HararyGraphNetworkReliability Algorithm

Brief description

Given the use of optical transport networks growth, it is important that these networks are reliable, in other words, it is needed to create mechanisms that allows to identify ways to increase the reliability of optical transport networks. In this context, the reliability is related to the capacity of the network to be online in case of link or node failure.

The algorithm maximizes the network reliability without change his base structure, i.e, it keeps the same node and links number. It redistribute the links and looks for higher level of reliability using a subclass of Hararys graph.

This algorithm calculates the reliability of the user network and for the new graph generated using the Hararys subclass graph, when its connectivity is $\frac{2m}{n} \geq 3$, where m is the number of links and n is the number of nodes. Posteriorly, a comparison between the networks is realized. Furthermore, the cutting set of both graphs with their cost is compared.

Algorithm description table

Algorithm inputs	Requires a netPlan object with graph links. Algorithm parameters: - pho: The probability of a link rupture. Default 0.05.
Algorithm outputs	HTML report
Required libraries	None
Keywors	Harary, Reability
Authors	Emerson Dallagnol, Mauricio Henrique Secchi, Roberto Walter
Date	November 2015

Detailed description

The physical network topology can be represented as a graph G with a set $V(G)$ of N nodes representing the optical cross connects, and a set $E(G)$ of L links representing the bidirectional optical fibers. Two nodes u and v are adjacent if there is a link in $E(G)$ interconnecting them. The degree of a node v is the number of nodes adjacent to v in G. The average node degree is given by $\langle \delta \rangle = \frac{2L}{N}$.

A set of links $S \in E(G)$ is a link cut set if $G \backslash S$ is disconnected, i.e., if the removal of S from G disconnects G. The link connectivity of a graph G, $k'(G)$, is the size of the smallest link cut set of G.

Reliable networks refers to highly connected networks. It is needed to have the most reliable network using all the available resources as the main goal. So, how can we make a more reliable topology of a network, using all the same nodes and links resources from the original network?

In 1969, Harary [3] solved this issue by developing an algorithm to design highly connected networks given the number of nodes N and the node connectivity k as input. Such graphs are the so-called Harary graphs, denoted here as $H_{k,N}$. Harary's algorithm builds a k-node-connected graph on N nodes with $L = \left\lceil \frac{kN}{2} \right\rceil$ links, which is extremal with respect to the number of links, and such that $k(H_{k,N}) = k'(H_{k,N}) = \left\lfloor \frac{2L}{N} \right\rfloor$.

The Figure 1 show a Harary graph with $n = 6$ and $k = 3$.

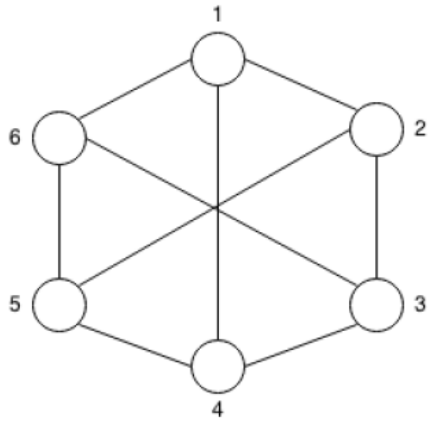

Figure 1. Harary graph with $n = 6$ and $k = 3$.

So Hararys graphs are build following the way presented above to have maximum connectivity of vertices and edges. But for some values of n and m, some graphs can not be generated by Harary procedure [6]. Bauer et al. [7] developed a procedure that solves this question, sorting the Harary graph in two categories: Elementary Graph Harary and General Graph Harary.

Based on Harary's idea, Hakimi [1] designed an algorithm to build Harary graphs when L and N are given previously. Both algorithms have order of N^2 time complexity. Hakimi's procedure builds some graphs that cannot be obtained by Harary's procedure.

It turned out that a subset of the Harary graphs are not only highly connected but also reliable; see Deng et al. [4].

Assuming that links may fail with equal and independent probability $\rho \in (0,1)$, we consider the reliability of a topology G as its probability to remain connected after link failures. We assume ρ in this range (approaching to 0) because as reported in [8], is more reasonable small values forρ. The exact calculation of $R_G(\rho)$ increases exponentially with network size. In this context, Kelmans [2] defined the network reliability as follows:

$$R_G(\rho) = 1 - \sum_{i=k'(G)}^{L} S_i \rho^i (1-\rho)^{L-i}$$

where S_i is the number of link cut sets of size i.

We have implemented an algorithm to build an Harary graphs network given the number of links and nodes of a network such that $\langle \delta \rangle \geq 3$. This algorithm reproduces the study produced by Pavan et al. [5].

The algorithm maximizes the network reliability without change his base structure, i.e, it keeps the same node and links number. It redistribute the links and looks for higher level of reliability using a subclass of Hararys graph. Posteriorly, a comparison between the networks is done. Furthermore, the cutting set of both graphs with their cost is compared.

References:

1 - S. L. Hakimi, “An algorithm for construction of the least vulnerable communication network or the graph with the maximum connectivity,” IEEE Trans. Circuit Theory, vol. 16, no. 2, pp. 229–230, 1969.

2 - A. Kelmans, “Connectivity of probabilistic networks,” Autom. Remote Control, vol. 29, no. 3, pp. 444–460, 1967.

3 - F. Harary, Graph Theory. Addison-Wesley, 1969.

4 - H. Deng, J. Chen, Q. Li, R. Li, and Q. Gao, “On the construction of most reliable networks,” Discr. Appl. Math., vol. 140, no. 1–3, pp. 19–33, 2004.

5 – Pavan, C., de Lima, L. S., Paiva, M. H. M., & Segatto, M. E. V. (2015). How Reliable Are the Real-World Optical Transport Networks?. *Journal of Optical Communications and Networking*, *7*(6), 578-585.

6 - Teixeira, L. d. S., de Abreu, N. M. M., and da Silva Lima, L. (2008). Grafos que modelam redes confiáveis.

7 - Bauer, D., Boesch, F., Suffel, C., and Tindell, R. (1985). Combinatorial optimization problems in the analysis and design of probabilistic networks. Networks, 15(2):257–271.

8 - K. Lee, H.-W. Lee, and E. Modiano, “Reliability in layered networks with random link failures,” IEEE/ACM Trans. Netw., vol. 19, no. 6, pp. 1835–1848, 2011.