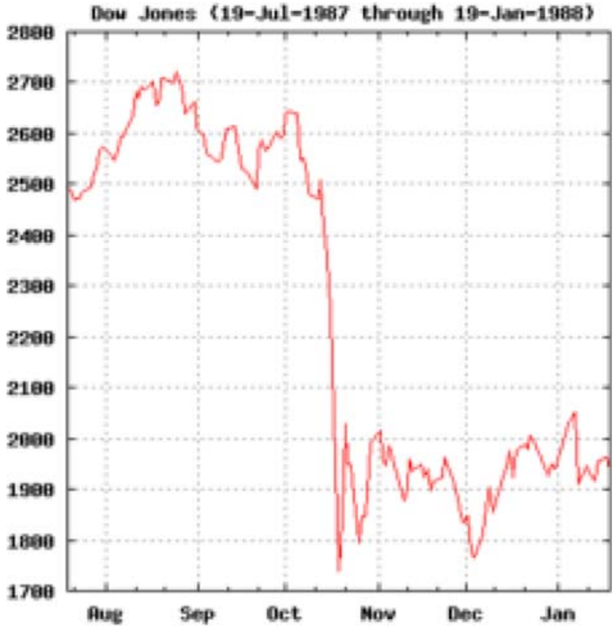

Fig. 1 The stock market crash of 1987

Once again there is nothing "irrational" about this, particularly for those with short horizons.

Now one could argue that this was just a one-off example due to the peculiar functioning of the market. But now look at Fig. 2 which illustrates the evolution of the Dow Jones industrial average over the period from 1990 to 1999.

It would not be unreasonable to think of this series as having been generated by a process such as that given in (5). This would suggest that "Black Monday" was just an aberration and that after the index reverted to its geometrical Brownian motion reflecting a series of shocks contained in the arrival of economic news. Notice, however, one drawback to this interpretation there seems to be very little relation between the "fundamental shocks" as measured by the news, carefully annotated by Dow Jones and the changes in the index. This impression is consistent with the findings of Cutler et al. (1988) that there is no correlation between major economic news and movements in stock exchange indices.[2] Leaving this on the side for a moment, consider Fig. 3.

This gives the evolution of the same index from the end of 1999 onwards. How are we to explain the sudden change in 2000? As is clear from the annotations

[2] There are, of course notable exceptions such a the largest 1-day point drop which occurred on September 17, 2001, the first day of trading after the September 11, 2001 attacks, when the Dow fell 684.81 points or 7.1%. By the end of the week of September 17th, the Dow had fallen 1369.70 points, or 14.3%. Yet, there is no clear indication as to what the economic consequences of that event would be so this reflects a collectively negative reaction but not necessarily a rational collective revision of economic expectations.

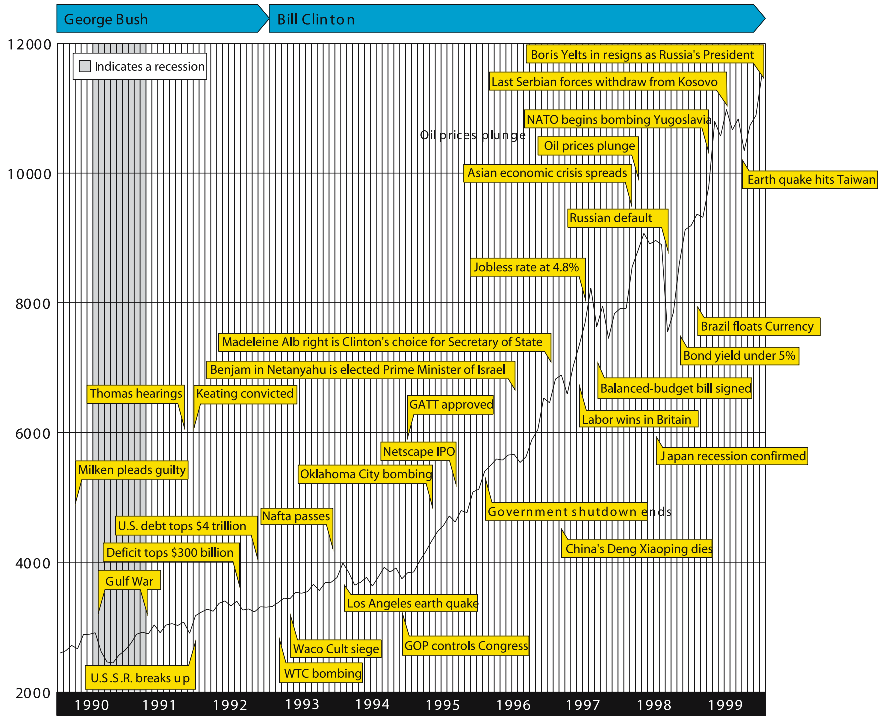

Fig. 2 The Dow Jones industrial average 1990–1999

there is no significant news at that point. Yet, if expectations drive demand, and hence prices, there was a radical change in, at least, collective expectations. The interpretation that I will give, in the context of a simple example, is that the agents in the market, "herded" on a different forecast, or forecasting rule and that this led to the change. The argument boils down to saying that individuals tend to switch the way they view the future from time to time and that these switches are coordinated. The coordination is not irrational and as is well known in the literature, "rational herding" can occur and although it is rational it may generate perverse outcomes, (for a complete survey of this topic, see Chamley 2004).

The example I will give is in the context of the foreign exchange market where the same sort of problems as those outlined for stock exchange indices. When one wishes to explain sudden and substantial changes in exchange rates, the same type of question arises, do they really simply reflect modifications in collective expectations about future fundamentals? If that were the case, how does one reconcile the two ideas frequently expressed by traders that on the one hand, "fundamentals matter in the long run" but, on the other hand, they do not drive exchange rates in the short run?[3] A simpler explanation would be

[3] See Goodhart et al. (1991).

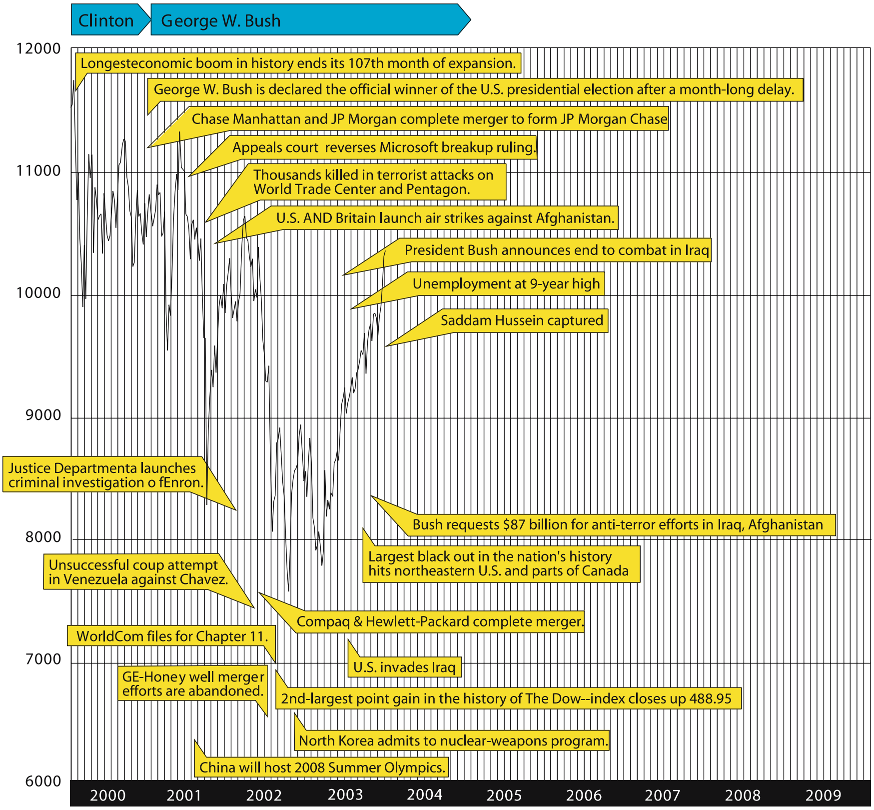

Fig. 3 The Dow Jones industrial average 1999–2004

that, as the result of the mechanism that I have just outlined, bubbles occur and periodically collapse but that the timing of the crash is unpredictable. In fact, in the sort of model I will describe, the expected time before a collapse is known but, independent of, where the exchange rate is at any point in time.

This raises another problem. Suppose that agents are not heterogeneous in terms of their forecasts but rather in terms of their horizons. In this case, traders with short horizons, even if they are convinced that, at some point, prices will return to the level consistent with fundamentals will continue to follow the current trend, that is, they will act as trend followers or "chartists". This will reinforce the bubble and is far from being irrational. For as Keynes remarked,

> "*It is better (...) to fail conventionally than to succeed unconventionally.*" Keynes (1936).

This is in contradiction with those who argue that bubbles are simply due to the irrationality of traders and even further from the view of others like Garber (2000) who argue that the major historical bubbles can be attributed to fundamentals. Thus, in his view, there are no bubbles in the sense of prolonged departures from fundamentals. Yet, consider the following observations by Frankel and Froot (1990b) who note, when discussing the dollar exchange rate, the following:

> "*At times, the path of the dollar has departed from what would be expected on the basis of macroeconomic fundamentals. The most dramatic episode is the period from June 1984 to February 1985. The dollar appreciated another 20% over this interval, even though the real interest differential had already begun to fall. The other observable factors that are suggested in standard macroeconomic models (money growth rates, real growth rates, the trade deficit) at this time were also moving in the wrong direction to explain the dollar rise.*"

Whatever the appropriate definition, and the usual one is a "prolonged departure from the value that could be deduced from underlying fundamentals", this would seem to be a good example of a bubble. Indeed, there is a long history concerning bubbles and there is a substantial body of evidence for their existence, (see for example, Blanchard and Watson 1982; Flood and Garber 1980; Meese 1986; Tirole 1985; West 1988; Woo 1987; Stiglitz 1990; Flood and Hodrick 1990; Donaldson and Kamstra 1996; Avery and Zemsky 1998; Shiller 2000; Kelly and O'Grada 2000; Brooks and Katsaris 2003).

The sort of explanation that I will suggest for these phenomena will depend on the idea that agents, at least some of the time, adopt technical trading or "chartist" forecasting rules and this is an important contributor to bubbles. There is substantial evidence for this and for example Taylor and Allen (1992) who made a survey of foreign exchange market participants, say,

> "*... at least 90% of the respondents place some weight on this form of non-fundamental analysis when forming views at one or more time horizons.*"

An important message here is that the type of view taken may depend on the agents' horizons, and that these may be a significant source of heterogeneity. Here again, as I will explain, there is empirical evidence that shows a clear correlation between horizons and expectations and the rule for forming those expectations.

However, before proceeding, it is worth dwelling for a moment on the received wisdom that, empirically, we do observe different expectations, or rather that we do observe choices which reflect different expectations. The usual objection to this is that expectations are not, per se, observable. Nor indeed are the horizons on which expectations may depend. Yet, to get round this, many surveys have been made to ascertain the expectations of investors and I will come back to this shortly. Once we accept the idea that expectations can be heterogeneous, even if temporarily, it seems that we have to move away from the standard "rational expectations" framework. One idea is simply to introduce agents who systematically have "wrong" expectations but who may survive nevertheless. Such models were pioneered by De Long et al. (1989, 1990) who introduced the notion of "noise traders". Such a solution to the problem is not very appealing and, if one takes account of the idea that agents may learn, it is difficult to accept that certain actors will persist in their error.

Another alternative is to introduce dispersed information into the model and one approach suggested by Townsend (1983) is to have symmetrically dispersed information and to analyse the consequences of "higher order expectations", expectations about others' expectations and the information that generated them. The idea here is that a small amount of non-fundamental trade may generate considerable volatility since traders perceive movements in asset prices as conveying information about future values of fundamentals; see Allen et al. (2003). Again, despite the more sophisticated reasoning attributed to agents, a certain degree of irrational behaviour is needed to generate the results. What is important to note here is, that if there is a return to fundamental it should not be, and is not, in our model, permanent. If agents learn about the true fundamentals and the prices then adhere to those fundamentals as in Bacchetta and van Wincoop (2005), bubble-like departures would cease to occur. In our model such bubbles will continue to develop and to burst. This gives a more realistic meaning to the widely held view, that "in the long run fundamentals matter". Prices will inevitably return to those implied by the fundamentals but will just as inevitably leave them again. This is, of course consistent with the fact, that on average, in the short run fundamentals are poor predictors of prices.

Heterogeneous expectations are also necessary to explain other phenomena. Without them, one could not explain as Bacchetta and van Wincoop (2005) point out, the enormous volume of trade on financial markets. Spot trading on foreign exchange markets in 2001 was approximately $1.2 trillion per day, for example.

A number of models have been constructed in which agents change their views as time passes and in some of these the changes can be self reinforcing. This will generate heterogeneous expectations both across the traders and over time. A particularly relevant contribution is that of Brock and Hommes (1997) in which agents can use one of two predictors. One of them is costly but when all agents use it the process is stable. The other is cheaper but when used by many individuals induces unstable behaviour of the price process. Thus their model has periods of stability interspersed with bubble like behaviour. They present analytical results to characterise this possibly chaotic behaviour of their system. There are many other models in which agents have different forecasting rules, and these are often summarized, as in our example, as "chartist" and "fundamentalist" views. Many such models, when simulated, generate time paths of prices which switch from one expectations regime to another and which generate "realistic" time series, but, in general, no analytical characterization of their properties is provided.

5 Chartism, expectations and horizons

That chartism is used extensively in currency trade is confirmed in several questionnaire surveys made at foreign exchange markets around the world. Examples are Cheung and Chinn (2001), for the U.S. market; Lui and Mole

(1998), for the Hong Kong market; Menkhoff (1997), for the German market, Oberlechner (2001) for the markets in Frankfurt, London, Vienna and Zurich; and Taylor and Allen (1992), for the London market, (see also Frankel and Froot 1990a,b; Shiller 1987) A detailed study of the role of psychology in currency trading may also be found in Oberlechner (2004). That study is based on surveys of expectations on European and North American markets. The basic conclusion of all these surveys, is once more that the relative importance of technical versus fundamental analysis in the currency market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, or chartism, while more weight is placed on fundamental analysis for longer horizons.

A typical observation is that of Frankel and Froot (1990a) who conclude that

> "*... short-term and long-term expectations behave very differently from one another. In terms of the distinction between fundamentalists and chartists views, we associate the longer-term expectations, which are consistently stabilizing, with the fundamentalists, and the shorter term forecasts, which seem to have a destabilizing nature, with the chartists expectations. Within each of the above tables, it is as if there are actually two models of expectations operating, one at each end of the forecasting horizons, and a blend in between. Under this view, respondents use some weighted average of the chartist and fundamentalist forecasts in formulating their expectations for the value of the dollar at a given future date, with weights depending on how far off that date is.*"

As Hommes (2006) explains, Frankel and Froot (1990a,b) use three different sources for their survey data on exchange rate expectations of financial specialists, bankers and currency traders. Some of the surveys go back to 1976 and include telephone interviews. Those interviewed were asked for their exchange rate expectations at time horizons varying from 1 week to 12 months. What is interesting is that respondents' short-term expectations were frequently quite different from their long-term expectations. Their main finding is that agents are, then using chartist, fundamentalist forecasting rules or some mixture of them. But we still need an explanation for the sudden changes that occur. Frankel and Froot (1990b), conclude that:

> "*... it may indeed be the case that shifts over time in the weight that is given to different forecasting techniques are a source of changes in the demand for dollars, and that large exchange rate movements may take place with little basis in macroeconomic fundamentals.*"

In the example I will present, I will suggest an extreme version of this idea. This is that agents are following either a fundamentalist or chartist rule at a given point in time and that they shift, stochastically, depending on the evolution of the market and the rules being used by the other agents.

6 An example, the market for a foreign asset[4]

I will look at a market for a foreign asset and for a domestic safe investment. The demand for each of these will be based on the price expectations or forecasts of the traders. Agents choose one of two types of rule, *fundamentalist* and *chartist* but, any finite number of rules would fit into the same framework.

First let me look at the investor who wishes to forecast the value of the exchange rate s_{t+1} in order to make her investment decision. If she is a *fundamentalist* she believes there is an equilibrium value $\bar{s}_t$ to which the exchange rate will revert. For the sake of simplicity give the following form to these beliefs:

$$E^f\left(s_{t+1}\,|I_t\right) = \bar{s}_t + \sum_{j=0}^{M_f} \nu_j\left(s_{t-j+1} - \bar{s}_{t-j}\right) \quad \text{with} \sum_{j=0}^{M_f} \nu_j = 1, \tag{6}$$

where M_f is the finite memory length of the fundamentalists and I_t is all the relevant information available to the agent. In other words the fundamentalist has observed price deviations from those that she forecast in the past and believes that the deviation this time will be a weighted sum of those.

If she is a *chartist* her forecast as to the future value of the exchange rate will be an extrapolation of its past values. Think of these extrapolations as being a moving average of past prices and having the following form:

$$E^c\left(s_{t+1}\,|I_t\right) = \sum_{j=0}^{M_c} h_j s_{t-j}, \tag{7}$$

where the h_j are positive constants and M_c is the finite memory length of the chartists.

Before being able to talk about the evolution of this market we need to know precisely what is being traded. There is a domestic asset with a safe rate of return and a foreign asset which earns a dividend which is fixed. However, the foreign asset is risky from the point of view of the domestic investor since its value is denominated in foreign currency and the exchange rate is uncertain. To make this clearer define the following variables at time t:

- ρ the dividend in foreign currency paid on one unit of foreign currency,
- s_t the exchange rate at time t,
- f_t^i the demand by individual i for foreign currency
- d_t^i the demand by individual i for domestic currency
- r the interest rate on domestic assets.

[4] The model presented here is a simplified version of ones developed with Hans Foellmer, Gilles Teyssiere, Ulrich Horst and Richard Topol. For more details see Kirman and Teyssiere (2005), Kirman et al. (2006), and Foellmer et al. (2005).

The individual's wealth at time t is determined by her investments in foreign and domestic assets and what she earned on each of them respectively. That is,

$$W_t^i = (1+r)d_{t-1}^i + s_t(1+\rho)f_{t-1}^i. \tag{8}$$

At each point in time the individual's demands for foreign and domestic assets must satisfy the budget constraint:

$$W_t^i = d_t^i + s_t f_t^i. \tag{9}$$

The gain for an individual in period t is given by:

$$g_t^i = W_t^i - (1-\omega)W_{t-1}^i, \tag{10}$$

where ω is the discount factor and the cumulative gain is given by:

$$G_t^i = W_t^i - (1-\omega)^{t-1}W_1^i, \tag{11}$$

where W_1^i is individual i's wealth at the beginning of period one (before she chooses the rule to be used).

However, the gains at each period are determined by the demands for domestic and foreign currency in the previous period and these are in turn determined by the forecast that the individual made as to the exchange rate in the next period. The latter depends on whether she was following the fundamentalist or chartist rule.

Now the question is which rule is chosen? The particular assumption made here is that the choice of rule is probabilistic and the probabilities of choosing a rule can be determined in a number of ways. Here I shall describe an individual behavioural learning process. In this case, the choice will depend on the success of the rules in the past in terms of the profit that was obtained when using them. Once again the probabilities could depend on other measures of success, such as the accuracy of the forecasting rules in the past, but in any particular model we have to decide on what governs the choice of rule. Recall the relation that I mentioned earlier for reinforcement learning.

$$P_t^{ik} = \frac{\mathrm{e}^{\beta W_t^{ik}}}{\sum_j \mathrm{e}^{\beta W_t^{ij}}}$$

Now if we want, as in this case, the probabilities to depend on the profitability of the rules in the past, we need to keep a total of the gains obtained by individual i up to period t by following the fundamentalist rule and similarly for the chartist rule. At each point the choice of which rule to use will be random and therefore we have to assign the profit at that period to the rule chosen in that period. 'To do this define a random variable θ_t^i which will take on two values F and C, that is:

$$\theta_t^i = F \text{ with probability } p_t^i(F)$$
$$\theta_t^i = C \text{ with probability } 1 - p_t^i(F) = p_t^i(C).$$

Now we can define an indicator function for the random variable and this is simply:

$$\begin{array}{ll} \mathrm{I}_t(F) = 1 & \text{if } \theta_t^i = F \text{ and } 0 \text{ if } \theta_t^i = C \\ \mathrm{I}_t(C) = 1 & \text{if } \theta_t^i = C \text{ and } 0 \text{ if } \theta_t^i = F. \end{array} \tag{12}$$

This leads to the gains for an individual when she was using each forecasting rule as:

$$\begin{aligned} G_t^i(F) &= \sum_{\mathrm{r}=1}^{\mathrm{t}-1} \mathrm{I}_\mathrm{r}(F)\left(G_r^i - G_{r-1}^i\right) \\ G_t^i(C) &= \sum_{\mathrm{r}=1}^{\mathrm{t}-1} \mathrm{I}_\mathrm{r}(C)\left(G_r^i - G_{r-1}^i\right) \end{aligned} \tag{13}$$

with $G_0^i(F) = G_0^i(C) = 0$.

At any point in time the investor's knows the rule θ_t^i she has drawn. the past values of her demands for foreign and domestic assets in the previous periods, the vector of observed exchange rates up to period t–l and the cumulated gains that she has realised from using each of the two forecasting rules, fundamentalist and chartist. The *total information* available to the individual i once her forecasting rule has been determined is given by I_t^i.

Next we wish to specify the agent's demand.

7 The demand of an investor for foreign currency

Assume that we can write the demand for agent i as:

$$f_t^i \equiv f_t^i\left(s_t, \theta_t^i\right) = KE\left(\tilde{s}_{t+1} \left| \mathrm{I}_t^i, \theta_t^i\right.\right) - Vs_t \tag{14}$$

where K and V are constants. This is the form originally proposed by Frankel and Froot. Now, it can be shown that a demand function of this form can be derived from the maximisation of a mean-variance or CARA utility function (see Kirman and Teyssiere 2005 or Kirman 2006 for example).[5]

$$f_t^i \equiv f_t^i\left(s_t, \theta_t^i\right) = K'(1+\rho)E\left(\tilde{s}_{t+1} \left| \mathrm{I}_t^i, \theta_t^i\right.\right) - (1+r)s_t \tag{15}$$

[5] This will only be an approximation since to derive this exact form one has to assume that the variance of is constant which is not, in fact, the case. It suffices however, to justify this, to assume that the agent considers it to be constant.

Clearly the expectations in this expression are conditioned on which rule the agent is following and recalling the definition of the indicator function for the random variable given in (12) we can write:

$$f_t^i\big(s_t,\theta_t^i\big) = f_t^i(s_t,F)\mathrm{I}_t(F) + f_t^i(s_t,C)\mathrm{I}_t(C) \tag{16}$$

8 The market equilibrium

Here we consider that the market clears at each period and hence that we are looking at a series of *temporary equilibrium* prices. This is just to simplify matters, agents could be less myopic and furthermore, some economists would insist that their expectations should be consistent with the stochastic price process. Doing this would change the dynamics completely and moreover, would remove the source of the fluctuations that characterise the dynamics of the prices in our model.

So, in order to establish our temporary equilibria, we have first to consider the aggregate demand for foreign currency of the investors. This is given by:

$$\Phi_t \equiv \sum_i f_t^i\big(s_t,\theta_t^i\big) = \sum_i f_t^i(F)I_t^i(F) + \sum_i f_t^i(C)I_t^i(C) \tag{17}$$

or if we define the number of investors who use the fundamental rule at time t as $N_t(F)$ and the total number of investors as N and $k_t = N_t(F)/N$ then we can rewrite aggregate demand as:

$$\Phi_t = N\big[f_t(F)k_t + f_t(C)(1-k_t)\big]. \tag{18}$$

Now to complete the model assume that there is a random liquidity supply X_t of foreign currency coming from underlying trade for example. Then, *the equilibrium exchange rate* s_t^* is given by:

$$\Phi_t - X_t = 0. \tag{19}$$

Hence, we obtain the following expression for the equilibrium exchange rate:

$$s_t^* = N\frac{(1+\rho)}{(1+r)}\big(\big[s_{t+1}^a(F)k_t + s_{t+1}^a(C)(1-k_t)\big] - X_t\big), \tag{20}$$

where, $s_{t+1}^a(F)$ and $s_{t+1}^a(C)$ represent the expectations under each of the two rules, fundamentalist and chartist as to the next exchange rate s_{t+1} that is:

$$s_{t+1}^a(F) = E\big(\tilde{s}_{t+1}\,|\mathrm{I}_t,F\big) \quad \text{and} \quad s_{t+1}^a(C) = E\big(\tilde{s}_{t+1}\,|\mathrm{I}_t,C\big). \tag{21}$$

and recall that X_t is the exogenous supply of foreign exchange, which we will take to be a constant with additive noise. Now if we substitute the expressions for the expectations under the two rules we have:

$$s_t^* = N\frac{(1+\rho)}{(1+r)}\left(\left[k_t\left[\bar{s}_t + \sum_{j=0}^{M_f} \nu_j \ \left(s_{t-j+1} - \bar{s}_{t-j}\right)\right] + (1-k_t)\sum_{j=0}^{M_c} h_j s_{t-j}\right] - X_t\right). \quad (22)$$

Examination of (22) shows that when the fundamentalists dominate the process will be mean reverting whereas when the chartists dominate it can become explosive. Given the rules that the agents use to choose between the rules the proportion of fundamentalists at any one time will be concentrated around 0 or 1 and will switch between the two. Thus the process will exhibit periods of "bubble" like behaviour alternating with periods where it will revert to the fundamentals.

9 Discussion

The point of the example I have just given, is that agents will change their expectations as they learn and, given the self reinforcing aspect of their forecasts, prices will move with them. What the example shows is that it may be the case that, as a result of the self-reinforcing character of the process, agents will cease for some periods to believe in any fundamentals and that they will become "chartists" or trend chasers. For a while heterogeneity disappears. In this case markets detach themselves from any fundamentals for a period but then return, [for a good overview of theoretical aspects and empirical evidence for this see Hunter et al. 2003]. Such a vision fits well with the burgeoning econometrics literature on "switching regimes" initiated by Hamilton (1989) and provides an economic framework for this phenomenon. During the critical periods in which agents switch from one belief to another there will be considerable heterogeneity of expectations, consistent with empirical observations; see Chionia and Mac Donald (2002). Yet, over time, agents cannot be said to be behaving irrationally. Curiously, the collectively rational aspect can be associated with periods where agents coordinate on a common view of the future. But this view will change over time.

What I am suggesting then, is that changes in the distribution of expectations as a result of the mutual influences of agents, play a key role in explaining the evolution of prices. Of course, in the standard "representative agent" model there is no place for any such interaction and resultant heterogeneity of expectations. Indeed, many authors have suggested that this is the reason for the poor predictive power of such models. This argument has been made in particular for the foreign exchange market and evidence for this is given by Meese and Rogoff (1988), Frankel and Rose (1995) and Cheung et al. (2002). It is worth emphasising again, in the sort of model proposed here, heterogeneity is largely a temporary phenomenon, and for much of the time people have similar expectations.

One further observation is in order. The example that I have given suggests that there will be alternation of bubbles and reversion to fundamentals. However, one might legitimately ask if there is any real structure to such a temporal evolution. To know that the process oscillates between extremes of opinions is interesting in itself but one would like to know if there is any recognisable structure. In Foellmer et al. (2005) we analysed a similar model and showed under what conditions the price and profits process had a "limit distribution". In other words, we showed under which conditions the time averages of the process converge, or the proportions of time spent in each price range converge. This means that the process is shown to be ergodic and to have a unique limit distribution. The essential condition for this is that the probability of an agent becoming a chartist must not be too high. This result means that, even though one cannot predict when the price process is going to return to the fundamentals, one does know what the structure of prices looks like in the long run. This is of course, a different notion of equilibrium from the standard one, there is no convergence to a steady state, the prices change all the time but in the long run there is a clear pattern. The form of this pattern is conditioned by the fluctuations between periods of almost no heterogeneity and periods of flux in which the heterogeneity increases. When the heterogeneity disappears it is because the agents have herded on a particular type of expectation but this, itself, changes over time. This seems more appropriate as an equilibrium notion for the sort of process that I have outlined and which can be generated by many versions of this sort of model and details of which can be found in Hommes (2006).

10 Conclusion

Heterogeneity plays an important role in the evolution of the economy and I have discussed three aspects here. In the first place, faced with the problems posed by the SMD results for the General Equilibrium model some economists such as Grandmont, argued that imposing heterogeneity of the characteristics of economic agents might solve the problems of uniqueness and stability of equilibrium. This is for the moment, in abeyance, but it did raise the question of precisely what heterogeneity means in this context. Heterogeneity of parameters of utility functions does not necessarily mean heterogeneity of behaviour. A more promising route is that followed by Hildenbrand (1994), who shows that increasing dispersion of individual demands as income increases will lead to aggregate demand satisfying the "Law of Demand" and hence to uniqueness and stability of equilibrium. This type of reasoning is not usual in economics and has not, so far, met with wide acceptance. Showing that a certain empirically observable fact will lead by deductive reasoning to a certain characteristic of aggregate demand for example, without ever worrying about underlying utility maximisation is too pragmatic an approach for many economists. Nevertheless, it has much to be said for it, if one is really interested in explaining economic phenomena.

The second section was devoted to the problem of evolution and the role of learning and adaptation. Heterogeneity is an essential feature of evolution, otherwise there would be no locus for selection. To what extent this heterogeneity is reduced as individuals adapt and learn more profitable behaviour remains an open question. To a large extent, the problem is posed in terms of the relative speed of adaptation and of the environment. In economics where the environment is essentially made up of the other agents who will also, presumably be learning and adapting, it is not clear that this adaptation will converge, nor, even if it does, that it will correspond to an optimum. The nature and speed of innovation, in comparison with the speed of adaptation and learning, is what is important here and it has to be shown that even short term optima will be reached. The alternative view is that of a large set of agents with differing characteristics producing aggregate regularities, but with no convergence to any steady state, through their adaptation and local interaction and in many ways this view is both less demanding and more appealing.

The last section was devoted to an examination of the role of heterogeneity in financial market models. Here the emphasis was on the fact that in reality a very large volume of trade occurs on these markets and this should be a direct result of individuals' behaviour and expectations. Clearly in the typical "representative agent' model one cannot talk about trade. Yet trade occurs when expectations or forecasts are different and the sort of model that I outlined can generate diversity of expectations for periods interspersed with periods in which expectations coincide. In this way, increased volatility is associated with agents who have previously been herding, shifting from one forecasting rule to another. This sort of changing heterogeneity is not purely haphazard and one can give a notion of stochastic equilibrium and the conditions that ensure that it will be achieved.

In conclusion heterogeneity plays a role in all aspects of the economy and it does not seem reasonable to dismiss it as unimportant or to assume it away.

References

Adjare SO (1990) Beekeeping in Africa. FAO Agricultural Services Bulletin 68/6Food and Agriculture Organisation of the United Nations Rome

Allen F, Morris S, Song Shin H (2003) Beauty contests. Bubbles and iterated expectations in asset markets. Cowles Foundation discussion paper no 1406, Yale University

Anderson S, de Palma A, Thisse J (1993) Discrete choice theory of product differentiation. MIT Press, Cambridge

Avery C, Zemsky P (1998) Multidimensional uncertainty and herd behaviour in financial markets. Am Econ Rev 88:724–748

Bacchetta P, van Wincoop E (2005) Can information heterogeneity explain the exchange rate determination puzzle?. Am Econ Rev (forthcoming)

Billette de Villemeur E (1999) Aggregation of demand and distribution of characteristics: A difficulty in modelling behavioural heterogeneity. Working papers 99–38, THEMA

Blanchard OJ, Watson MW (1982) Bubbles, rational expectations and financial markets. In: Wachtel P (ed) Crisis in the economic and financial system. Lexington Books, Lexington, pp 295–315

Brock WA (1993) Pathways to randomness in the economy: emergent nonlinearity and chaos in economics and finance. Working Papers 93-02-006, Santa Fe Institute

Brock W, Hommes C (1997) A rational route to randomness. Econometrica 65(5):1059–1096

Brock W, Durlauf S (2001a) Interactions-based models. In: Heckman J, Leamer E (eds) Handbook of econometrics, vol 5. Amsterdam, North-Holland

Brock WA, Durlauf SN (2001) Discrete choice with social interactions. Rev Econ Stud 68:235–60

Brooks C, Katsaris A (2003) Rational speculative bubbles: an empirical investigation the London stock exchange. Bull Econ Res 55:319–346
Chamley C (2004) Rational herds. Cambridge University Press, Cambridge
Cheung Y-W, Chinn MD (2001) Currency traders and exchange rate dynamics: a survey of the US market. J Int Money Finance 20:439–471
Cheung Y-W, Chinn MD, Pascual AG (2002) Empirical exchange rate models of the nineties: are they fit to survive? NBER WP 9393
Chiappori P-A (1985) Distribution of income and the law of demand. Econometrica 53:109–127
Chionia D, Mac Donald R (2002) Aggrégate and disaggregate measures of the foreign exange risk premium. Int Rev Econ Finances 11:57–84
Crane E (1999) The World history of beekeeping and honey hunting. Routledge, London
Cutler DH, Poterba JM, Summers LH (1988) What moves stock prices?. Working papers 487, Massachusetts Institute of Technology (MIT), Department of Economics
Debreu G (1974) Excess demand functions. J Math Econ 1:15–23
De Long JB, Schleifer A, Summers LH, Waldmann RJ (1989) The size and incidence of the losses from noise trading. J Finance 44:681–699
De Long JB, Shleifer A, Summers LH, Waldmann RJ (1990) Noise trader risk in financial markets. J Polit Econ 98:703–738
Donaldson RG, Kamastra M (1996) A new dividend procedure that rejects bubbles in asset prices: the case of the 1920's stock crash. Rev Financ Stud 9:333–383
Fehr E, Gächter S (2000) Fairness and retaliation: the economics of reciprocity. J Econ Perspect 14:159–181
Flood RP, Garber PM (1980) Market fundamentals versus price level bubbles: the first tests. J Poli Econ 88:7497
Flood RP, Hodrick RJ (1999) On testing for speculative bubbles. J Econ Perspect 4:85–102
Foellmer H, Horst U, Kirman A (2005) Equilibrium in financial markets with heterogeneous agents: a new perspective. J Math Econ 41:123–155
Frankel JA, Froot KA (1990a) Chartists, fundamentalists and the demand for dollars. In: Courakis AS, Taylor MP (eds) Private behaviour and government policy in interdependent economies. Oxford University Press, Oxford, pp 73–126
Frankel JA, Froot KA (1990b) Exchange rate forecasting techniques, survey data, and implication for the foreign exchange market. IMF Working Paper, WP/90/43
Frankel JA, Rose AK (1995) Empirical research on nominal exchange rates. In: Grossman GM, Rogoff K (eds) Handbook of international economics, vol lll. North Holland, Amsterdam
Friedman M (1953) The methodology of positive economics. In: Essays in positive economics. University of Chicago Press, Chicago, pp 3–43
Garber PM (2000) Famous first bubbles. MIT Press, Cambridge
Grandmont J-M (1987) Distributions of preferences and the 'Law of Demand'. Econometrica 55(1):155–161
Grandmont J-M (1992) Transformations of the commodity space, behavioural heterogeneity, and the aggregation problem. J Econ Theory 57:1–35
Hamilton JD (1989) A new approach to economic analysis of nonstationary time series and the busiiness cycle. Econometrica 16:357–384
Hardle W, Hildenbrand W, Jerison M (1991) Empirical evidence on the law of demand. Econometrica 59:1525–1549
Hichri W, Kirman A (2005) Do individuals learn to play Nash? Evidence from public goods games. GREQAM working paper Marseille
Hildenbrand W (1994) Market demand: theory and empirical evidence. Princeton University Press, Princeton
Hildenbrand W (1983) On the Law of Demand. Econometrica 51:997–1019
Hildenbrand W, Kirman A (1988) Equillibrium Analysis. North Holland, Amsterdam
Hildenbrand W, Kneip A (1993) Family expenditure data, heteroscedasticity and the law of demand. Richerche Econ 47:137–165
Hildenbrand K, John R (2003) On parametrization in modelling behavioral heterogeneity. University of Copenhagen Economic Discussion Paper, 03-27
Hildenbrand W, Kneip A (2005) On behavioral heterogeneity. Econ Theory 25:155–169

Hommes C (2006) Heterogeneous agent models (HAMs) in economics and finance. In: Judd KL, Tesfatsion L (eds) Handbook of computational economics, vol 2, Agent-based computational economics. North Holland, Amsterdam
Hunter WC, Kaufman GG, Pomerleano M (2003) Asset Price Bubbles. MIT Press, Cambridge
Johnson H (1968) The economic approach to social questions. Economica 35:1–21
Kelly M, Grada CO (2000) Market contagion: evidence from the panics of 1854 and 1857. Am Econ Rev 90:1110–1125
Keynes JM (1936) The general theory of employment, interest and money. Macmillan, London
Kirman A (2006) The microeconomic foundations of instability in financial markets. In: Kirman A, Teyssiere G (eds) Long memory in economics. Springer, Berlin Heidelberg New York (forthcoming)
Kirman A, Ricciotti R, Topol R (2006) Bubbles and foreign exchange markets: it takes two to tango. Macroecon Dyn (forthcoming)
Kirman A, Teyssiere G (2005) Testing for bubbles and change points. J Econ Dyn Control 29:765–799
Kirschner M, Gerhart J (2005) The plausibility of life. Yale University Press, New Haven
Ledyard J (1997) Public goods: a survey of experimental research. In: Kagel John H, Alvin E. Roth (eds) Handbook of experimental economics. Princeton University Press, Princeton
Lindgren K (1991) Evolutionary phenomena in simple dynamics. In: Langton CG, Taylor C, Farmer JD, Rasmussen S (eds) Artificial life. Addison-Wesley, Redwood City
Lucas R (1988) Adaptive behaviour and economic theory. J Business 59:S401–S426
Lui Y-H, Mole D (1998) The use of fundamental and technical analyses by foreign exchange dealers: Hong Kong evidence. J Int Money Finance 17:535–545
Mantel R (1974) On the characterisation of aggregate excess demand. J Econ Theory 7:348–353
McFadden D (1974) Conditional logit analysis of qualitative choice behavior. In: Zarembka P (ed) Frontiers in econometrics. Academic Press, New York, pp 105–142
Meese RA (1986) Testing for bubbles in exchange markets. J Political Econ 94:345–373
Meese R, Rogoff K (1988) Was it real? the exchange rate – interest differential relationship over the modern floating-rate period. J Finance 43:933–948
Menkhoff L (1997) Examining the use of technical currency analysis. Int J Finance Econ 2:307–318
Morishima M (1964) Equilibrium, stability, and growth; a multi-sectoral analysis. Clarendon Press, Oxford
Nelson R, Winter S (1982) An evolutionary theory of economic change. Harvard University Press, Cambridge
Oberlechner T (2001) Importance of technical and fundamental analysis in the European foreign exchange market. Int J Finance Econ 6:81–93
Oberlechner T (2004) The psychology of the foreign exchange market. West Sussex, England
Quah J (1997) The law of demand when income is price dependent. Econometrica 65(6):1421–1442
Shiller R (2000) Irrational exuberance. Princeton University Press, Princeton
Stiglitz JE (1990) Symposium on bubbles. J Econ Perspectives 4:13–18
Sonnenschein H (1972) Market excess demand functions. Econometrica 40:549–556
Taylor MP, Allen H (1992) The use of technical analysis in the foreign exchange market. J Int Money Finance 11:204–214
Tirole J (1985) Asset bubbles and overlapping generations. Econometrica 53:1490–1528
Townsend RM (1983) Forecasting the forecasts of others. J Polit Econ 91:546–588
von Hayek F (1989) Spontaneous ('grown') order and organized ('made') order. In: Modlovsky N (ed) Order – with or without design? Centre for Research into Communist Economies, London, pp 101–23
Weisbuch G, Chenevez O, Kirman A, Nadal J-P (1998) A formal approach to market organization: choice functions, mean field approximations and maximum entropy principle. In: Lesourne J, Orlean A (eds) Advances in self-organization and evolutionary economics. Economica, pp 149–159
West KD (1988) Bubbles, fads, and stock price volatility tests: a partial evaluation. J Finance 43:639–655
Winter S (1962) Economic "natural selection" and the theory of the firm. Yale Econ Essays 4:225–272
Woo WT (1987) Some evidence of speculative bubbles in the foreign exchange markets. J Money Credit Bank 19:499–514