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Some new results on the Levy, Levy and Solomon microscopic stock market model

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Abstract

We report some findings from our simulations of the Levy, Levy and Solomon microscopic stock market model. Our results cast doubts on some of the results published in the original papers (i.e., chaotic stock price movements or emergence of a power-law distribution of returns). We also point out the possibility of sensitive dependence on initial conditions of the emerging wealth distribution among agents. Extensions of the model set-up show that with varying degrees of risk aversion, the less risk averse traders will tend to dominate the market. Similarly, when introducing a new trader group (or even a single trader) with a constant share of stocks in their portfolio, the latter will eventually take over and marginalize the other groups. The better performance of the more sober investors is in accordance with traditional perceptions in financial economics. Hence, the survival of 'noise traders' looking at short-term trends and patterns remains as much of a puzzle in this framework as in the traditional Efficient Market Theory. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The artificial stock market model by Levy, Levy and Solomon [1–5], whose first printed version appeared in 1994, is an early example of physicists' interest in modeling interaction in financial markets and the application of concepts of statistical physics to economic problems. It is also an interesting example of combination of ideas from both fields: it applies a standard economic utility maximization approach for the derivation of market activities of individual investors which is augmented by stochastic factors

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for, what the authors call, a modulation of the 'temperature of the market' – the latter idea being obviously borrowed from physical models in quite different areas. The model is successful in generating very spectacular (but not entirely realistic) crashes and upheavals of the market price [1,2]. Furthermore, varying combinations of time horizons among traders lead to diverse and sometimes surprising results for the time development of the share of wealth owned by different groups [4]. The existing papers are, however, silent on the scaling laws for prices and returns, but Levy and Solomon [3] claim that realistic power laws (in accordance with Pareto's famous law) for the distribution of wealth among agents can be obtained from the model.

In this note, we report some of our insights from a reinvestigation of the Levy, Levy, Solomon model: first, there is an interesting finding of an extreme form of sensitivity with respect to initial conditions. Namely, we found, that with exactly the same simulation design in terms of trader's time horizons and other relevant quantities, a small change in the initial conditions (i.e., the random price history given at the start of the simulations), may result in totally different long-term outcomes. Second, experiments with alternative utility functions showed that when we allow for different attitudes towards risk, traders with a lower degree of risk aversion will tend to dominate the market. If, additionally, we introduce traders with a constant proportion of shares in their portfolio, the latter will eventually dominate all other groups and over time gain the lion's share of the available wealth. Third, on a more critical note, statistical analyses raise questions about the finding of chaotic dynamics emphasized in Levy, Levy and Solomon [2] and Levy et al. [4]. Rather, we present evidence and intuition that what has been denoted 'chaos' is in fact pretty close to pure randomness.

2. The model set-up and previous results

2.1. Model structure

At the beginning of every period each investor i needs to divide his entire wealth $W(i)$ into shares and bonds. His wealth, therefore, is split into a fraction $X(i)$ held in shares at time t and a fraction $1 - X(i)$ held in bonds. Since credit or short sales of stock are not allowed, $X(i)$ is bound away from 0 and 1, i.e., $0.01 \leqslant X(i) \leqslant 0.99$. Additionally, the model assumes that the number of investors, n, as well as the supply of stocks, N_A, are fixed. Apart from an identical utility function $U(W_{t+1})$ investors at the beginning also have the same amount of wealth and the same number of stocks. Whereas the riskless bond pays a fixed interest rate r, the stock return H_t is time varying and subject to uncertainty. It is composed of two components: first, it includes capital gains or losses resulting from changes of the market price (P_t). Second, shareholders receive a dividend payment D_t per period which is assumed to follow a stochastic growth path. Returns of the risky asset are, therefore, given by

$$H_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} .$$

In the basic version of the model, preferences of investors are described by a logarithmic utility function which is in accordance with the usual assumption of diminishing marginal utility of wealth and risk aversion of investors. Since future returns appear in the utility maximization problem of investors, their expectations about future prices and returns have to be considered. In Levy, Levy and Solomon, the pertinent assumption is that investors have a limited memory span of k periods and expect the returns observed within this interval to occur in the next period with equal probability of $1/k$. Given these expectations, the expected utility $E[U(X(i))]$ can be maximized with respect to the number of stocks demanded by the agent. In their simulations, Levy, Levy and Solomon consider one or more groups of investors with identical memory span k. Once the optimum number of shares has been calculated for each investor group, each individual investor's demand is computed by adding a normally distributed random number ε_i to the outcome of the maximization process. This leads to heterogeneity within groups with fluctuations around the average size of share demand, which may result from idiosyncratic factors that are not captured in the overall framework. With all individual demand functions given, the new stock price is computed as the market equilibrium price (i.e., a price which leads to identity of demand and supply).

2.2. Previous results

Simulations with only one investor group show periodic stock price fluctuations (cf. Fig. 1) whose period depends on the memory span k. This price development can be explained as follows: Assume that, at the start of the simulation, a random history of total stock returns H_t is drawn that encourages investors to increase the share of risky assets in their portfolio. The increase in demand causes an increase in prices and, therefore, a new positive return emerges. This causes the investor group to raise their stock shares successively up to a maximum of 99%. At this situation accompanied by a high price level the outcome remains constant for slightly more than k periods until the extremely positive returns of the boom period are 'forgotten'. Since, however, the real dividend is small because of the considerably high stock price, a relatively small (negative) total stock return is sufficient (caused by the noise term ε) in order to make the riskless bond appear to be more attractive than stocks. The demand for shares and together with it the stock price breaks down and a crash occurs. If such a crash happens its extremely negative contribution to the history of returns makes the fraction of stock drop to a minimum of 1%. Again, it takes about another k periods for the investors to forget about the extremely negative returns.

If two groups with different memory spans are considered, quite often time series also have a periodic appearance. However, depending on the choice of memory span, changes in dynamics can appear. Interestingly, looking at the distribution of total wealth, dominance over share price development then does not necessarily mean dominance over total wealth. The model outcome becomes more diverse with three

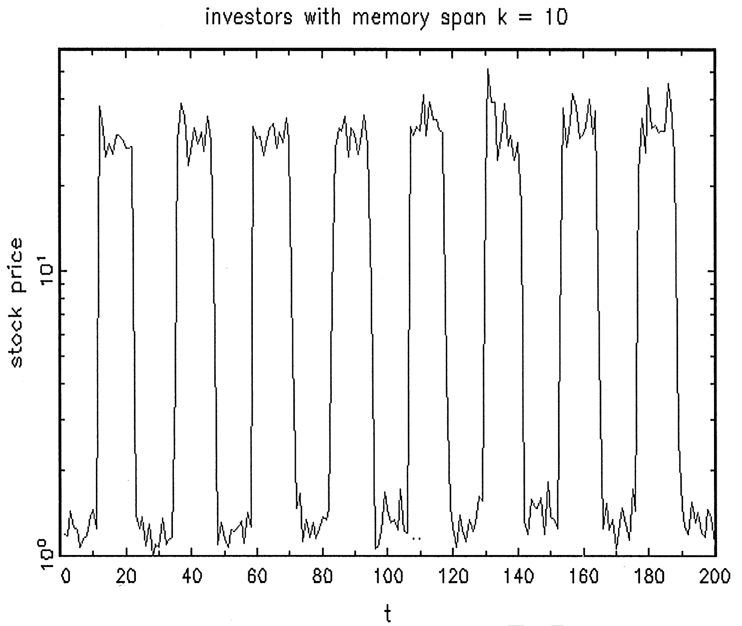

Fig. 1. With only one type of traders and a logarithmic utility function, the typical outcome of the Levy, Levy and Solomon model is a cyclic development of stock prices with periodic booms and crashes. As concerns the visual appearance of the price patterns, we were able to reproduce all the results shown in Refs. [1–3].

(or more) investor groups. In these more complex cases, Levy and Solomon repeatedly found what they call 'chaotic' results in stock price development.

3. New results

3.1. Sensitive dependence of long-term development in market shares

One example of a complicated dynamics with multiple investor groups analyzed in some detail by Levy, Persky and Solomon has the combination $k = 10$, 141, and 256 with equal number of traders in each group. It appeared from their experiments that the group with $k = 256$ usually was the dominant one. As shown in Fig. 2, this latter result could only partially be confirmed by our research. Multiple runs of this simulation scenario, in fact, showed significant deviations from the results reported by Levy, Levy and Solomon. Interestingly, we observed distributions of wealth that were dominated by the group with $k = 256$ as well as others with dominance by the one with time horizon $k = 141$. We also found that the outcome in terms of dominance of one group or the other depends on the overall number of agents: increasing the number of

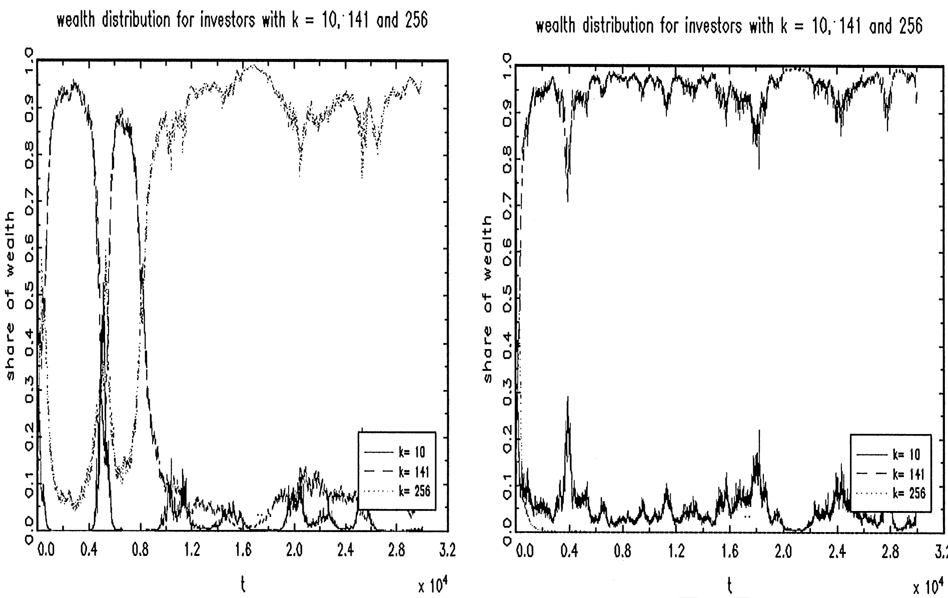

Fig. 2. Development of the distribution of wealth with three groups characterized by $k = 10$, 141 and 256, respectively. Depending on the initial conditions, either the group with memory span 256 (left-hand side) or the group with $k = 141$ (right-hand side) may happen to dominate the market.

agents from the baseline scenario of $n = 100$ with otherwise identical simulation design leads to repeated changes of the dominance pattern.

Hence, the overall outcome of the model shows an extreme dependence on initial conditions in this case (and probably in other cases as well). It seems worth emphasizing that the initial set-up differs only very slightly between different runs: since investors need to be equipped with past prices at time $t = 0$ in order to compute their optimal share of assets, we have to generate a vector of realizations of random numbers with the number of entries equal to the maximum k among groups. The only difference in initial conditions is, then, that we used different random draws in our simulations. However, in the scenario above with the highest k equal to 256, the differences in the mean returns over various draws should really be minute. Nevertheless, they appear to be sufficient to give an initial advantage to one or the other group that cannot be overcome any more in the following. Hence, what we find, in some sense, is that 'success breeds success', even if it occurred by chance.

3.2. Alternative utility functions

In order to get a clue to the sensitivity of the model, we also performed simulation runs with some alternative utility function as well as with their combinations. In economic literature, utility functions are often characterized by what is called the absolute and relative degree of risk aversion. These numbers give information about how the absolute or the relative share of invested stock capital changes with increasing wealth of the investor. Absolute risk aversion $A(W_t)$ and relative risk aversion $R(W_t)$ are given by: $A(W_t) = -U''(W_t)/U'(W_t)$ and $R(W_t) = -A(W_t)W_t$. For the logarithmic utility function, $U(W_t) = \ln(W_t)$, relative risk aversion is 1 and absolute risk aversion is W_t^{-1}. In many economic applications, the more flexible exponential, $U(W_t) = \mathrm{e}^{-\alpha W_t}$, or power-law utility function, $U(W_t) = (1/(1-\alpha))W_t^{\alpha-1}$ can be found. As can be easily computed, the former has a constant absolute degree of risk aversion equal to α, while for the latter, the relative degree of risk aversion is given by this parameter (note, that for the logarithmic utility function, introduction of an additional parameter α would make no difference).

Interestingly, with both the power-law and exponential utility functions, one can add another layer of heterogeneity because of the different degrees of risk aversions depending on α. In fact, different degrees of risk aversions of trader groups have a strong impact upon the distribution of total wealth. The higher α, the more risk averse the investors are and, accordingly, the smaller is the investor group's optimum share of assets. Consequently, the respective memory span is no longer the only decisive factor anymore, but investors' decisions depend on both the parameters α and k. If, for example, two groups have the same memory span but different risk aversion, then, in many cases the group with the higher α will eventually come to dominate the market (cf. Fig. 3).

Levy, Levy and Solomon [1] have, in fact, also introduced a power-law utility function and give the results from some simulation runs with groups distinguished by their degree of risk aversion. Looking more systematically at the interplay of risk aversion and memory span, it seems to us that the former is the more relevant factor, as with different α's we frequently found a reversal in the dominance pattern: groups which were fading away before became dominant when we reduced their degree of risk aversion. A typical example is shown in Fig. 3 where we revisit the scenario of the groups characterized by $k = 10$, 141, and 256, respectively, but instead of the logarithmic assume a power-law utility function. For the risk aversion parameter, we first choose $\alpha = 0.4$ for the groups with $k = 256$ and 10, while the remaining group, $k = 141$, was assigned a slightly higher degree of risk aversion, $\alpha = 0.6$. It turned out that with this scenario, a clear dominance of the group with the longest time horizon was obtained which held a rather constant fraction around 90% of total wealth with the $k = 141$ group owning the remaining 10% and the short-horizon investors ($k = 10$) fading away almost instantaneously. Interestingly, when we reversed the pattern of risk aversion for $k = 256$ and 141, the latter group (now less risk averse) gained dominance and the fraction of wealth of the $k = 256$ investors shrinked to slightly more than 10%. Although the $k = 10$ investors do a bit better here than in the first scenario, they are way behind the other investors.

It is worth emphasizing, that these simulation results are in remarkable agreement with standard perceptions in economic theory: those ready to accept a higher risk will also earn higher returns (on average) so that as a group (irrespective of individual failures) they will increase their share of wealth. However, note that the scenario in

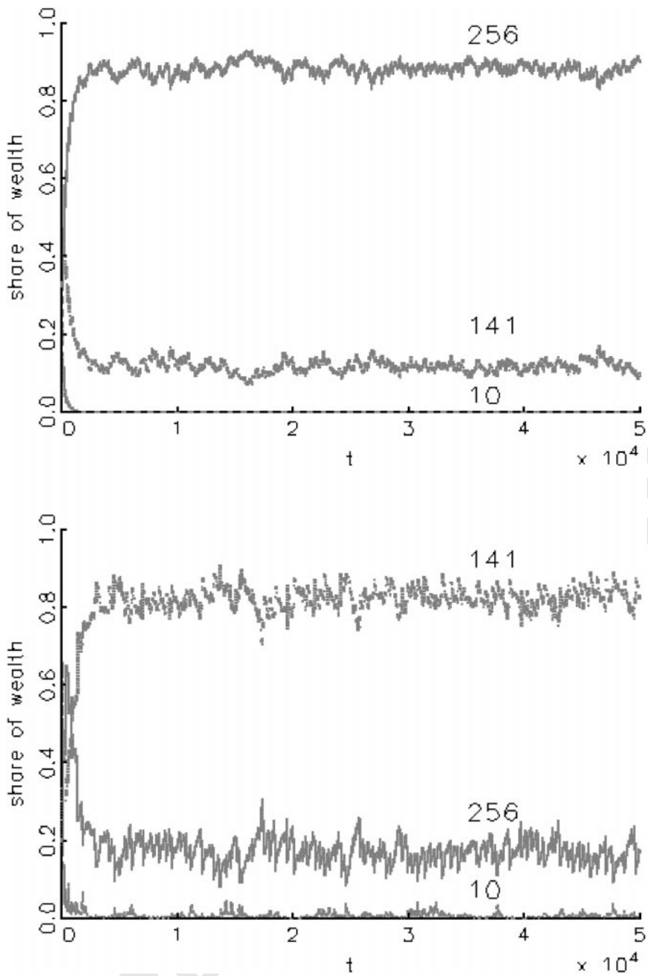

Fig. 3. Development of the distribution of wealth with three groups with different time horizons and different degrees of risk aversion. Upper panel: the three groups are characterized by k $(\alpha) = 10$ (0.4), 141 (0.6) and 256 (0.4), respectively. Here, we observe a clear dominance of the latter group. Lower panel: when exchanging the risk coefficients for the $k = 256$ and 141 groups, the dominance pattern is almost reversed. We also note, that exactly the same pictures appeared in multiple runs so that the sensitivity with respect to initial conditions reported above seems to vanish when specifying different degrees of risk aversion.

economics textbooks is one of given external (fundamental) sources of risk while here our traders have to deal with *endogenous price risk* generated by their own activities. It also appears that when adding different degrees of risk aversion, the differences of time horizons are not decisive any more, provided the time horizon is not too short. The reason for this is, probably, investors with a very short horizon will suffer more than others from the bubbles and crashes as they still have a very high fraction of assets in their portfolio when crashes occur and predominantly hold riskless bonds shortly before the boom. In contrast, the behavior of investors with a longer horizon (who overlook several cycles of booms and crashes) is less sensitive to the immediate price history so that the fluctuations in their portfolio holdings are less pronounced.

Similar results were found using the exponential utility function which yields constant absolute risk aversion. However, it is also worth mentioning that an increase of the overall number of investors to $n = 1000$ is sufficient to destroy the interesting irregular price dynamics producing definitely periodic stock price developments in models with power-law utility functions (cf. Refs. [6,7]) and almost linear price paths with the exponential utility function.

3.3. Dominance of investors with constant share of assets

The investors in the Levy, Levy and Solomon framework try to forecast future stock price movements on the basis of past observation. The standard efficient market viewpoint would, however, question the forecastability of future stock prices and the profitability of such a behavior. Since in efficient markets, prices follow a random walk, a buy-and-hold strategy should be at least as profitable as any (useless) attempts at forecasting future prices. Of course, the present simulation design does not necessarily lead to an efficient market, as we have already seen that the interaction of agents may lead to both irregular and regular cycles in stock prices. It is nevertheless interesting to explore the performance of agents who are agnostic towards the development of prices. Since we are dealing with a rising stock market (because of the assumed dividend growth), we implement this strategy by introducing investors with a constant proportion of stocks in their portfolio (which would result from an exponential utility function together with the expectation of a stationary distribution of returns, H_t). As it turns out, the introduction of such traders even in very small numbers and with very small shares of initial wealth significantly affects the overall results: In all our analyzed cases these traders steadily increased their fraction of wealth and eventually came to dominate the market. An example (with a somewhat spectacular pattern of prices and shares of wealth) is given in Fig. 4. These results are valid for all types of utility functions of the other agents. The only difference is that the higher the degree of risk aversion α of the other agents, the longer the investors with a constant share of stock will need to achieve dominance.

The reason for this result is quite simple: As we know, those investors with variable fractions of both assets are causing stock-price cycles by their trading activities. Since they are also the ones who buy stocks at relatively high prices and sell at low prices they achieve a poor performance in the end. As the constant portfolio investors are on the opposite side of the market in both cases, they, in fact, have an additional advantage from the procyclic reactions of the other traders, so that their own share in total wealth rises. Hence, it turns out, that the advantages of certain trader groups over others as pointed out in Levy, Levy and Solomon only prevail in an environment with *all* trading groups pursuing similar forecasting strategies. Once the market is invaded by a small minority with a (traditional) ‘strategy’ of a constant share of stocks in

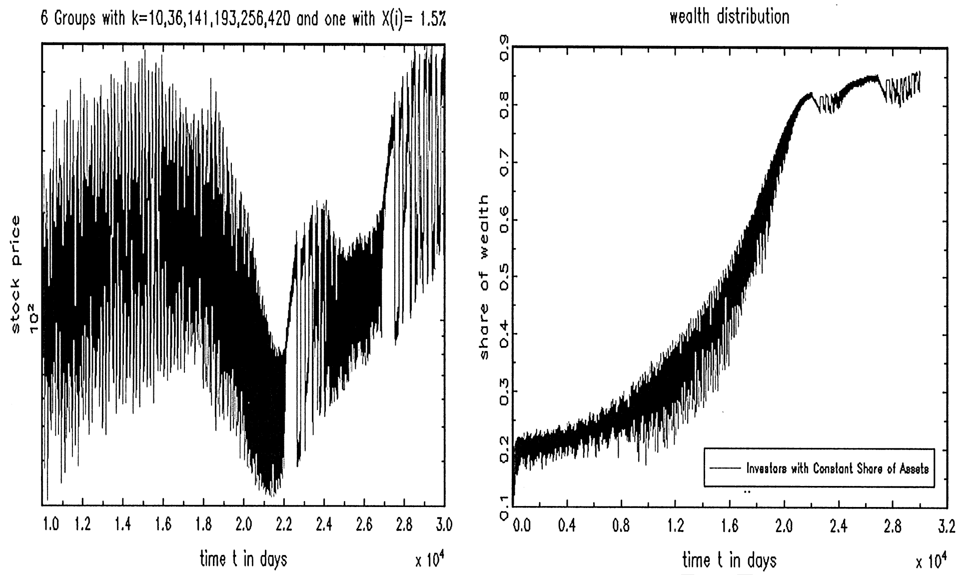

Fig. 4. One constant portfolio investor is added to market with seven groups (k = 10, 36, 141, 193, 256 and 420). Although the constant portfolio investor only holds 1.5% of his portfolio in risky assets and he also starts with a small fraction of the overall wealth, he eventually achieves dominance and asymptotically gains 100% of the available wealth.

their portfolio, the latter eventually takes over and the other strategies will vanish. Again, this is a result which confirms standard wisdom in economics: the more sober investors outperform those paying attention to short-term trends in the market. Hence, the survival of such strategies in real-life markets remains a puzzle within the Levy, Levy and Solomon microscopic simulation framework as it does within the Efficient Market Theory (cf. Ref. [8]).

3.4. *What is a 'chaotic' stock price development?*

As outlined in the Introduction, Levy, Levy and Solomon often describe scenarios in which an almost regular periodic motion appears to bifurcate into what they call a 'chaotic' stock price evolution when increasing the degree of heterogeneity among traders (by, e.g., adding additional groups). However, our re-examination of a number of such scenarios casts some doubts on the appropriateness of the term 'chaotic' for these results (which were, therefore, put in quotation marks in the preceding parts of the paper). Quite surprising on a first view, we found stock returns are often normally distributed in these cases. A typical case is shown in Fig. 5, in which the distribution of returns (relative price changes) from an extended simulation run shows an almost Gaussian shape. This visual impression is supported by statistical evidence from the standard Jarque–Bera test of normality which does not reject its null hypothesis despite

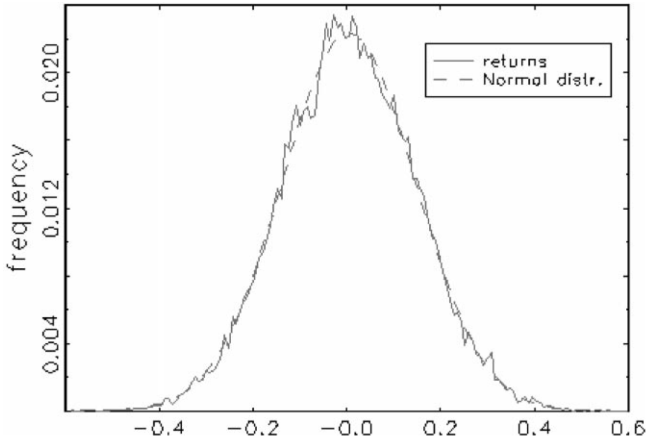

Fig. 5. Distribution of returns form a simulation with six groups characterized by memory spans $k = 10$, 36, 141, 193, 256 and 420. This is an example with a stock price development described as 'chaotic' or 'complex' in Ref. [4]. However, it seems that the result is rather close to pure randomness. The histogram is drawn for 20,000 observations after an initial transient of 100,000 time steps. The close similarity to the normal distribution is confirmed by statistical measures: Kurtosis is 0.043 and skewness is −0.003. This yields a Jarque–Bera statistic of 1.55 which does not allow to reject the normal distribution (significance is 0.46%).

the large sample of 20,000 observations used. More examples are given in Ref. [9]. From these experiments, it seems that instead of producing chaos (in the mathematical sense), in the more complicated simulation scenarios the model becomes a quite efficient random number generator! The reason is probably, that with no pronounced cycles and emerging dominance of one group with relatively long-time horizon, the behavior of most investors becomes very similar. Differences in their demanded quantities are, then, mainly due to the random component added to the maximization result. As the price formation process aggregates these n (number of traders) independent random quantities, the central limit law should apply – which apparently is the case.

4. Conclusions

This paper has reported some findings from our re-investigation of the Levy, Levy and Solomon model of a microscopic stock market. While some of these may provide interesting additional details (e.g., the perplexing sensitivity of the long-term outcome to slight changes in initial conditions), other findings may serve to question some earlier results (e.g., with respect to the chaotic properties of certain time paths). However, from our (the economists') point of view, the most interesting new result is the failure of the strategies of the original Levy, Levy and Solomon model in the presence of traders with the simple device of holding shares as a constant fraction of their wealth. Thus, the outcome of the model is much closer to the traditional viewpoint on financial markets than its authors thought: The spectacular booms and crashes only occur as long as the market is dominated by people focusing on relatively short histories of observed returns. As soon as we allow for long-term investors who disregard such short-term trends, the latter will gradually take over which, as a consequence, will also lead to a smoother development of the market. The prevalence of 'short-termism' in real-life markets remains a puzzle that cannot be solved within the current framework. In our opinion, this poor performance of the speculators is probably due to their relatively unsophisticated forecasting procedures. It remains to be shown whether the results will change with more elaborate and realistic strategies.

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