UNR ECON 741 – Fall 2020
Homework 1

Mauricio Solorio

September 14, 2020

You may collaborate with as many people as you like, but you can turn in your assignments in groups of no more than three. Please submit your answers, along with all source code (Stata, R, LaTeX) that you are using.

Bonus (15 points)

Do your work in LaTeX for up to 15 bonus points.

Chapter 2 (27 points)

Please find the derivatives for the following functions:

1. $f(x) = 3x^4$ (1 point)
    - $f'(x) = 12x^3$
2. $f(x) = \alpha x^4$ (1 point)
    - $f'(x) = 4\alpha x^3$
3. $f(x) = \alpha x^\beta$ (1 point)
    - $f'(x) = \beta\alpha x^{\beta-1}$
4. $f(x) = ln(x)$ (1 point)
    - $f'(x) = \frac{1}{x}$

5. $f(x) = x^{\beta} \times ln(x)$ (2 points)
   - $f'(x) = \beta x^{\beta-1} ln(x) + x^{\beta} \frac{1}{x}$
6. $f(x) = exp(4x)$ (2 points)
   - $f'(x) = 4e^{4x}$
7. $f(x) = exp(x^5)$ (2 points)
   - $f'(x) = 5x^4 e^{x^5}$
8. $f(x) = exp(x^5) ln(4x^4)$ (3 points)
   - $f'(x) = 5x^4 e^{x^5} ln(4x^4) + e^{x^5} \frac{2}{x}$
9. With respect to y: $f(x, y) = x^{\alpha} \times y^{\beta}$ (3 points)
   - $f'(x) = \beta x^{\alpha} y^{\beta-1}$
10. With respect to x: $f(x, y) = x^{\alpha} \times y^{\beta}$ (3 points)
    - $f'(x) = \alpha x^{\alpha-1} y^{\beta}$
11. Find the value(s) of x that maximizes or minimizes this function: $f(x) = x^3 - 3x$ (4 points)
    - $f'(x) = 3x^2 - 3 = 0; x = \sqrt{1} = +/-1$
12. Note whether the point(s) that you described above are maximums or minimus (4 points)
    - $f''(x) = 6x = 0$; when $x = 1$ is a local minimum, and when $x = -1$ is a local maximum.

Chapter 3 (56 points):

1. Use the Example Data posted under the Week 2 module to answer the following questions. **Limit your data to women who are employed and age 30 to 39**.

   (a) Find the average income in the data for these women and report it. We will treat this as our population parameter, μ. (2 points)
   - Average income population set: $48,348.34

   (b) Take a random sample of 1000 women. (25 points)

   i. Give the sample mean (which is your estimate of μ. (3 points)
   - Average Income sample: $47,355.63

   ii. Give the sample standard deviation. (3 points)
   - Sample Standard Deviation: $39,169.62

   iii. Give the standard error of your estimate of μ. (4 points)
   - Standard Error: 1238.65

   iv. Give the 95% confidence interval on your estimate of μ. (5 points)
   - Lower end of the range: $44,924.97
   - Upper end of the range: $49,786.29

   v. Test a hypothesis that $\mu = 50,000$. Do you reject or fail to reject? What is the α value of the test? (5 points)
   - T test = 2.13 which is greater than t statistic, thus reject null alpha between 0.025 0.01

   vi. Test a hypothesis that $\mu > 55,000$. Do you reject or fail to reject? What is the α value of the test? (5 points)
   - T test = 6.17 which is greater than t statistic, thus reject null alpha infinitely small.

   (c) Now take 10000 random samples of 1000 women. Record the sample mean for each sample and save it to a matrix. (14 points)

   i. Produce a graph of the distribution of these 10,000 sample means. (6 points)
   - See figure 1 at the end of this section. The red horizontal and vertical lines are the population mean which serves as reference.

ii. Report the average of these 10,000 sample means. (4 points)

- The average of these 10,000 samples is: $48,326.1

iii. Report the standard deviation of these 10,000 sample means. (4 points)

- Standard Deviation is: 1463.62

(d) Describe how the results from your last two answers relate to the law of large numbers and the central limit theorem. (15 points)

- The Central limit Theorem states that when sample size tends to infinity, the sample mean will be normally distributed. As we can see on the histogram, these 10,000 sample means seem to be normally distributed, the means follow a bell-shape curve 1. The Law of Large Number states that when sample size tends to infinity, the sample mean equals to population mean. As we can see the overall sample mean for these 10,000 samples is &48,326.1 which is 0.05% off from the population mean of $48,348.34, thus the law of large numbers is respected.

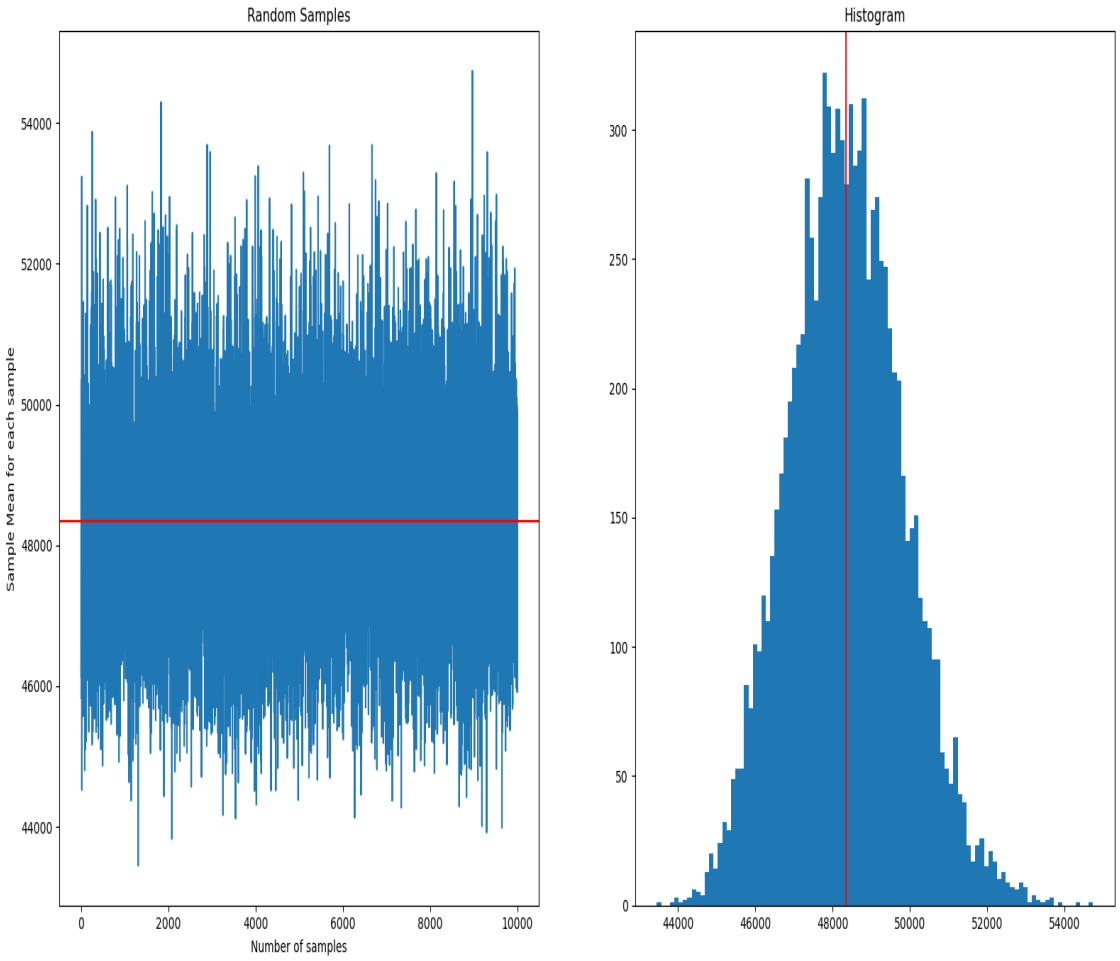

Figure 1: Distribution Graph.

Chapter 4 (32 points)

See the following definitions of matrices A,B,C,D, and E.

$$[A] = \begin{bmatrix} 1, 1, 5 \\ 1, 2, 3 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 1, 0 \\ 0, 1 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 3, 3 \\ 1, 6 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 5, 2, 7 \\ 1, 9, 5 \\ 1, 4, 5 \end{bmatrix}$$

$$[E] = \begin{bmatrix} 4, 5, 6 \\ 3, 4, 1 \\ 6, 7, 5 \end{bmatrix}$$

Use these definitions to answer the following questions. If a question is not answerable, please tell me why.

1. Find B^{-1} (3 points)
   - $B^{-}1 = \begin{bmatrix} 1, 0 \\ 0, 1 \end{bmatrix}$
2. What is C^{-1} (3 points)
   - $C^{-}1 = \begin{bmatrix} 0.4, -0.2 \\ -0.06, 0.2 \end{bmatrix}$
3. What is A'? (2 points)
   - $A' = \begin{bmatrix} 1, -1 \\ 1, 2 \\ 5, 3 \end{bmatrix}$
4. What is B'? (2 points)
   - $B' = \begin{bmatrix} 1, 0 \\ 0, 1 \end{bmatrix}$
5. What is C'? (2 points)
   - $C' = \begin{bmatrix} 3, 1 \\ 3, 6 \end{bmatrix}$
6. What is $A \times C$? (4 points)
   - $A \times C$ = ValueError: shapes (2,3) and (2,2) not aligned: 3 (dim 1) != 2 (dim 0). They are not the same dimension.

7. What is $A' \times C$? (4 points)

   - $A' \times C = \begin{bmatrix} 4, 9 \\ 5, 15 \\ 18, 33 \end{bmatrix}$

8. What is $B \times C$? (4 points)

   - $B \times C = \begin{bmatrix} 3, 3 \\ 1, 6 \end{bmatrix}$

9. What is $B + C$? (2 points)

   - $B + C = \begin{bmatrix} 4, 3 \\ 1, 7 \end{bmatrix}$

10. What is D^{-1}? You can use software for this. (4 points)

    - $D^{-1} = \begin{bmatrix} 2.77777778e^{-01}, 2.00000000e^{-01}, -5.88888889e^{-01} \\ -2.00815534e^{-18}, 2.00000000e^{-01}, -2.00000000e^{-01} \\ -5.55555556e^{-02}, -2.00000000e^{-01}, 4.77777778e^{-01} \end{bmatrix}$

11. What is $D \times E$? You can use software for this. (4 points)

    - $D \times E? = \begin{bmatrix} 68, 82, 67 \\ 61, 76, 40 \\ 46, 56, 35 \end{bmatrix}$