Working and Saving Informally

The Link between Labor Market Informality and Financial Exclusion

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Introduction

Motivation

Developing countries are characterized by high informal employment and by low saving rates.

- High Informal employment:
  - In the Latin America and the Caribbean region (LAC), about half of labor force is informal.
  - Informality:
    - may introduce some useful flexibility
    - but lowers workers' protections, increases employment risks, hinders productivity growth.

  [World Bank, 2013; Perry et al., 2007; La Porta and Shleifer, 2014]

- Low Saving rate:
  - In LAC, savings are 17% of GDP compared to 30% in High-Income regions.
  - Low savings:
    - make individuals more vulnerable to shocks
    - but they are not simply due to many individuals "too poor to save".

  [Cavallo et al., 2016; Karlan and Morduch, 2010; Dupas and Robinson 2013; Bond et al. 2015.]

This Paper: Why

If both high levels of informality and low levels of saving are problems in themselves, this paper studies how they feed each other to generate even worse outcomes.

- Informality increases the need for precautionary savings because of higher employment risk;
- but the informality status also cause financial exclusion and sub-optimal saving levels;
- which in turn may induce workers to accept informal jobs with higher frequency because they cannot finance an effective labor market search.

This Paper: How

Since the deep linkages prevents from studying each problem in isolation, we develop a model that integrates all the crucial elements giving rise to both phenomena:

- Agents search on- and off-the-job for both formal and informal work;
- save through both formal financial institutions and informal ones.
- But informal workers face higher costs of accessing formal financial institutions (financial exclusion.)

To provide a quantitative assessment and evaluate policy interventions, we estimate the model on Colombia:

- It belongs to a region where both issues are particularly acute (Colombia is the fourth economy in LAC).
- It collects good quality data on both savings and labor market behavior (rare among developing countries).

Preview of Preliminary Results

1. The link labor market informality and financial exclusion is confirmed:
   - Our estimates confirm that informal workers face higher cost to access formal financial institutions.
   - Our equilibrium-based counterfactual show that granting full financial access to informal workers would increase savings by 3% a month and formal assets by 21%. It would also decrease inequality in assets and consumption.
2. Specific policy experiments for Colombia:
   - The recent fiscal reform that lowered the payroll contribution for formal workers may be responsible for increasing saving by 10% a year.
3. Methodological contributions:
   - First paper to successfully estimate a search model of the labor market with savings and borrowing where two assets are allowed.

     [Rendon (2006); Lentz (2009); Lise (2013); Garcia-Perez and Rendon (2020); Abrahams (2022]
   - First paper to successfully estimate a search model of the labor market with both informality and savings.

     [Bobba et al. 2022, 2021; Megir et al. 2015; Bosch and Esteban-Pretel (2012)]; Charlot et al. 2013; Albrecht et al 2009]

Our paper in Context

1. Large literature on savings in developing countries shows that low savings not only due to too poor to save individuals but also to institutions:
   - Lack of a safe place to save [Dupas and Robinson 2013]; Capital market segmentation [Batini et al. 2010]; Not-working financial intermediation [Ogbuabor et. 2013]; Labor market informality [Lorenzo and Osimani (2001); Granda and Hamann (2015)]
   - Our contribution provides:
     - A joint model of both financial exclusion and labor market informality that can rationalize previous seemingly contradictory results
     - Novel estimates of the importance of these institutions.
2. Growing literature using models with frictions to explain labor market informality is able to study:
   - firm productivity [Megir et al. 2015]; human capital accumulation [Bobba et al. 2022, 2021]; cyclical reallocations [Bosch and Esteban-Pretel (2012)]; different contracts in equilibrium [Albrecht et al 2009; Charlot et al. 2013]

   but ignores the link with saving behavior and financial access which our contribution is the first to provide in this context.

Our paper in Context

3. Small but established literature analyzing saving with idiosyncratic risk in search models of the labor market
   - early and seminal [Danforth (1979); Acemoglu and Shimer (1999)] ; macro literature [Krusell et al. (2010); Bils et al. (2011); Ji (2021)] ; closer to us because estimate model's parameters on microdata [Rendon (2006); Lentz (2009); Lise (2013)
   - Our contribution:
     - New application on developing countries with high informality.
     - New methodology which allows for two assets, one risk-free and the other risky.
4. Tiny literature analyzing labor market informality and saving behavior [Esteban-Pretel and Kitao, 2021; Granda and Hamann, 2015] cannot study financial exclusion because allows for only one asset.

The Model

Workers' optimization problem

The model environment can be summarized in the following optimization problem:

$$\max_{c,\phi} \quad E_0 \int_0^\infty e^{-(\rho+\theta)t} \left[\frac{c^\delta}{\delta} + \epsilon f \right]$$

$$s.t. \quad da = \begin{cases} \left[(r_1\phi + r_2(1-\phi))(1+\nu l_{a^-})a + b - c - \frac{\psi^u}{2}\phi^2 \right] dt & \text{u} \\ \left[(r_1\phi + r_2(1-\phi))(1+\nu l_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2 \right] dt & \text{f=1,0} \end{cases}$$

$$a \geq \underline{a}$$

$$dr_2 = \kappa(\bar{r}_2 - r_2)dt + \sigma dz \qquad r_2 \sim \mathcal{N}\left(\bar{r}_2, \frac{\sigma^2}{2\kappa}\right)$$

$$di = \begin{cases} dq_{\lambda_1^u} \mathbf{I}_1 w(1) + dq_{\lambda_0^u} \mathbf{I}_0 w(0) - b & u \\ dq_{\eta_1} b + dq_{\lambda_1^e} \mathbf{I}_1 w'(1) + dq_{\lambda_0^e} \mathbf{I}_0 w'(0) - w(1) & f = 1 \\ dq_{\eta_0} b + dq_{\lambda_1^e} \mathbf{I}_1 w'(1) + dq_{\lambda_0^e} \mathbf{I}_0 w'(0) - w(0) & f = 0 \end{cases}$$

where $w(f)$ are draws form $F(w|f)$ and f are a draws from a Bernoulli distribution with $p(f)$.

Decisions rules

- Let $U(a, r_2)$ and $W(a, r_2, w, f)$ be the value of being unemployed and employed, respectively. The optimal consumption and portfolio decision rules are derived from the first order conditions of the value functions:

$$\begin{aligned} u'(c) &= \partial_a U(a, r_2) \\ (r_1 - r_2)(1 + \nu I_{a^-})a &= \psi^u \phi \\ u'(c) &= \partial_a W(a, r_2, w, f) \\ (r_1 - r_2)(1 + \nu I_{a^-})a &= \psi^e(f)\phi \end{aligned}$$

- The optimal labor market decision rules concern accepting or rejecting job offers:

$$\begin{cases} \text{accept an offer } \{w, f\} \text{ if } W(a, r_2, w, f) \geq U(a, r_2) & \text{unemployed} \\ \text{accept an offer } \{w', f'\} \text{ if } W(a, r_2, w', f') \geq W(a, r_2, w, f) & \text{employed with } \{w, f\} \end{cases}$$

Definition
Given the primitive parameters $\{\rho, \theta, \lambda^u, \lambda^e(1), \lambda^e(0), \eta(1), \eta(0), \psi^u, \psi^e(1), \psi^e(0), b\}$, the instantaneous utility function $u(c)$, the distributions of wage offers $F(w|1)$, $F(w|0)$, $p(1)$ the *steady state equilibrium* is a set of values $U(a, r_2)$ and $W(a, r_2, w, f)$ that satisfy the value functions equations, together with the invariant distributions of individuals across labor market states and the invariant distributions of total assets $\Lambda(a)$.

Note:

- Endogenous:
  - hazard rates
  - accepted wages distributions
  - distribution over labor market states
  - assets distribution
- Exogenous
  - wage offers distributions
  - Poisson rates (mobility parameters, effective discount rate)
  - utility function and institutional parameters

Estimation

Data description

We combine information from two data sources: GEIH and ELCA.

Gran Encuesta Integrada de Hogares (GEIH): Monthly household survey focusing on labor market outcomes.

- Individual characteristics (gender, age, years of schooling)
- Labor market states:
  - Unemployed.
  - Formal employment (full-time employees who contribute to the social security).
  - Informal employment (full-time informal employees + self-employed working 48+ hours a week (top coded at 100 hours)).
- On going durations in unemployment and employment states (in months).
- Labor income and weekly hours worked:
  - Real monthly wages (in US dollars of December 2016).

Data description

Encuesta Longitudinal Colombiana (ELCA): Longitudinal survey (2010, 2013, and 2016) focusing on individuals' saving behavior.

- Individual characteristics (gender, age, years of schooling)
- Labor market outcomes except durations (same definitions as GEIH).
- Savings behavior
  - Average monthly savings (in US dollars of December 2016).
  - Formal savings (formal financial institutions like banks and employees funds/credit unions)
  - Informal savings (cash, group savings, chit funds, etc).

Sample: male, between 25 and 55 years old, living in urban areas, with only secondary education completed ("unskilled")

Descriptive statistics on labor market outcomes

Descriptive Statistics on Labor Market Outcomes

	Formal Employment	Informal Employment	Unemployment
	Labor Market States		
Proportion	0.395	0.527	0.077
	Wages (hundred of US$ per month)		
Mean	3.284	2.429	–
Standard Deviation	1.395	1.126	–
Ratio of Average Wages	1.352	1.000	–
	Ongoing Duration (months)		
Mean	67.535	89.507	4.034
Standard Deviation	78.689	100.191	6.858
	Sample		
Number Obs.	31709	42307	6195

Descriptive statistics on saving behavior

Descriptive Statistics on Saving Behavior

	Formal Employment	Informal Employment	Unemployment
Proportion of Individuals who save			
At all	0.271	0.211	0.036
Mainly in formal institutions	0.493	0.185	0.333
Savings amount among savers (hundred of US$)			
Mean	0.601	0.508	0.443
Standard Deviation	0.721	0.748	0.480
Saving rate among savers (savings/labor income)			
Mean	0.133	0.151	-
Standard Deviation	0.123	0.122	-
Sample Size			
Number Obs.	517	589	83

Estimation method

- We estimate the model primitive parameters using the Method of Simulated Moments (MSM).

$$\hat{\Xi}_{N,T}(W) = \text{argmin}_{\Xi} \frac{1}{2} \left[M_N^D - M_T(\Xi) \right]' W_N \left[M_N^D - M_T(\Xi) \right]$$

- Parametric assumption:

$$\log(w)|f \sim \mathcal{N}(\mu(f), \sigma(f))$$

- Parameters to estimate:

$$\Xi \equiv \{b, \epsilon, \lambda^u, \psi^u, \kappa, p(1)\} \cup \{\lambda^e(f), \eta(f), \mu(f), \sigma(f), \psi^e(f)\}_{f \in \{0,1\}}$$

- we calibrate the parameters $\{r_1, \bar{r}_2, s, \rho, \theta\}$ and we fix at their institutional value the parameters $\{\tau, \nu\}$.

Identification

1. Wage offers distributions:
   - We follow the literature by assuming lognormality [Eckstein and Van den Berg 2007; Flinn and Mullins 2015; Bobba et al. 2022]: $\log(w)|f \sim \mathcal{N}(\mu(f), \sigma(f))$, with $f = 0, 1$.
   - Under the assumption and the model, we can identify the primitive distribution by observing their truncations.
2. Mobility parameters:
   - Unemployment durations identify (contribute to the identification of) λ^u
   - Employment durations jointly identify $\{\eta(f), \lambda^e(f)\}$
   - Steady state proportions in each labor market state separately identify $\eta(f)$ from $\lambda^e(f)$
3. Portfolio cost functions:
   - Savings and proportion of them in formal assets identify ψ^u and $\psi^e(f)$.
   - But we have to assume all agents start life with no assets.
4. Rate of returns:
   - Formal assets: We set $r_1 = 0.075$ based yearly return of the 10-year Colombian Government Bond.
   - Informal assets:
     - We use Eeckhout and Munshi(2010) to set an upper bound for r_2 (2.1 times r_1) and assume a lower bound of 0
     - This support condition + distributional assumptions lead to: $\bar{r}_2 = 0.079$ and $\frac{\sigma_Z}{\sqrt{2\kappa}} = 0.031$
     - κ is then identified by information on the savings and portfolio allocations

5. Effective discount rate and unemployed flow income
   - They are jointly identified [Flinn and Heckman 1982]: we fix $\tilde{\rho}$ and identify b through equilibrium equations.
   - We calibrate the two components of $\tilde{\rho}$ as:
     - $\rho = 0.012$, based on the discount rate recommended for LAC by multilateral development banks;
     - $\theta = 0.013$, based on Colombia's life expectancy of 77 years.
6. Relative risk aversion parameter:
   - Notoriously difficult to identify in this class of models without additional information [Dey and Flinn 2008; Flabbi and Mabli 2018]
   - Lacking additional information, we set it at a consensus value of 1.5, implying $\delta = -0.5$. [Bond et al. 2015]
7. Institutional parameters:
   - $\tau = 0.16$ based on 2016 payroll contributions in Colombia [Fernandez and Villar 2017]
   - $\nu = 1.14$ based on the average markup that financial institutions charge when lending with respect to what they pay on (worker's) saving [IMF *International Financial Statistics*]

Estimation results

Estimated Parameters

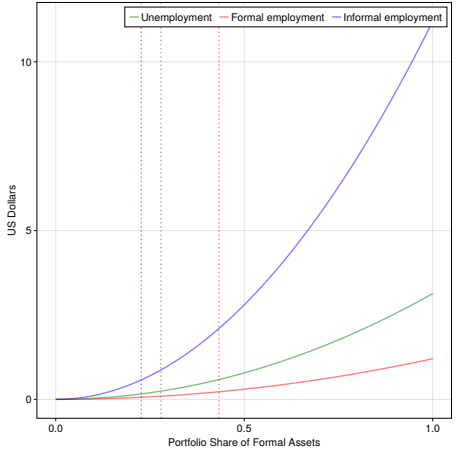
Definition	Parameter	Est. Value	Std. Error
Mobility Shocks			
Job offer rate - unemployment	λ^u	0.178	(0.0072)
Job offer rate - formal employment	$\lambda^e(1)$	0.034	(0.0054)
Job offer rate - informal employment	$\lambda^e(0)$	0.015	(0.0040)
Job separation rate - formal employment	$\eta(1)$	0.017	(0.0039)
Job separation rate - informal employment	$\eta(0)$	0.014	(0.0027)
Job offers			
Proportion formal job offers	$p(1)$	0.455	(0.0038)
Location wages distribution - formal employment	$\mu(1)$	1.056	(0.0519)
Scale wages distribution - formal employment	$\sigma(1)$	0.394	(0.0147)
Location wages distribution - informal employment	$\mu(0)$	0.800	(0.0369)
Scale wages distribution - informal employment	$\sigma(0)$	0.408	(0.0205)

Estimation results

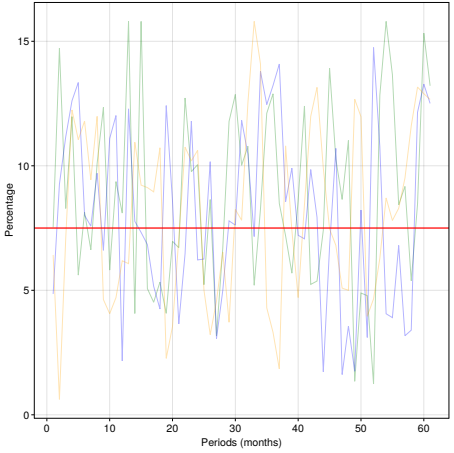
Estimated Parameters (cont...)

Definition	Parameter	Est. Value	Std. Error
Portfolio costs			
Cost function parameter - unemployment	ψ^u	0.063	(0.0045)
Cost function parameter - formal employment	$\psi^e(1)$	0.024	(0.0027)
Cost function parameter - informal employment	$\psi^e(0)$	0.224	(0.0314)
Rate of return informal assets			
Persistence	κ	0.701	(0.0218)
Std. Dev. of shock	σ_z	0.037	(0.0006)
Unemployment income			
Flow	b	0.197	(0.0230)
Utility Value of Formal Jobs			
Value	ϵ	0.026	(0.0012)

Estimation results



(a) Portfolo Cost Function



(b) Assets Returns

Counterfactual experiments

Definitions

We perform two sets of counterfactual experiments:

1. **Financial inclusion**: Equal portfolio costs $\psi^e(0) = \psi^e(1) = 0.024$.
2. **Lower labor market informality**: Proportion of informal job offers drops from the baseline 54% to 33% (almost 40% reduction).
3. **Payroll tax policy** Increase of the payroll to from 16% to 29.5% (level set prior to the 2012 reform).

We evaluate the impact on labor market and financial outcomes and on wealth and consumption inequality taking into account the endogenous adjustment in individual's optimal behaviors.

Counterfactual Experiments - Labor Market and Financial Outcomes

	Benchmark	$\psi^e(0) = \psi^e(1)$		$1 - p(1) = 0.325$		$\tau = 0.295$	
	Value	Value	Ratio	Value	Ratio	Value	Ratio
Labor market states (proportion)							
$e(1)$	0.394	0.393	0.996	0.610	1.548	0.342	0.867
$e(0)$	0.566	0.565	0.997	0.349	0.617	0.615	1.086
u	0.039	0.043	1.077	0.040	1.024	0.043	1.097
Wages (hundred of US$ per month)							
$E[w\|e(1)]$	3.759	3.753	0.999	3.813	1.014	3.772	1.004
$E[w\|e(0)]$	2.854	2.871	1.006	2.926	1.025	2.861	1.003
Savings (hundred of US$ per month)							
$E[s\|s > 0]$	0.189	0.195	1.030	0.206	1.087	0.170	0.900
$E[s\|s > 0, e(1)]$	0.221	0.225	1.019	0.224	1.015	0.176	0.797
$E[s\|s > 0, e(0)]$	0.172	0.177	1.030	0.179	1.042	0.170	0.990

NOTE: Benchmark's values are: $\psi^e(0) = 0.224$; $\psi^e(1) = 0.024$; $p(0) = 0.545$; $\tau = 0.160$.

Counterfactual Experiments - Labor Market and Financial Outcomes

	Benchmark	$\psi^e(0) = \psi^e(1)$		$1 - p(1) = 0.325$		$\tau = 0.295$	
	Value	Value	Ratio	Value	Ratio	Value	Ratio
Total Assets (hundred of US$)							
$E[a]$	6.149	6.365	1.035	6.746	1.097	5.519	0.898
$E[a\|e(1)]$	7.362	7.412	1.007	7.564	1.027	5.768	0.783
$E[a\|e(0)]$	5.495	5.862	1.067	5.719	1.041	5.557	1.011
Formal Assets (hundred of US$)							
$E[\phi a]$	2.241	2.705	1.207	2.617	1.168	1.921	0.857
$E[\phi a\|e(1)]$	3.264	3.223	0.987	3.226	0.988	2.404	0.736
$E[\phi a\|e(0)]$	1.598	2.461	1.540	1.718	1.075	1.704	1.066
Portfolio (proportion of total assets which is formal)							
$E[\phi]$	0.310	0.415	1.338	0.353	1.138	0.297	0.957
$E[\phi\|e(1)]$	0.433	0.430	0.994	0.423	0.978	0.401	0.926
$E[\phi\|e(0)]$	0.227	0.415	1.831	0.241	1.064	0.239	1.054

NOTE: Benchmark's values are: $\psi^e(0) = 0.224$; $\psi^e(1) = 0.024$; $p(0) = 0.545$; $\tau = 0.160$.

Counterfactual Experiments - Inequality

General Entropy Indexes	Benchmark Value	$\psi^e(0) = \psi^e(1)$ Value	Ratio	$p(0) = 0.325$ Value	Ratio	$\tau = 0.295$ Value	Ratio
Total Assets							
$GE(0)$ Mean log deviation	0.277	0.240	0.869	0.254	0.919	0.277	1.001
$GE(1)$ Theil index	0.224	0.196	0.878	0.201	0.900	0.223	0.997
$GE(2)$ Coefficient of variation/2	0.247	0.216	0.872	0.218	0.881	0.241	0.975
Formal Assets							
$GE(0)$ Mean log deviation	0.794	0.359	0.453	0.614	0.774	0.799	1.007
$GE(1)$ Theil index	0.434	0.232	0.533	0.331	0.762	0.451	1.039
$GE(2)$ Coefficient of variation/2	1.625	1.135	0.699	1.344	0.827	1.678	1.033
Consumption							
$GE(0)$ Mean log deviation	0.128	0.126	0.986	0.126	0.989	0.128	1.002
$GE(1)$ Theil index	0.110	0.107	0.971	0.105	0.958	0.109	0.990
$GE(2)$ Coefficient of variation/2	0.113	0.108	0.957	0.106	0.938	0.110	0.977

NOTE: Benchmark's values are: $\psi^e(0) = 0.224$; $\psi^e(1) = 0.024$; $p(0) = 0.545$; $\tau = 0.160$.

Concluding remarks

Concluding remarks

- We develop and estimate a model able to replicate the crucial features of developing countries economies:
  1. High level of labor market informality
  2. Low level of savings
  3. High proportion of assets held in informal institutions
- Our claim that working informally is linked to saving informally is confirmed:
  - Informal workers face partial financial exclusion from formal financial institutions
  - If full financial access were guaranteed to them:
    - Savings would increase 3% a month and formal assets 21%
    - Asset inequality would decrease 13% and consumption inequality 4%
- Colombia-specific policies:
  - A recent reform reducing formal payroll contribution had the potential to increase savings by 10% a month.
- We also provide two methodological contributions in the labor market search literature:
  1. We add saving and borrowing to search models with informality.
     [Bobba et al. 2022, 2021; Megir et al. 2015; Bosch and Esteban-Pretel (2012)]; Charlot et al. 2013; Albrecht et al 2009]
  2. We allow for two assets in search models with saving.
     [Rendon (2006); Lentz (2009); Lise (2013); Danforth (1979); Acemoglu and Shimer (1999); Krusell et al. (2010); Bils et al. (2011)]

Thank You!!

Additional Slides

Model environment

- Time is continuous and the environment is assumed to be stationary.
- Individuals discount the future at ρ and face common probability of death (with Poisson rate θ).
- Individuals are ex-ante homogeneous in every aspect.
- Individuals objective function:

$$E_0 \int_0^{\infty} e^{-\tilde{\rho} t} \left[u(c) + \epsilon f \right]$$

- The labor market is characterized by three states: unemployment, employment in a formal job, and employment in an informal job.
- Both off- and on-the-job is allowed.
- Unemployed workers receive a flow income b (unemployment benefits, transfers and subsidies).

Model environment

- A job offer is a pair wage and type of job: (w, f). Jobs arrive at rate λ^u and $\lambda^e(f)$.
- Wages are draws form $F(w|f)$ and f is a draw from $p(f)$ with $f = \{0, 1\}$.
- Jobs are terminated at exogenous rate $\eta(f)$.
- Two assets: a_1 risk-less *formal asset* with return r_1 and a_2 risky *informal asset* with return r_2.
- r_2 follows a Ornstein-Uhlenbeck process:

$$dr_2 = \kappa(\bar{r}_2 - r_2)dt + \sigma dz$$

  z is a standard Brownian motion and in steady state $r_2 \sim \mathcal{N}\left(\bar{r}_2, \frac{\sigma^2}{2\kappa}\right)$
- Total wealth $a = a_1 + a_2$ and the share of formal assets $\phi = \frac{a_1}{a}$. There is a convex cost of portfolio ϕ: $\frac{\psi^u}{2}\phi^2$ and $\frac{\psi^e(f)}{2}\phi^2$.

- Budget constraint:

$$da = \begin{cases} \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + b - c - \frac{\psi^u}{2}\phi^2\right] dt & \text{if unemployed} \\ \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] dt & \text{if employed} \end{cases}$$

- Individuals can borrow, however markets are incomplete. Self-imposed borrowing limit for a permanent state of unemployment:

$$\underline{a} = -\frac{b}{\bar{\bar{r}}_2(1+\nu)}$$

where $\bar{\bar{r}}_2$ is the upper bound of the C.I. that contains 99% of the r_2 draws, and the interest rate spreads are νr_i, $i = 1, 2$.

Value functions: Steady state value of unemployment

$$
\begin{aligned}
\tilde{\rho} U(a, r_2) &= \max_{0 \leq c \leq \bar{c}, 0 \leq \phi \leq 1} \{ u(c) \\
&+ \lambda^u \sum_{f=0}^{1} \int_w \max\{W(a, r_2, w, f) - U(a, r_2), 0\} dF(w|f) p(f)\} \\
&+ \partial_a U(a, r_2) \left[(r_1 \phi + r_2(1-\phi))(1 + \nu I_{a^-}) a + b - c - \frac{\psi^u}{2} \phi^2 \right] \\
&+ \partial_{r_2} U(a, r_2) \kappa(\bar{r}_2 - r_2) + \frac{1}{2} \partial_{r_2}^2 U(a, r_2) \sigma_z^2 \}
\end{aligned}
$$

where $(1 + \nu)$ is the markup over the savings rate that financial institutions charge and $I_{a^-} = 1$ if $a < 0$ (borrowing).

Value functions: Steady state value of employment

$$
\begin{aligned}
\tilde{\rho} W(a, r_2, w, f) &= \max_{0 \leq c \leq \bar{c}, 0 \leq \phi \leq 1} \{u(c) + \epsilon f \\
&+ \lambda^e(f) \sum_{f=0}^{1} \int_{w'} \max\{W(a, r_2, w', f') - W(a, r_2, w, f), 0\} dF(w'|f') p(f') \\
&+ \eta(f) \left[U(a, r_2) - W(a, r_2, w, f)\right] \\
&+ \partial_a W(a, r_2, w, f) \left[(r_1 \phi + r_2(1-\phi))(1 + \nu I_{a^-})a + w(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] \\
&+ \partial_{r_2} W(a, r_2, w, f) \kappa(\bar{r}_2 - r_2) + \frac{1}{2}\partial^2_{r_2} W(a, r_2, w, f) \sigma_z^2\}
\end{aligned}
$$

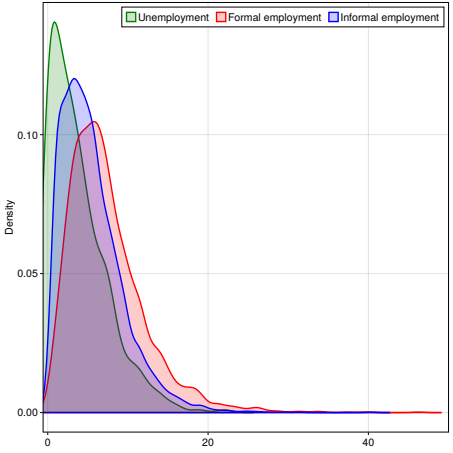
Fit of the model

Moments Fit

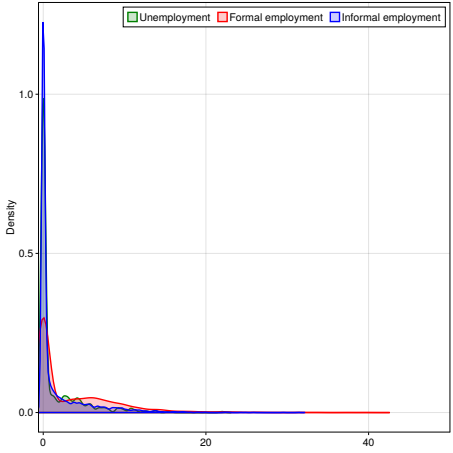
Statistic	Data	Model	Statistic	Data	Model	Statistic	Data	Model
$e(1)$	0.395	0.394	$E[I_{s>0} \times s \mid e(1)]$	0.163	0.097	$\Pr[\phi > 0.5 \mid e(1)]$	0.493	0.434
$e(2)$	0.527	0.566	$SD[I_{s>0} \times s \mid e(1)]$	0.460	0.223	$\Pr[\phi > 0.5 \mid e(1)]$	0.493	0.434
u	0.077	0.039	$E[I_{s>0} \times s \mid e(0)]$	0.107	0.080	$\Pr[\phi > 0.5 \mid e(0)]$	0.185	0.208
$E[w(1)]$	3.284	3.759	$SD[I_{s>0} \times s \mid e(0)]$	0.400	0.183	$\Pr[\phi > 0.5 \mid u]$	0.333	0.314
$SD[w(1)]$	1.395	1.465	$E[I_{s>0} \times s \mid u]$	0.016	0.001	$\Pr[\phi > 0.5 \mid e(1), Q_1]$	0.312	0.397
$E[w(0)]$	2.429	2.854	$SD[I_{s>0} \times s \mid u]$	0.112	0.003	$\Pr[\phi > 0.5 \mid e(1), Q_2]$	0.458	0.436
$SD[w(0)]$	1.126	1.153	$E[I_{s>0} \times s \mid e(1), Q_1]$	0.061	0.029	$\Pr[\phi > 0.5 \mid e(1), Q_3]$	0.368	0.450
$P5[w(1)]$	2.289	1.790	$E[I_{s>0} \times s \mid e(1), Q_2]$	0.065	0.067	$\Pr[\phi > 0.5 \mid e(1), Q_4]$	0.623	0.454
$P5[w(0)]$	0.867	1.348	$E[I_{s>0} \times s \mid e(1), Q_3]$	0.145	0.106	$\Pr[\phi > 0.5 \mid e(0), Q_1]$	0.000	0.047
$E[t \mid e(1)]$	5.628	5.950	$E[I_{s>0} \times s \mid e(1), Q_4]$	0.393	0.187	$\Pr[\phi > 0.5 \mid e(0), Q_2]$	0.107	0.176
$SD[t \mid e(1)]$	6.557	6.316	$E[I_{s>0} \times s \mid e(0), Q_1]$	0.026	0.029	$\Pr[\phi > 0.5 \mid e(0), Q_3]$	0.194	0.257
$E[t \mid e(0)]$	7.459	7.653	$E[I_{s>0} \times s \mid e(0), Q_2]$	0.056	0.051	$\Pr[\phi > 0.5 \mid e(0), Q_4]$	0.353	0.353
$SD[t \mid e(0)]$	8.349	8.107	$E[I_{s>0} \times s \mid e(0), Q_3]$	0.096	0.087			
$E[t \mid u]$	4.034	4.954	$E[I_{s>0} \times s \mid e(0), Q_4]$	0.310	0.152			
$SD[t \mid u]$	6.859	5.922						

NOTE: $s = da/dt$ is the amount saved, $I_{s>0}$ is an indicator variable that takes the value of 1 if the individual saves a positive amount and zero otherwise, and Q_i represents the quartile i in the observed wages distribution.

Steady state distributions

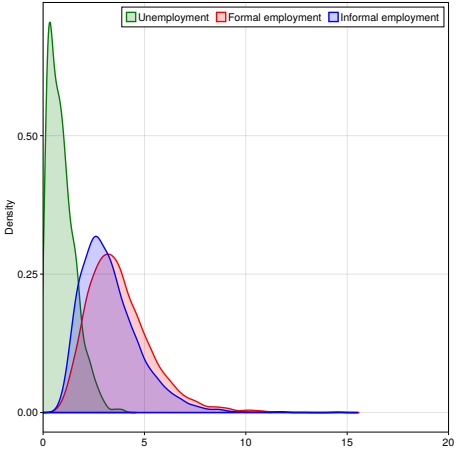


(c) Total Assets

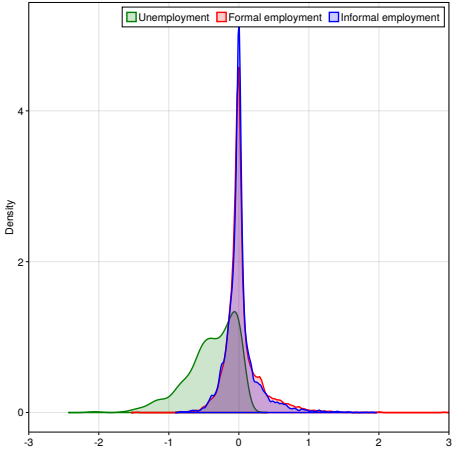


(d) Financial Assets

Steady State Distributions



(e) Consumption



(f) Savings

- Value functions iteration with a discretized state space and an upwind finite difference method to approximate the derivatives (Achdou et.al., 2017).
- Define $W_{i,j,k,f}$ and $U_{i,j}$ for the grids a_i, $r_{2,j}$, w_k.

$$\partial_a U(a, r_2) \approx \begin{cases} \frac{U_{i+1,j} - U_{i,j}}{a_{i+1} - a_i} & da > 0 \\ \frac{U_{i,j} - U_{i-1,j}}{a_i - a_{i-1}} & da < 0 \end{cases}$$

$$\partial_a W(a, r_2, w, f) \approx \begin{cases} \frac{W_{i+1,j,k,f} - W_{i,j,k,f}}{a_{i+1} - a_i} & da > 0 \\ \frac{W_{i,j,k,f} - W_{i-1,j,k,f}}{a_i - a_{i-1}} & da < 0 \end{cases}$$

- Upwind to approximation $\partial_a U(a, r_2)$ and $\partial_a W(a, r_2, w, f)$
- The upwind approximation $\partial^2_{r_2} U(a, r_2)$ and $\partial_{r_2} W(a, r_2, w, f)$ is similar, use *forward difference* when $dr_2 > 0$ and *backward difference* when $dr_2 < 0$.

- We use again finite differences to approximate the second derivative.

$$\partial^2_{r_2} U(a, r_2) \approx \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(r_{2,j+1} - r_{2,j})^2}$$
$$\partial^2_{r_2} W(a, r_2, w, f) \approx \frac{W_{i,j+1,k,f} - 2W_{i,j,k,f} + W_{i,j-1,k,f}}{(r_{2,j+1} - r_{2,j})^2}$$

- Boundary conditions in a-dimension are needed for the backward approximation:

$$\begin{aligned}\partial_a U(\underline{a}, r_2) = u'(r_1\phi(\underline{a}, r_2, 0) + r_2(1 - \phi(\underline{a}, r_2, 0)))\underline{a} + b - c^u(\underline{a}, r_2) \\ - \frac{\psi^u}{2}\phi(\underline{a}, r_2, 0)^2)\end{aligned}$$
$$\begin{aligned}\partial_a W(\underline{a}, r_2, w, f) = u'(r_1\phi(\underline{a}, r_2, w, f) + r_2(1 - \phi(\underline{a}, r_2, w, f)))\underline{a} + w - c^u(\underline{a}, r_2, w, f) \\ - \frac{\psi^e(f)}{2}\phi(\underline{a}, r_2, w, f)^2)\end{aligned}$$

- Boundary conditions in r_2-dimension:

$$\partial_{r_2} U(a, \underline{r_2}) = 0 \Rightarrow U_{i,0} = U_{i,1}$$
$$\partial_{r_2} U(a, \bar{r}_2) = 0 \Rightarrow U_{i,J+1} = U_{i,J}$$
$$\partial_{r_2} W(a, \underline{r_2}, w, f) = 0 \Rightarrow W_{i,0,k,f} = W_{i,1,k,f}$$
$$\partial^2_{r_2} W(a, \bar{r}_2, w, f) = 0 \Rightarrow W_{i,J+1,k,f} = W_{i,J,k,f}$$