On the Interaction between Public Sector Employment and Minimum Wage in a Search and Matching Model[*]

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Abstract

We incorporate a minimum wage to a search and matching model of the labor market with private and public sectors and workers with different human capital levels. The model is structurally estimated using data for Chile, a country with a large fraction of employment in the public sector and a binding minimum wage. Results suggest a sizable productivity gap in favor of the private sector. Closing this gap would have a large aggregate and distributive impact, significantly reducing the fraction of minimum wage earners. Our results highlight unexplored margins of the effect of the minimum wage on the labor market.

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1   Introduction

There are two interesting facts of the labor markets in developing economies: first, the public sector accounts for a large fraction of employment (Mizala et al., 2011), and second, there is a large mass of workers earning wages around the minimum wage levels (Maloney and Mendez, 2004; Boeri et al., 2008; Boeri, 2012). The Chilean labor market is particularly interesting because not only the public sector is a large employer but also the wage distributions in

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both the public and the private sector have a large density around the minimum wage. The minimum wage in Chile is particularly relevant compared with other OECD countries[1]. Chile has a national minimum wage that applies to all workers between 18 and 65 years and it is adjusted once a year. Based on the sample of prime aged full-time urban employed male workers used in this paper, 13.5% of the workers are employed in the public sector. In addition, the minimum wage seems to be binding, as 31% and 18% of the employed in the private and public sectors, respectively, earn up to 1.2 minimum wages. This suggests that increases in the minimum wage have at least a direct impact on the wage bill in both sectors. In addition, if public sector jobs are determined according to rules different from those applying to the private sector, employment in both sectors would respond differently to changes in the minimum wage. Therefore, a question that arises is: To what extent does the existence of a public sector employer affect the impact of the minimum wage in the labor market? In particular, how would the minimum wage policy affect the number and composition of workers and the distribution of wages across the private and the public sectors?

In this paper, we empirically analyze the interaction between public sector employment and the minimum wage using recent data for Chile, a country with a large fraction of employment in the public sector and a binding minimum wage. We do so by developing a search and matching model with private and public sector employment and a minimum wage policy. We then structurally estimate the model to match the Chilean data and perform different policy and counterfactual experiments.

There is a vast literature on the effects of the minimum wage on the labor market. Neumark and Wascher (2008) provide a thorough review of the literature and conclude that there is a lack of consensus on the employment effects of the minimum wage. This has motivated a resurgence of interest on this topic (Harasztosi and Lindner, 2015; Meer and West, 2015). Regarding the effect of the minimum wage on the public sector, the literature is restricted to a few empirical papers and it is also inconclusive. Lemos (2007) shows that minimum wage increases have a strong compression effect on the wages distribution of public sector workers in Brazil but do not have adverse effects on employment. On the contrary, Gindling and Terrell (2007) findings for Costa Rica suggest that a raise in the minimum wage increases the average wage and lowers employment in the public sector. Using data for Honduras, Gindling and Terrell (2009) find a positive effect of the minimum wage on average government wages but no effect on public sector employment. Alaniz et al. (2011) study the effect of the minimum wage on different labor outcomes using longitudinal data for Nicaragua. Their results show evidence of transitions to work in the public sector by

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[1]See OECD (2018) report on minimum wages in the OECD countries..

2

workers who lose their formal private sector jobs as a result of a higher legal minimum wage. There is also a related literature on spillover effects between wages in the public and the private sectors. Using individual data for Hungary, Telegdy (2014) finds evidence of public to corporate wages spillover effects. Using macro data, Lamo et al. (2012) study private-public sector wage leadership in OECD countries. Their results suggest that private sector wages lead public sector wages rather than the other way around. Apart from the empirical evidence, there is interesting information suggesting that governments are aware of the impact of raising the minimum wage on their labor costs and employment levels. This is indeed what it reads from two official reports of the states of New Jersey (2012) and the District of Columbia (2013) in the US calculating the estimated effects of state minimum wage increases on their budgets and employment[2].

A few papers introduce public sector employment in search and matching models. Burdett (2011) and Bradley et al. (2014), develop search models with on-the-job search (à la Burdett and Mortensen, 1998) and public sector employment. Quadrini and Trigari (2007), Michaillat (2014), Gomes (2015a), Gomes (2015b) and Albrecht et al. (2017) use instead Diamond-Mortensen-Pissarides type models. None of these papers consider the effects of a minimum wage policy in a labor market with private and government sectors. There are though two-sector search models with a minimum wage, e.g. Acemoglu (2001). More recently, Meghir et al. (2015) estimate a wage posting model à la Burdett and Mortensen (1998) with heterogeneous firms and an informal sector for Brazil, where they consider the effect of the minimum wage and other policies.

Our paper is related to Albrecht et al. (2017), a model with public and private sectors and a continuum of worker types. While they focus on the effects of public sector employment policies on the labor market, we focus instead on the effects of an overall labor market policy on the wage/productivity distributions and employment across the sectors. We find that a larger minimum wage increases low-skill workers incentives to accept jobs in the public sector, where aside from offering more stable jobs, hiring requirements and employment may not be affected by the minimum wage policy. The existence of a public sector employer where workers apply, in turn, alters firms incentives to create jobs in the private sector when the minimum wage increases. As a result, the effects of the minimum wage on job creation and overall labor market outcomes are affected by the existence of a public sector employer. We develop a search and matching model with private and public sector employment and a legal minimum wage. Specifically, we introduce a minimum wage policy (Flinn, 2006, 2011) to a labor market model with private and public sectors and with heterogeneous workers in their

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[2]See   www.governing.com/topics/mgmt/gov-cities-that-raise-the-minimum-wage-would-have-to-pay-some-public-employees-more.html

human capital level, high-skilled and low-skilled workers. We assume that productivity is match specific. That is, when two parties meet in the private sector, there is a productivity draw from a distribution of productivity. Depending on the productivity draw and the minimum wage, the match is realized or not and wages are eventually determined. If the minimum wage is binding but the productivity draw is such that it is in the interest of the parties to match, then the worker is paid the minimum wage. For low enough productivity levels the match is not formed and for high enough productivity levels, the minimum wage is not binding and wages are determined by Nash Bargaining. Matches in the public sector are formed similarly, but the minimum productivity requirement is a policy parameter that can differ from the minimum wage. We also follow Gomes (2015a) to assume that public sector wages are a constant type-specific fraction of private sector wages. A free entry condition determines private sector vacancies and public sector vacancies are determined according to an employment target in the public sector.

We estimate our model using the Simulated Method of Moments (SMM) procedure to match the Chilean data from the National Socio-Economic Characterization Survey (CASEN) of 2013. We follow the strategy of Flinn (2006) to estimate a search model with endogenous contact rates using only supply side data. The estimation is performed in two steps and uses the fact that the model can be decoupled in two parts, the supply and the demand sides. We do this since we can exploit the fact that the only link in the model between both sides of the market are the contact rates. According to the estimation results, the expected productivity is larger for high skilled workers in both sectors and both worker types are more productive in the private sector than in the public sector. The sectorial productivity difference is much larger for low skilled workers. We perform several policy and counterfactual experiments. We find that, assuming a constant hiring rule and employment level in the public sector, increasing the minimum wage shifts the composition of employment in the public sector from high to low skilled workers, reducing aggregate public sector productivity. We also find that the effects of the minimum wage can be affected by both public sector employment and the government hiring rule. Most of our results are driven by the large estimated productivity differences between the sectors. Indeed, counterfactual experiments show that increasing expected productivity levels in the public sector (to equalize the private sector levels), has a large distributive impact and reduces the fraction of minimum wage earners in the private sector almost by half.

The paper is organized as follows. In the next section, we present the model and characterize the equilibrium. In section 3, we discuss the estimation procedure and the identification strategy, present the estimation results and evaluate the model fit. Section 4 presents the results of policy and counterfactual experiments, and finally section 5 concludes.

2 The Model

The model used in this paper extends the basic Diamond, Mortensen and Pissarides model (DMP model thereafter)[3] in two directions. First, in line with Albrecht et al. (2017), two sectors are introduced in the model, the private and the public sectors. Private sector firms create vacancies and look for workers to fill them and once they are filled a bargained wage is paid to the workers. On the contrary, in the public sector the government adjusts vacancies to reach a constant public employment target and workers are paid according to a premium over the private sector bargained wage. Second, there is a minimum wage as in Flinn (2006), which imposes a restriction to the bargaining process in both sectors.

2.1 Environment

Time is continuous and the economy is populated by a unit mass of workers, which are heterogeneous in their human capital levels (ex-ante heterogeneity). As in Dolado et al. (2007) we only consider two types of workers, those with high human capital H (skilled workers) and those with low human capital L (unskilled workers).[4] We assume that there is a mass κ of H type workers, which is exogenously determined, and that there are no transitions across workers groups. The search process is random and only unemployed workers search for a job. We define the proportion of unemployed worker who are type H as η. While unemployed, workers receive a flow value of z_y, with $y = H, L$, which can be interpreted as the utility (or disutility) of leisure net of any unemployment benefits. Finally, agents discount the future at rate ρ.

There are two sectors in the economy, the private (p) and the public (g) sectors. Private sector firms and the government search randomly for workers and when they meet, a match specific productivity is realized (ex-post heterogeneity). This productivity level is a draw from a distribution that is sector and skill level specific, that is $x \sim G_s^y(x)$ with $s = p, g$ and $y = H, L$, and it is constant for the duration of the job if both parts reach an agreement. While searching for workers, private sector firms pay a cost c. This cost is zero for public sector vacancies.

Search frictions are characterized by a constant returns to scale matching function $Q(v, u)$, where u is the unemployment rate and $v = v_p + v_g$ is the total number of vacancies as a proportion of the labor force. It is assumed that v_g is determined by the government to reach a public-sector employment target, while v_p is endogenously determined by a standard free entry condition. Defining the overall market tightness as $\theta = \frac{v}{u}$ and the probability of meeting

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[3]See for example Pissarides (2000).
[4]This assumption does not limit the results and facilitates the estimation procedure.

a private employer as $\phi = \frac{v_p}{v_p + v_g}$, we can characterize the arrival rates of prospective employers as $\alpha_w^p = \phi q(\theta)$ and $\alpha_w^g = (1 - \phi)q(\theta)$ for the private and the public sectors, respectively. Additionally, the arrival rate of prospective employees is $\alpha_e^H = \eta \frac{q(\theta)}{\theta}$ and $\alpha_e^L = (1 - \eta) \frac{q(\theta)}{\theta}$ for high and low skilled workers, respectively. Finally, once a job is formed it can be terminated exogenously at rates δ_p and δ_g in the private and the public sectors, respectively.

2.2 Value Functions

At any point in time, a worker can be in one of the following three states: unemployed, working for a private sector firm (private sector job) or working for the government (public sector job). Let U_y denote the value of unemployment for a worker type $y = H, L$, $N_{y,p}(x)$ denote the value of employment for a type y worker in a private sector job with match specific productivity x, and $N_{y,g}(x)$ denote the analogous of the previous value but for the case in which the worker is employed in a public sector job. Therefore, the flow value of an unemployed worker is given by:

$$\rho U_y = \left[\ z_y + \alpha_w^p \int \max[N_{y,p}(x) - U_y, 0] dG_p^y(x) + \alpha_w^g \int \max[N_{y,g}(x) - U_y, 0] dG_g^y(x) \ \right], \ \ y = H, L \tag{1}$$

While unemployed, type y workers receive a (dis)utility z_y and private and public sector jobs arrive at Poisson rates α_w^p and α_w^g, respectively. If a private sector job arrives, a match specific productivity is realized and the job is formed if $N_{y,p}(x) > U_y$. Analogously, if a public sector job arrives the match is formed if $N_{y,g}(x) > U_y$. The flow value of a type y worker employed in a private sector job with current productivity x can then be written as:

$$\rho N_{y,p}(x) = w_{y,p}(x) + \delta_p \left[U_y - N_{y,p}(x) \right], \ \ y = H, L \tag{2}$$

while the flow value of a type y worker employed for the government with current productivity x can be expressed as:

$$\rho N_{y,g}(x) = w_{y,g}(x) + \delta_g \left[U_y - N_{y,g}(x) \right], \ \ y = H, L \tag{3}$$

In equation (2), a type y employee with productivity x in a private sector job receives a wage rate $w_{y,p}(x)$ and a termination shock arrives with its consequent capital loss of $U_y - N_{y,p}(x)$ at a Poisson rate δ_p. Equation (3) has an analogous interpretation for the case of public sector employees. In this case, the wage and the Poisson rate of the termination shock are $w_{y,g}(x)$ and δ_g, respectively.

Private sector firms create vacancies which can be filled or unfilled at any point in time . The production process occurs only if the vacancy is filled. Defining the value of a filled

and unfilled vacancy as $J_{y,p}(x)$ and V_p, respectively, we can write the flow value of a private sector job filled by a y type worker with current productivity x as:

$$\rho J_{y,p}(x) = x - w_{y,p}(x) + \delta_p \left[V_p - J_{y,p}(x) \right], \quad y = H, L \tag{4}$$

Productive matches generate a flow output per worker of x and firms pay a wage rate $w_{y,p}(x)$. If the termination shock occurs, the vacancy becomes unfilled and there is a capital loss of $V_p - J_{y,p}(x)$ to the firm. At this point firms start their search process again. In turn, the flow value of an unfilled vacancy in the private sector can be expressed as:

$$\rho V_p = \begin{array}{l} -c + \alpha_e^H \int \max[J_{H,p}(x) - V_p, 0] dG_p^H(x) \\ + \alpha_e^L \int \max[J_{L,p}(x) - V_p, 0] dG_p^L(x) \end{array} \tag{5}$$

Firms searching for workers to fill their vacancies pay a flow cost c and can meet high or low skilled unemployed workers. High skilled workers are met at a Poisson rate α_e^H and the match is formed only if it is worth it, that is $J_{H,p}(x) > V_p$ for a match specific productivity x. In the same way, low skilled workers are met at a Poisson rate α_e^L and the match is formed if $J_{L,p}(x) > V_p$.

Finally, in the case of the public sector we assume that the government has an employment rate objective and chooses the number of vacancies to reach that goal. Therefore, the endogenous number of public sector vacancies will depend on the rest of the equilibrium objects of the model.[5]

2.3 Wages Determination

Wages in the private sector are determined by Nash Bargaining. Following Flinn (2006), the mandatory minimum wage (m) is incorporated as a side constraint in the worker-firm bargaining problem. Therefore, wages are the solution of the following problem:

$$w_{y,p}(x) = \arg \max_{w \geq m} \left[N_{y,p}(x) - U_y \right]^\beta \left[J_{y,p}(x) - V_p \right]^{1-\beta}$$

where β is interpreted as the bargaining power of the worker. The solution of the problem, ignoring the side constraint, is the standard wage equation, where workers are paid a weighted average (according to the bargaining power) between the match productivity and

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[5]Table A.1 in appendix A summarizes the notation of the value functions and the flow values in the different states of the labor market.

their outside option (the unemployment flow value):

$$w_{y,p}(x) = \beta x + (1 - \beta)\rho U_y$$

Under this wage rule, the productivity that would imply that the worker is paid exactly the minimum wage m is:

$$\widetilde{x}_{y,p} = \frac{m - (1 - \beta)\rho U_y}{\beta}$$

It is possible to distinguish two cases. On one hand, when $\widetilde{x}_{y,p} \leq m$ it holds that $m \leq \rho U$ and therefore all matches would generate wage offers higher than m and thus the solution to the constrained problem is the same as the unconstrained one. That is, the minimum wage is not binding because it is lower than the worker's outside option. On the other hand, when $\widetilde{x}_{y,p} > m$ the minimum wage is binding and three sub-cases arise. First, in the interval $x_{y,p} \in [m, \widetilde{x}_{y,p})$ the offers in the wage equation would be below the minimum wage and therefore the minimum wage is binding and the firms pay m.[6] Second, for any $x_{y,p} \geq \widetilde{x}_{y,p}$ the Nash bargaining wage equation determines the wage. Finally, no match will be formed for productivity draws such that $x < m$. Therefore, the complete wage schedule in the private sector is:

$$w_{y,p}(x) = \begin{cases} m & if \; \widetilde{x}_{y,p} \leq m \\ \beta x + (1 - \beta)\rho U_y & otherwise \end{cases} \tag{6}$$

For the case of the public sector wages, we tried to keep a structure as simple as possible assuming, as in Gomes (2015a), that workers in this sector are paid a "public sector premium" over the private sector wage:

$$w_{y,g}(x) = \lambda_y w_{y,p}(x)$$

where $\lambda_y > 0$ is the "public sector premium" which we assume is skill specific. Note that if $\lambda_y > 1$ public sector wages can never be below the mandatory minimum wage. Using the Nash bargaining equation of the private sector, the public sector wage equation can be written as:

$$w_{y,g}(x) = \lambda_y \beta x + \lambda_y (1 - \beta)\rho U_y$$

It is also assumed that the government sets an exogenous minimum productivity level $\underline{x}_y \geq 0$ as a hiring rule for public sector workers. We can also define the public sector

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[6]It is not difficult to show that if $\widetilde{x}_{y,p} > m$ then $m > \rho U_y$.

productivity such that the modified Nash bargained wage equals the minimum wage as:

$$w_{y,g}(\widetilde{x}_{y,g}) = m \Rightarrow \widetilde{x}_{y,g} = \frac{\frac{m}{\lambda_y} - (1-\beta)\rho U_y}{\beta}.$$

A similar discussion to the one for private sector workers follows. That is, only if $\widetilde{x}_{y,g} > m$ is the minimum wage binding, and three cases arise. First, for productivity in the interval $x_{y,g} \in [\underline{x}_y, \widetilde{x}_{y,g})$ the government pays m. Second, for any $x_{y,g} \geq \widetilde{x}_{y,g}$ the wage equation (7) determines the wage, and finally, no match will be formed for productivity draws such that $x < \underline{x}_y$. These wage schedules are shown graphically in figure 1 assuming that $\lambda_y > 1$. Thus the complete wage schedule in the public sector is:

$$w_{y,g}(x) = \begin{cases} m & if \ \widetilde{x}_{y,g} \leq m \\ \lambda_y \beta x + \lambda_y (1-\beta)\rho U_y & otherwise \end{cases} \tag{7}$$

2.4 Equilibrium

Depending on the values of the parameters and the size of the mandatory minimum wage, three cases could arise in equilibrium; (i) the minimum wage is not binding for either of the two types of workers, (ii) the minimum wage is binding only for low skilled workers, and (iii) the minimum wage is binding for both types of workers. Three cases are discussed below.

2.4.1 Equilibrium 1: No Binding Minimum Wage

In this case the minimum wage is irrelevant because workers and firms will form the match according to the reservation productivities. In the case of the private sector, the reservation productivity $x^*_{y,p}$ satisfies $U_y = N_{y,p}(x^*_{y,p})$. Using the wage equation (6) we have:

$$x^*_{y,p} = \rho U_y, \quad y = H, L \tag{8}$$

In a similar way, the public sector reservation productivity $x^*_{y,g}$ satisfies $U_y = N_{y,g}(x^*_{y,g})$ and we assume that $\underline{x}_y = x^*_{y,g}$. Using the wage equation (7) we have:

$$x^*_{y,g} = (1 + \varphi_y)\rho U_y, \quad y = H, L \tag{9}$$

where $\varphi_y = \frac{1}{\beta}\left(\frac{1-\lambda_y}{\lambda_y}\right)$ is the reservation productivity wedge across sector. If $\lambda_y > 1$ then the wedge is negative implying $x^*_{y,g} < x^*_{y,p}$. That is, a positive premium implies that the public sector has a less restrictive hiring rule (in terms of productivity). Using the reservation values $x^*_{y,p}$ and $x^*_{y,g}$ and definitions of $N_{y,p}(x)$, $N_{y,g}(x)$, $w_{y,p}(x)$ and $w_{y,p}(x)$ it is possible to

rewrite equation (1) as:

$$\rho U_y = \left[z_y + \alpha_w^p \int_{\rho U_y} \beta \left(\frac{x - \rho U_y}{\rho + \delta_p} \right) dG_p^y(x) \right. \tag{10}$$

$$\left. + \alpha_w^g \int_{(1+\varphi_y)\rho U_y} \lambda_y \beta \left(\frac{x - (1 + \varphi_y) \rho U_y}{\rho + \delta_g} \right) dG_g^y(x) \right], \quad y = H, L$$

Note that these two Bellman equations solve for the outside option values ρU_H and ρU_L given α_w^p and α_w^g (or ϕ, θ, η) for $y = H, L$. On the private sector firms side, profit maximization requires that all rents from new job creations should be exhausted such that the value of one more vacancy is zero, that is $V_p = 0$ (Mortensen and Pissarides, 1994). Using again the reservation values $x_{y,p}^*$ and $x_{y,g}^*$ and definitions of $N_{y,p}(x)$, $N_{y,g}(x)$, $w_{y,p}(x)$ and $w_{y,p}(x)$, this condition (also referred to as the free entry condition) implies:

$$c = \alpha_e^H \int_{\rho U_H} \frac{(1 - \beta)(x - \rho U_H)}{\rho + \delta_p} dG_p^H(x) + \alpha_e^L \int_{\rho U_L} \frac{(1 - \beta)(x - \rho U_L)}{\rho + \delta_p} dG_p^L(x) \tag{11}$$

where $\alpha_e^H = \eta \frac{q(\theta)}{\theta}$ and $\alpha_e^L = (1 - \eta) \frac{q(\theta)}{\theta}$. This last equation solves for the market tightness, θ, given ρU_H and ρU_L (together with η and ϕ).

2.4.2 Equilibrium 2: Binding Minimum Wage only for Low Skilled Workers

This case occurs when $x_{L,p}^* \leq m < x_{H,p}^*$, where $x_{L,p}^*$ and $x_{H,p}^*$ are the reservation productivities in the no binding minimum wage case. For high skilled workers equation (10) holds and determines their unemployment flow value (and therefore the reservation productivities in the private and the public sector). The case of low skilled workers is different. According to the previous discussion on wage determination, in the private sector a match will be formed if and only if the productivity draw is greater than the minimum wage. If that is the case, workers earn the minimum wage if their productivity is in the interval $[m, \widetilde{x}_{L,p})$, with $\widetilde{x}_{L,p} = \frac{m - (1 - \beta)\rho \tilde{U}_L}{\beta}$. We use $\rho \tilde{U}_L$ instead of ρU_L to denote the flow unemployment value for the case in which the minimum wage is binding. If productivity is greater than $\widetilde{x}_{L,p}$ the wage rate is defined by equation (6).

Meanwhile, in the public sector a match will be formed if the match-specific productivity draw is greater than the hiring minimum productivity $\underline{x}_L$. Public sector minimum wage earners have productivity in the range $[\underline{x}_L, \widetilde{x}_{L,g})$, where $\widetilde{x}_{L,g} = \frac{\frac{m}{\lambda_L} - (1 - \beta)\rho \tilde{U}_L}{\beta}$ is such that $w_{L,g}(\widetilde{x}_{L,g}) = m$. For productivity greater than $\widetilde{x}_{L,g}$, wages are determined according to equation (7). It is easy to show that if $\rho \tilde{U}_L < m$ then $\widetilde{x}_{L,g} > x_{L,g}^*$.

The value of $\underline{x}_L$ could be also set at $x_{L,g}^*$, the reservation productivity in the non binding

minimum wage case. In estimating the model, however, we leave this parameter free and estimate its value from the data. Taking into account what we described above, it is possible to modify the value function of low skilled unemployed agents in the following way:

$$\rho\tilde{U}_L = \begin{bmatrix} z_L + \alpha_w^p \left\{ \int_m^{\widetilde{x}_{y,p}} \left(\frac{m-\rho\tilde{U}_L}{\rho+\delta_p}\right) dG_p^L(x) + \int_{\widetilde{x}_{y,p}} \beta \left(\frac{x-\rho\tilde{U}_L}{\rho+\delta_p}\right) dG_p^L(x) \right\} \\ +\alpha_w^g \left\{ \int_{\underline{x}_L}^{\widetilde{x}_{L,g}} \left(\frac{m-\rho\tilde{U}_L}{\rho+\delta_g}\right) dG_g^L(x) + \int_{\widetilde{x}_{L,g}} \lambda_L\beta \left(\frac{x-(1+\varphi_L)\rho\tilde{U}_L}{\rho+\delta_g}\right) dG_g^L(x) \right\} \end{bmatrix} \quad (12)$$

with $\widetilde{x}_{L,p} = \frac{m-(1-\beta)\rho\tilde{U}_L}{\beta}$ and $\widetilde{x}_{L,g} = \frac{\frac{m}{\lambda_L}-(1-\beta)\rho\tilde{U}_L}{\beta}$. This Bellman equation solves for $\rho\tilde{U}_L$ given α_w^p and α_w^g (or ϕ, θ, η). As was previously mentioned, ρU_H is solved as in the unconstrained case. Figure 1 graphically shows the wage schedule, for the private and the public sectors in the case of workers with a binding minimum wage, and the reservation productivities. The assumption maintained in the graph is that $\lambda_L > 1$, however the analysis with the opposite case is analogous. This parameter will also be estimated from the data.

The free entry condition is also modified taking into account the fact that the minimum wage is now binding for low skilled workers:

$$c = \alpha_e^H \int_{\rho U_H} \frac{(1-\beta)(x-\rho U_H)}{\rho+\delta_p} dG_p^H(x)$$

$$+ \alpha_e^L \left[\int_m^{\widetilde{x}_{L,p}} \frac{x-m}{\rho+\delta_p} dG_p^L(x) + \int_{\widetilde{x}_{L,p}} \frac{(1-\beta)\left(x-\rho\tilde{U}_L\right)}{\rho+\delta_p} dG_p^L(x) \right] \quad (13)$$

where again $\widetilde{x}_{L,p} = \frac{m-(1-\beta)\rho\tilde{U}_L}{\beta}$ and $\alpha_e^H = \eta\frac{q(\theta)}{\theta}$ and $\alpha_e^L = (1-\eta)\frac{q(\theta)}{\theta}$. Therefore, the last equation solves for θ given ρU_H and $\rho\tilde{U}_L$ (together with η and ϕ).

2.4.3   Equilibrium 3: Binding Minimum Wage for Both Workers

This case occurs when $x_{L,p}^* < x_{H,p}^* \leq m$ where $x_{y,p}^*$ with $y = H, L$ are the reservation productivities in the no binding minimum wage case. All ideas described in case 2 apply now to both types of workers. In particular, a worker will earn the minimum wage if the match specific productivity is in the interval $[m, \widetilde{x}_{y,p})$ and the wage rate (6) if productivity is greater than $\widetilde{x}_{y,p}$. For the public sector case, a worker with productivity higher than $\underline{x}_y$ will be hired and paid m, and he/she will be paid the wage rate in equation (7) if $x_{y,g} \geq \widetilde{x}_{y,g}$. The modified value function for the flow value of unemployment for both types of unemployed

agents is.

$$\rho\tilde{U}_y = \begin{bmatrix} z_y + \alpha_w^p \left\{ \int_m^{\tilde{x}_{y,p}} \frac{m-\rho\tilde{U}_y}{\rho+\delta_p} dG_p^y(x) + \int_{\tilde{x}_{y,p}} \frac{\beta\left(x-\rho\tilde{U}_y\right)}{\rho+\delta_p} dG_p^y(x) \right\} \\ +\alpha_w^g \left\{ \int_{\underline{x}_y}^{\tilde{x}_{y,g}} \left(\frac{m-\rho\tilde{U}_y}{\rho+\delta_g} \right) dG_g^y(x) + \int_{\tilde{x}_{y,g}} \lambda_y\beta\frac{x-(1+\varphi_y)\rho\tilde{U}_y}{\rho+\delta_g} dG_g^y(x) \right\} \end{bmatrix}, \quad y = L, H \quad (14)$$

with $\tilde{x}_{y,p} = \frac{m-(1-\beta)\rho\tilde{U}_y}{\beta}$ and $\tilde{x}_{y,g} = \frac{\frac{m}{\lambda_y}-(1-\beta)\rho\tilde{U}_y}{\beta}$. As before these Bellman equations solve for $\rho\tilde{U}_H$ and $\rho\tilde{U}_L$ given α_w^p and α_w^g (or ϕ, θ, η). Meanwhile, the free entry condition in this case is:

$$c = \alpha_e^H \left[\int_m^{\tilde{x}_{H,p}} \frac{x-m}{\rho+\delta_p} dG_p^H(x) + \int_{\tilde{x}_{H,p}} \frac{(1-\beta)\left(x-\rho\tilde{U}_H\right)}{\rho+\delta_p} dG_p^H(x) \right] \qquad (15)$$

$$+\alpha_e^L \left[\int_m^{\tilde{x}_{L,p}} \frac{x-m}{\rho+\delta_p} dG_p^L(x) + \int_{\tilde{x}_{L,p}} \frac{(1-\beta)\left(x-\rho\tilde{U}_L\right)}{\rho+\delta_p} dG_p^L(x) \right]$$

with $\tilde{x}_{L,p} = \frac{m-(1-\beta)\rho\tilde{U}_L}{\beta}$ and $\alpha_e^H = \eta\frac{q(\theta)}{\theta}$ and $\alpha_e^L = (1-\eta)\frac{q(\theta)}{\theta}$. The last equation solves for θ given $\rho\tilde{U}_H$ and $\rho\tilde{U}_L$ (together with η and ϕ).

2.4.4 Steady State Conditions

To close the model we use the notion of steady state equilibrium, that is the in-flows and the out-flows of each state are equalized:

$$\delta_p e_{y,p} = \phi q(\theta)\widetilde{G}_p^y(\max\{m, x_{y,p}^*\})u_y, \quad y = H, L \qquad (16)$$

$$\delta_g e_{y,g} = (1-\phi) q(\theta)\widetilde{G}_g^y(\max\{\underline{x}_y, x_{y,g}^*\})u_y, \quad y = H, L$$

$$u_H + e_{H,p} + e_{H,g} = \kappa$$

$$u_L + e_{L,p} + e_{L,g} = (1-\kappa)$$

$$u = u_H + u_L$$

where $\widetilde{G}_p^y(\cdot) = 1 - G_p^y(\cdot)$. Note that the equations in (16) cover the three cases previously described depending on whether the reservation productivities $x_{y,p}^*$ and $x_{y,g}^*$ are higher or lower than the hiring rule in the case of binding minimum wage, m and $\tilde{x}_{y,g}$ for the private and public sectors, respectively. Solving the above-mentioned system of equations, it is possible to find a closed form solution for the unemployment and the employment rates in

12

both sectors:

$$u_y = \frac{\delta_p \delta_g \left(I(y=H)\kappa + I(y=L)(1-\kappa)\right)}{\delta_p \delta_g + \delta_g \phi q(\theta)\widetilde{G}_p^y(\max\{m, x_{y,p}^*\}) + \delta_p \left(1-\phi\right) q(\theta)\widetilde{G}_g^y(\max\{\underline{x}, x_{y,g}^*\})}$$

$$e_{y,p} = \frac{\delta_g \phi q(\theta)\widetilde{G}_p^y(\max\{m, x_{y,p}^*\}) \left(I(y=H)\kappa + I(y=L)(1-\kappa)\right)}{\delta_p \delta_g + \delta_g \phi q(\theta)\widetilde{G}_p^y(\max\{m, x_{y,p}^*\}) + \delta_p \left(1-\phi\right) q(\theta)\widetilde{G}_g^y(\max\{\underline{x}, x_{y,g}^*\})}$$

$$e_{y,g} = \frac{\delta_p \left(1-\phi\right) q(\theta)\widetilde{G}_g^y(\max\{\tilde{x}_{y,g}, x_{y,g}^*\}) \left(I(y=H)\kappa + I(y=L)(1-\kappa)\right)}{\delta_p \delta_g + \delta_g \phi q(\theta)\widetilde{G}_p^y(\max\{m, x_{y,p}^*\}) + \delta_p \left(1-\phi\right) q(\theta)\widetilde{G}_g^y(\max\{\underline{x}, x_{y,g}^*\})}$$

Finally, the proportion of unemployed workers who are type H is:

$$\eta = \frac{u_H}{u_H + u_L} \tag{17}$$

In turn, the proportion of private sector vacancies can be written as a function of the unemployment rate, the labor market tightness and the vacancy rate in the public sector:

$$\phi = \frac{u\theta - v_g}{u\theta} \tag{18}$$

where v_g is chosen to target a level of $e_g = e_{Hg} + e_{Lg}$.

Putting all the ingredients together it is possible to define the equilibrium in this model in the following way:

Definition. *Given a vector of parameters $(\kappa, z_H, z_L, \rho, \beta, c, \delta_p, \delta_g, \lambda_H, \lambda_L, m, \underline{x}_L)$, a matching function $q(\theta)$, and probability distribution functions for the productivity $G_p^y(x)$ and $G_g^y(\cdot)$ for $y = H, L$, a steady-state equilibrium in the economy with private and public sectors and a mandatory minimum wage is a labor market tightness θ, a proportion of vacancies in the private sector ϕ and a proportion of unemployed worker who are high skilled η, together with the unemployment flow values ρU_y (or $\rho\tilde{U}_y$) for $y = H, L$, the unemployment rate u and the employment rates $e_{y,p}$ and $e_{y,g}$ for $y = H, L$ such that:*

1. *Given η, ϕ and θ, and therefore α_w^p and α_w^g, ρU_H and ρU_L solve the equation in (10) in equilibrium 1; ρU_H and $\rho\tilde{U}_L$ solve equations (10) and (12), respectively, in equilibrium 2; or $\rho\tilde{U}_H$ and $\rho\tilde{U}_L$ solve equations in (14) in equilibrium 3.*

2. *Given η and ϕ, θ solve equation (11) in equilibrium 1, equation (13) in equilibrium 2, or equation (15) in equilibrium 3 and it is consistent with ρU_y (or $\rho\tilde{U}_y$) in (1).*

3. *η and ϕ solve equations (17) and (18) (using the steady state conditions in (16)) and they are consistent with ρU_y (or $\rho\tilde{U}_y$) and θ in (1) and (2).*

The prevailing case (1, 2 or 3) in the economy is determined by comparing the flow value of unemployment (and hence the reservation productivity in the private sector) in case 1 with the mandatory minimum wage.

The solution algorithm directly follows the equilibrium definition and it is presented in the Appendix B.

3 Estimation

This section describes the data used in the structural estimation of the model, the estimation method and the identification strategy. We exploit the possibility to decouple the supply and demand sides of the model to estimate and identify the corresponding parameters separately. At the end of this section we present the estimation results and analyze the fit of the model.

3.1 Data

We estimate the model for the Chilean labor market using a cross-section household survey, which is representative at the national level, namely the Socio-Economic Characterization Survey (CASEN).[7] We use the survey of 2013. In one of the fragments of this survey people are asked about their labor market status as well as their monthly labor income and hours worked in the week prior to the survey. Additionally, the survey contains information on individual characteristics such as gender, age and education. Since there are two types of workers (and within them they are ex-ante homogeneous) and there are no participation decisions in the model, it is necessary to impose a number of restrictions to the sample to ensure, to a certain degree, that those assumptions hold in the data. First, the group of skilled workers is defined as workers with at least a university degree, which means they have at least 17 years of schooling. Second, in Chile the female participation rate is low (below 50%) compared with the male counterpart (around 75%), and therefore, in our sample, we use only male participants in the labor market. Third, given that the mandatory minimum wage affects different age groups in the population in different ways, we are interested in keeping in the sample only those who we believe are more likely to be structurally affected by this policy. Therefore, our sample is comprised of prime aged workers living in urban areas who are in the mature stage of their labor market history. We therefore consider males aged between 30 and 55 years. Finally, we include only full-time formal employees in both sectors, private and public, who have an explicit job contract in our sample. We thus exclude

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[7]The survey is conducted by the Ministry of Social Development since 1985 with a biennial or triennial frequency.

informal and self-employed workers of our sample. As a result of these data restrictions, the sample is reduced to 21,163 observations.

The sample size was further reduced due to problems with the data. On one hand, individuals with missing information on unemployment durations, hours worked or wages where dropped, resulting in a reduction of 26,8% of the valid sample observations. On the other hand, to avoid the effect of outliers in the estimation we dropped the upper and lower one percentile of the wages distribution by sector and skill level, resulting in 7.8% less observations. Thus, after these adjustments, the final sample used consists of 14,331 individuals. An additional adjustment to the data was necessary because some observations were below the mandatory minimum wage.[8] As in Flinn (2006) and Silva (2007) for the cases of the United States and Chile, respectively, we impute the minimum wage to those who earn less than that minimum wage (almost 10%). After the imputation, 21% of the unskilled workers earn the minimum wage in our final sample, and all the skilled workers wages are above the the minimum wage.[9]

In addition, for the identification strategy (described below) we need information on transitions from unemployment to employment in each sector (private and public). Because CASEN is a cross-section survey, it does not contain such type of information as individuals are observed either employed in the private sector, employed in the public sector or unemployed. To fill this data gap, we use the Social Protection Survey (2002-2009)[10] as an additional source of information.[11] This survey contains longitudinal data on labor market histories (status in the labor market, types of jobs for employed workers, and wages), which allow us to identify the destination sector of individual exits from unemployment spells. To avoid the data being affected by the political cycles in hiring and firing during elections and changes of government administrations, we calculate the percentage of unemployed workers who are hired by the private and the public sectors for each transition between 2002 to 2009 and then we compute the average for estimation purposes. It is worth mentioning that we do not attempt to estimate the model using the Social Protection Survey because the data is not only self reported but also retrospective and therefore may be subject to large measurement errors. We use CASEN instead because its sample size is larger and more accurate as the data is self-reported but not retrospective.

To summarize, the data available for the model estimation are: (1) distribution of worker

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[8]Note that these observations are probability zero outcomes conditional on the model.

[9]In Chile, the mandatory minimum wage is defined on a monthly basis, therefore to calculate the hourly minimum wage we used the legal working week of 45 hours.

[10]The survey is conducted by the Micro-data Center of the Economics Department at the University of Chile with the participation of academics of the University of Pennsylvania and the University of Michigan.

[11]It would have been desirable to use data for 2013 but unfortunately the 2013 Social Protection Survey is not available yet.

types, given by the indicator variables for high and low skilled workers $\{I(j = H), I(j = L)\}$; (2) hourly wages in the private and public sectors $\{w_j^p, w_j^g\}$, $j = H, L$, (3) unemployment durations (ongoing) $\{t_j^u, t_j^u\}$, $j = H, L$; (4) labor market status measured by indicator variables for unemployment, employment in the private sector and the public sector $\{I_j(u = 1), I_j(e_p = 1), I_j(e_g = 1)\}$, $j = H, L$; (5) the mass of low skilled workers earning the minimum wage measured by the indicator variables $\{I_j(w_L^p = m), I_j(w_L^g = m)\}$ and finally (6) exits from unemployment to both, the private and the public sectors, $\{\%_j(u \to e_p), \%_j(u \to e_g)\}$, $j = H, L$.

Table 1 shows selected descriptive statistics of the sample. It is interesting to note that for low skilled workers there is a wage premium (of about 14%) and that the wage distribution is less disperse in the public sector. On the contrary, for high skilled workers the average wage is 4% larger in the private sector than in the public sector, and the wage distribution is more spread out in both sectors. Also, the public sector has a larger fraction of high skilled workers. In fact, 33% of workers in the public sector are high skilled, as opposed to only 13% in the private sector. All these regularities are consistent with the evidence for other countries. Finally, on average low skilled workers leave the unemployment state quickly and it is more likely for a private sector job to arrive than one from the public sector (this is particularly notorious for unskilled workers).

3.2   Estimation Method

The first step in the estimation is to decide which model to estimate (1, 2 or 3) depending on how the minimum wage affects the different types of workers. According to the data, 21% of the low skilled workers earn the minimum wage in the private sector and no high skilled worker is observed earning the minimum wage. Therefore, for estimation purposes we use model 2, that is the model in which the mandatory minimum wage is binding only for low skilled workers. With respect to the estimation procedure, in principle it is possible to write the contribution of the duration and wages data to the likelihood as in Flinn (2006) and try to perform maximum likelihood estimation. However, this method cannot be applied here because the parameters λ_y for $y = H, L$ and $\underline{x}_L$ appear in the hiring rules, affecting in that way the truncation points of the accepted wages distributions, as well as the support of the likelihood function.[12] Instead, we use the Simulated Method of Moments (SMM) procedure that involves choosing the model parameters in order to minimize some distance between a set of defined moments simulated using the model and their sample counterparts (Gourieroux and Monfort, 2002).

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[12]Attempts to maximize the likelihood function by changing its support violates one of the regularity conditions of the method.

We follow the strategy of Flinn (2006) to estimate a search model with endogenous contact rates using only supply side data. In this strategy the estimation is performed in two steps and uses the fact that the model can be decoupled in two parts, the supply and the demand sides. The only link in the model between both sides of the market are the contact rates α_p and α_g. For the supply (workers) side, the relevant information in the decision making process are the resulting contact rates in equilibrium. In contrast, the demand side (firms) requires the information behind those contact rates; that is the matching function, market tightness and the number of vacancies in each sector. Using this feature, the estimation strategy is as follows: In the first step we estimate the supply side parameters using SMM assuming that the arrival rates of jobs are constant parameters. The parameters to be estimated in this step are then:

$$\Theta^{SS} = \left\{ \alpha_p, \alpha_g, \delta_p, \delta_g, z_H, z_L, \lambda_{g,H}, \lambda_{g,L}, G_p^H(x), G_g^H(x), G_p^L(x), G_g^L(x), \underline{x}_L \right\}$$

In step two, the demand side parameters are recovered from the estimates of α_p and α_g in step 1 and the equilibrium conditions of the vacancy creation problem, given the matching function $Q(v, u)$. The parameters to be estimated in this step are:

$$\Theta^{DS} = \{\phi, \theta, c, v_g\}$$

As it is usual in the literature that structurally estimates search models with supply side data, we do not attempt to estimate (identify) β and ρ, the worker's bargaining power and the discount rate, respectively (Eckstein and van den Berg, 2007). Instead, we fix those parameters using the following values: $\beta = 0.5$ and $\rho = 0.05$. Additionally, the condition for the estimation of the demand side parameters is the knowledge of the matching function. We do not attempt to estimate that function neither, instead we use a Cobb-Douglas function and set the matching elasticity to $\gamma = \beta$ following standard efficiency considerations.[13]

3.2.1 Supply Side Parameters Estimation and Identification

Recall from the data description that 21% of the low skilled workers earn the minimum wage in the private sector and that non of the high skilled workers are observed with earnings equal to the minimum wage. This implies that the estimation should be done using model 2, that is the model in which the minimum wage is binding only for low skilled workers. This can be clearly noticed in the observed wages distributions in figure 2, where there is a clear

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[13]We are aware that this condition would not necessarily restore full (constrained) efficiency in our model. That is, standard trade externalities are compensated as expected but compositional effect might prevent the decentralized equilibrium to reach the planner allocation.

mass point in the mandatory minimum wage (the truncation point of the distribution) in the private sector which correspond to low skilled workers.

Therefore, for the case of high skilled workers it is possible to use Flinn and Heckman (1982) strategy and estimate (identify) the reservation wage, and hence the reservation productivity, using the lowest observed wage in the private sector, that is $w_p(x^*_{H,p}) = \hat{x}^*_{H,p} = \min\{w^{obs}_{H,p}\}$. As Flinn and Heckman (1982) have shown, this is a strongly consistent estimator of the reservation wage. The remaining parameters are estimated using Simulated Method of Moments (SMM). This procedure implies simulating the model for a given set of parameters and calculating a set of moments from the simulated data, and then comparing those simulated moments with the moments calculated with the corresponding data sample. Assuming $G^j_i(x)$, with $j = H, L$ and $i = p, g$, is log-normal with parameters $(\mu_{j,i}, \sigma^2_{j,i})$, the set of parameters

$$\Theta^{SS} = \{\alpha_p, \alpha_g, \delta_p, \delta_g, z_H, z_L, \lambda_H, \lambda_L, \mu_{H,p}, \sigma_{H,p}, \mu_{L,p}, \sigma_{L,p}, \mu_{H,g}, \sigma_{H,g}, \mu_{L,g}, \sigma_{L,g}, \underline{x}_L\}$$

are estimated using SMM. This is done by minimizing the following loss function:

$$\hat{\Theta}^{SS} = argmin_{\Theta^{SS}} \left\{ \left(\Gamma\left(\Theta^{SS}\right) - \Gamma_N \right)' W \left(\Gamma\left(\Theta^{SS}\right) - \Gamma_N \right) \right\}$$

where $\Gamma\left(\Theta^{SS}\right)$ is a vector $K \times 1$ of simulated moments given Θ^{SS}, Γ_N is a vector $K \times 1$ of data sample moments, and W is a weighting matrix. The weighting matrix W is a diagonal matrix with elements equal to the inverse of the bootstrapped variances of the sample moments. To ensure that the estimated parameters are consistent with model 2 (that is, m binding only for L workers) and that they generate a mass of workers earning the minimum wage, we impose three restrictions in the minimization problem above: $m > \rho\tilde{U}_L$, $m < \tilde{x}_{L,p}$ and $\underline{x}_L < \tilde{x}_{L,g}$. The first implies that the minimum wage is binding for low skilled workers, while the last two ensure there is a mass of low skilled workers earning the minimum wage in both sectors. The moments we match are: the average observed wages in the private and public sectors by skill level (4 moments); the standard deviations of observed wages in the private and public sectors by skill level (4 moments);[14] the ratio between the average wages of the public and the private sectors by skill level (2 moments); the unemployment rate and its proportion by skill level (2 moments); the employment rates in the public and the private sectors by skill level (4 moments); the proportion of exits from unemployment to the private and the public sectors (2 moments); the unemployment duration by skill level (2 moments);

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[14]We do not observe drops that could correspond to a discontinuous support in the wages distributions, therefore the location and the scale parameters are enough to identify the wages distribution assuming log-normality.

and finally, the proportion of low skilled workers earning the minimum wage in the private and the public sectors (2 moments). We estimate 17 parameters on the Supply Side of the model using 22 moment conditions.

In the following we present a mostly heuristic discussion of the identification strategy. The bulk of the computation of the simulated moments are basically related with the unemployment durations and wages because labor market states (unemployment and employment rates) are directly derived from the steady state conditions of the model. In the case of the unemployment duration, we draw realization from the following durations distribution conditional on being unemployed:

$$f_y(t|u) = h_y \exp(-h_y t) \ \ t > 0, y = H, L$$

where h_y is the hazard rate out of unemployment, which can be divided according to the sector of destination (Bover and Gomez, 2004), that is $h_y = h_{y,p} + h_{y,g}$ with $h_{y,p} = \alpha_w^p \left(1 - G_p^y(\max\{m, x_{y,p}^*\})\right)$ and $h_{y,g} = \alpha_w^g \left(1 - G_g^y(\max\{\underline{x}_y, x_{y,g}^*\})\right)$. These are called the intensity of transitions. Conditional on the structure of the model, the transition intensities, and hence the hazard rate out of unemployment, are constant, justifying the exponential distribution for unemployment durations. In addition, to identify the exits to the private and the public sectors separately we can write the model counterpart of the proportion of exits from unemployment to the private ($\Phi_{y,u\to p}$) and the public sectors ($\Phi_{y,u\to g}$) by skill level:

$$\Phi_{y,u\to p} = \frac{h_{y,p}}{h_{y,p} + h_{y,g}}$$
$$\Phi_{y,u\to g} = 1 - \Phi_{y,u\to p}$$

As in Flinn and Heckman (1982), the average unemployment duration conditional on the skill level has information to identify the hazard rate out of unemployment and the proportion of exits from unemployment to the private and the public sectors allows to break down that hazard rate in the transitions intensities to both sectors. Additionally, this information and the equilibrium labor market states together identify the exogenous termination rates of both sectors.

On the wages side, it is necessary to find the observed wages distributions conditional on the skill level and the corresponding sector in order to have comparable moments with those obtained from the data. In the case of the private sector and for high skilled workers, the minimum wage is not binding and therefore the relevant truncation point in the distribution is the reservation productivity. Therefore, in this case wages are drawn from the following

conditional density:

$$f_p^H(w_p|w_p > x_{H,p}^*, e_p) = \frac{\frac{1}{\beta}g_p^H\left(\frac{w_p-(1-\beta)x_{H,p}^*}{\beta}\right)}{1 - G_p^H(x_{H,p}^*)}$$

where $g(\cdot)$ and $G(\cdot)$ are the log normal density function and its cumulative distribution, respectively. To construct the above distribution we use the wage equation to map wages to productivity and then combine that with the productivity distribution truncated at the reservation productivity. For low skilled workers in the private sector, in turn, the minimum wage is binding and, according to the model, workers with productivity in the interval $[m, \tilde{x}_{L,p})$ earn the minimum wage and those with productivity higher or equal than $\tilde{x}_{L,p}$ earn wages above the minimum wage. Therefore, wages in this case are drawn from the following conditional density:

$$f_p^L(w_p|w_p \geq m, e_p) = \begin{cases} \frac{\frac{1}{\beta}g_p^L\left(\frac{w_p-(1-\beta)\rho\tilde{U}_L}{\beta}\right)}{1-G_p^L(\tilde{x}_{L,p})} & w_p > m \\ G_p^L(\tilde{x}_{L,p}) - G_p^L(m) & w_p = m \end{cases}$$

The observed public sector wages density for high skilled workers, from which simulated wages are drawn, is found in an analogous manner to its private sector counterpart. That is, wages in the public sector for high skilled workers are drawn from:

$$f_g^H(w_g|w_g > x_{H,g}^*, e_g) = \frac{\frac{1}{\lambda_H\beta}g_g^H\left(\frac{w_g-\lambda_H(1-\beta)x_{H,p}^*}{\lambda_H\beta}\right)}{1 - G_p^H(x_{H,g}^*)}$$

while for low skilled workers employed in the public sector, wages are drawn from:

$$f_g^L(w_g|w_g \geq m, e_g) = \begin{cases} \frac{\frac{1}{\lambda_L\beta}g_g^L\left(\frac{w_g-\lambda_L(1-\beta)\rho\tilde{U}_L}{\lambda_L\beta}\right)}{1-G_g^L(\tilde{x}_{L,g})} & w_g > m \\ G_p^L(\tilde{x}_{L,g}) - G_p^L(\underline{x}_L) & w_g = m \end{cases}$$

Note that in the case of low skilled workers, the wages distribution indicates that workers in the productivity interval $[\underline{x}_L, \tilde{x}_{L,g})$ earn the minimum wage and those with productivity higher or equal than $\tilde{x}_{L,g}$ earn wages above the minimum wage determined by the government wage schedule.

The identification strategy using wages information is as follows. In the case of high skilled workers working in the private sector, the minimum observed wage and the mean and variance of the observed wages allow to identify the location and scale of the productivity

20

distribution ($\mu_{H,p}$ and $\sigma_{H,p}$). Moreover, the minimum observed wage is used, together with the Bellman equation that determines the equilibrium unemployment value for this type of workers, to recover the unemployment flow (dis)utility (z_H). In the case of high skilled workers working in the public sector, we use the mean and variance of the observed wages, which contains information of the location and scale of the productivity distribution in the public sector ($\mu_{H,g}$ and $\sigma_{H,g}$), which combined with the ratio of average wage in the public and the private sectors aids to identify the public sector premium for this type of workers (λ_H). The parameters associated with low skilled workers working in the private sector are identified from the mean and the variance of observed wages, the minimum wage and the proportion of workers earning the minimum wage. The former contains information of the location and scale of the productivity distribution ($\mu_{L,p}$ and $\sigma_{L,p}$), while the last two together provide information to identify the cut-off productivity $\tilde{x}_{L,p}$. This cut-off productivity, in turn, aids to recover the unemployment flow (dis)utility (z_L) using the Bellman equation that determines the equilibrium unemployment value for this type of workers ($\rho\tilde{U}_L$). Finally, as previously mentioned, the average wage in the public sector and variance of wages provides information on the location and scale of the productivity distribution in the public sector ($\mu_{L,g}$ and $\sigma_{L,g}$), while the proportion of workers earning the minimum wage and the ratio of average wages in the public and private sectors aids in the identification of the cut-off productivity ($\underline{x}_L$) and the public sector premium (λ_L), respectively. The latter depends on the unemployment flow (dis)utility (z_L), which is identified as described previously.

3.2.2 Demand Side Parameters Estimation and Identification

The second step corresponds to the estimation of the demand side parameters, which builds on the estimated values of the contact rates ($\hat{\alpha}_p$ and $\hat{\alpha}_g$) from the first step. Following Flinn (2006), without information on vacancies, v_p and v_g, we cannot identify any additional parameter in the matching function $Q(\cdot)$. This is relevant in our context because we know that the $Q(\cdot)$ function is a sufficient condition to recover and identify all the remaining parameters of the demand side of the model. There are two approaches we can follow to identify the matching function. The first approach, proposed by Flinn (2006), consists of using a matching function that does not contain any unknown parameters, like the exponential matching function $Q(u,v) = v(1 - e^{-u/v})$. The second approach consists of using external sources to obtain estimates of the parameter γ of a Cobb-Douglas matching function $Q(u,v) = u^{1-\gamma}v^{\gamma}$. According to Petrongolo and Pissarides (2001), the Cobb-Douglas matching function has had empirical success, while the exponential matching function generates implausible levels and duration of unemployment and hence is not, empirically, a good approximation. The drawback of the Cobb-Douglas function, however, is the lack of micro-foundations and the

use of external estimates. Here we weight more to fit the data and therefore we follow the second approach and set $\gamma = 0.5$.

Once the matching function is known, then $\hat{\theta}$ and $\hat{\phi}$ can be recovered from the solution to the following system of equations:

$$\hat{\alpha}_w^p = \phi q(\theta)$$
$$\hat{\alpha}_w^g = (1 - \phi) q(\theta)$$

Combining $\hat{\theta}$, the estimates from the supply side and the free entry condition in case 2 (equation 13) allow us to recover the search cost c:

$$c = \frac{q(\hat{\theta})}{\left(\rho + \hat{\delta}_p\right) \hat{\theta}} \left\{ \hat{\eta} \int_{\hat{x}_{H,p}^*} (1 - \beta) \left(x - \hat{x}_{H,p}^*\right) d\hat{G}_p^H(x) \right.$$
$$\left. + (1 - \hat{\eta}) \left[\int_m^{\hat{\tilde{x}}_{L,p}} (x - m) d\hat{G}_p^L(x) + \int_{\hat{\tilde{x}}_{L,p}} (1 - \beta) \left(x - \hat{\tilde{x}}_{L,p}\right) d\hat{G}_p^L(x) \right] \right\}$$

Finally, once all the above parameters are identified, v_g can be recovered using equation (18), that is:

$$v_g = (1 - \hat{\phi}) \hat{u} \hat{\theta}$$

3.3   Estimation Results

Table 2 shows the estimated parameters for the supply (top panel) and demand sides (middle panel) of the market. Also the parameters that are fixed in the estimation are presented in the bottom panel. The first eight rows of the top panel are the location and the scale parameters of the productivity distributions by sector and skill level. Given the log-normality assumption, the average productivity implied in those estimates are 17.82 USD and 3.43 USD per hour in the private sector (for skilled and unskilled workers) and 12.96 USD and 1.60 USD per hour in the public sector (for skilled and unskilled workers). These results indicate that high skilled workers are on average 37.5% more productive in the private sector than in the public sector, while low skilled workers productivity in the private sector more than doubles their productivity in the public sector. These productivity figures are shown in the top panel of table 3. However, the standard deviations indicate that the productivity distributions are substantially more spread out for high skilled than for low skilled workers (14.93 and 9.73 for the private sector vs. 3.73 and 3.16 for the public sector).

The estimated values for termination rates, in rows nine and ten, imply that the average durations of a job in the private and the public sectors are approximately 22 and 46 months,

respectively. Moreover, estimated parameters generate, in equilibrium, contact rates that imply that in the private sector job offers arrive almost every month while in the public sector they arrive every seven months (see table 3).

Wage premium in the public sector, presented in rows eleven and twelve of table 2, are 27 and 2.4% for high and low skilled workers, respectively. In addition, the fourth and fifth rows of table 3 show the reservation productivity for high skilled workers and indicates that the private sector is (75%) pickier than the public sector in choosing their workers, 2.41 USD vs 1.37 USD per hour. In contrast, the cut-off productivities for low skilled workers, shown in row thirteen of table 2, indicate that workers in the public sector need a slightly higher productivity, 1.96 vs 1.79, to be hired (note that in the private sector the cut-off productivity is the minimum wage).

The demand side parameters, shown in the middle panel of table 2 indicate that 86% of the unfilled vacancies are private sector vacancies and that the market tightness is relatively low because in this economy for every vacancy there are 1.33 workers looking to fill that vacancy. Moreover, 9.1% of the unemployed workers are high skilled and the search cost is 1.7 of the high skilled workers' average wage and 5.4 of the low skilled workers' average wage, both in the private sector. Finally, the bootstrap standard errors of the estimated parameters show that the estimation is quite precise.

To assess the fit of the estimates, table 3 compares the predictions obtained by simulating the model and their data sample counterpart. Three comments are worth mentioning. First, it is notorious that the overall fit of the model is good and this is particularly true for the moments related with the labor market status and the wages distributions. Second, where the model does badly is in fitting the average duration of the unemployment state, particularly for the skilled workers. This is related with the fact that the model overestimates the number of transitions of high skilled workers to the private sector, resulting in a lower unemployment duration.[15] Finally, four moments of particular interest, given the findings discussed in the data section before, are the ratio between the average wages of the public and the private sectors and the mass of workers earning the minimum wage in both the private and the public sectors. In the case of the former, the model estimates accurately and correctly capture the fact that, on average, high skilled workers earn more in the private sector while low skilled workers earn more in the public sector. For the latter, in turn, the model adequately captures the majority of mass observed in the data. Finally, in terms of the wages distributions, it can be observed that the simulated densities in figure 2 have

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[15]Since we weight each moment differently (using the weighted matrix W) in order to minimize the loss function in the SMM method, the moments related precisely with the duration and transitions are those that weigh less.

similar shapes as those obtained from the data.

4 Policy and Counterfactual Experiments

Using the estimated model as a starting point, we perform several policy and counterfactual experiments. Specifically, in our policy experiments we analyze the effect on the equilibrium outcomes and on total output of an increase in the minimum wage, an increase in the target size of public sector employment and a reduction in the hiring standards (rule) of the public sector. Total output is defined as aggregate production of all employed workers in the economy and it is equivalent to average productivity, since the economy is populated by a unit mass of workers. On the other hand, given that in the estimated model the results are driven by the large estimated productivity differences between the sectors, we explore in the counterfactual experiments the effects of setting the productivity parameters of the public sector equal to their private sector counterparts. Additionally, we consider the effect of setting the wage rule in the public sector equal to the private sector one.

To evaluate welfare effects of the policy experiments, we construct a welfare measure exploiting the distribution of workers in the steady-state equilibrium of the model and the values of each state of the labor market (Flinn, 2006). In particular, the welfare function for type H and L workers are respectively

$$W_H = u_H U_H + e_{H,p} \int_{x_{H,p}^*} N_{H,p}(x) dG_p^H(x) + e_{H,g} \int_{x_{H,g}^*} N_{H,g}(x) dG_g^H(x)$$

$$W_L = u_L U_L + e_{L,p} \int_m N_{L,p}(x) dG_p^L(x) + e_{l,g} \int_{\underline{x}_L} N_{L,g}(x) dG_g^L(x)$$

The total welfare function W_{TOT} can be calculated directly using the distribution of type H and L workers over the population, that is

$$W_{TOT} = \kappa W_H + (1 - \kappa) W_L$$

4.1 Policy Experiments

Results of policy experiments are presented in table 4. The first column shows, as benchmark, the simulation of the model using the estimated parameters of table 2. In the first policy experiment, presented in the second column of table 4, we increase the mandatory

minimum wage in 10% with respect to the baseline value (1.7978 US$) while keeping the rest of the estimated parameters in their point estimates. After the increase, the minimum wage continues to be binding only for low skilled workers. As can be noticed, the policy change has a large impact on the unemployment rate. The increase (by 10%) in the minimum wage increases (long run) unemployment by 0.5 percentage points (6.7% increase in relative terms) and unemployment durations are affected accordingly. There are also large composition effects as low skilled workers are more affected by the policy than high skilled workers. Indeed, the fraction of low skilled unemployed workers increases with the minimum wage while high skilled workers are much less affected. This is because, with a larger minimum wage, the expected costs of keeping jobs for both firms and low skilled workers in the private sector increase, hence reducing θ and the contact rate of vacancies in both sectors. Given the weaker labor market, reservation productivities for high skilled workers ($x^*_{H,p}$ and $x^*_{H,g}$) decrease and u_H is only slightly affected. Note that low skilled workers' reservation productivity is given by m in the private sector and by $\underline{x}_L$ in the public sector. With a lower contact rate and a higher reservation productivity for these workers, u_L increases greatly with the minimum wage, mainly driven by the reduction in private sector employment for low skilled workers. It is interesting that, given that both the government hiring rule $\underline{x}_L$ and employment remain constant, there is a slight increase of 1.2% in the number of low skilled workers in public sector jobs. The minimum wage policy also leads to a lower productivity in the public sector as there is a larger fraction of low skilled workers (who are less productive) and also a lower reservation productivity for high-skill workers, as mentioned above. In the private sector, on the other hand, productivity is driven up by the direct effect of m on the minimum productivity requirement. However, the unemployment effects are large enough to determine a negative effect of the minimum wage on total output, which decreases by 1.5%. In addition, as it is harder for private sector vacancies to get filled, the total cost of private sector vacancies ($c\phi\theta u$), not reported in the table, also increases greatly by 3.6%, reducing net output (of vacancy creation costs) by 2.6%. The increase in m also causes a large increase in the fraction of workers earning the minimum wage in both sectors, which is related to the large increases in $\tilde{x}_{L,p}$ and $\tilde{x}_{L,g}$. As a result of the combined effects on unemployment, employment and reservation productivities aggregate welfare drops by 4.6%, with the larger reduction for low-skill workers. Concluding, the effects of varying the minimum wage are large in unemployment, job creation, job composition, the wage distribution, with a considerable negative effect on total output.

In a second experiment, shown in the third column of table 4, we increase the target number of employed workers in the public sector in 10% with respect to the baseline. Large variations in the size of the public sector require large variations in public sector vacancies,

as expected. Most of the effects of increasing vacancies in the public sector operate through a sort of crowding out effect. Indeed, increasing e_g in 10% leaves the unemployment rate unaffected, with public sector employment increasing and private sector employment decreasing in a similar magnitude. This small overall effect on unemployment is consistent with the little variation in reservation productivities observed in the table. Labor market tightness is also slightly reduced. Average wages are almost unaffected. The major observed effect of increased public sector vacancies is on the composition of jobs. As expected, ϕ decreases and contact rates increase in the public sector and decrease in the private sector. When public sector employment increases, there is an increase in contact rates in the public sector and a small decrease in the contact rates for private sector vacancies. However, given that low skilled workers are in expected terms much less productive in public sector jobs than in private sector jobs, the expected value of unemployment $\rho \tilde{U}_L$ falls (and so $\tilde{x}_{L,i}$ increases). This determines that the range of productivity levels over which firms pay the minimum wage gets larger and so the fraction of workers earning m in both sectors. Finally, due to the expected productivity differences and the composition effects, aggregate output and welfare falls slightly as public sector employment increases.

In our third policy experiment, we vary the minimum productivity level required to be eligible for public sector jobs, $\underline{x}_L$. We consider the extreme case of no productivity requirements, that is $\underline{x}_L = 0$. The results are reported in the last column of table 4. We find that reducing $\underline{x}_L$ increases total output. In particular, with $\underline{x}_L = 0$ there is no rejection of public sector jobs and therefore fewer jobs are vacant in the public sector. As a result there is a benefit fo the economy in terms of reduced search frictions; firms create more jobs in the private sector and unemployment falls. Wages get lower in the public sector as more low-skill (less productive) workers are hired and a much larger fraction of these workers earns the minimum wage. The private sector benefits from a better composition of the unemployment pool due to the reduced search frictions. In the private sector, the pool of low-skill workers becomes more productive (because a higher fraction of low productive workers are hired by the public sector) and the minimum wage becomes less binding. Average high-skill employed workers productivity also increases because of the larger reservation productivities ($x^*_{H,p}$ and $x^*_{H,g}$) consistent with the better labor market prospects. Consequently total output and welfare largely increase.

We also analyze the effect of interacting the minimum wage with the two public sector employment policies discussed before. Specifically, we consider the effect of increasing m by 10% in the baseline economy, in an economy with e_g increased by 10% (third column of table 4) and in an economy with $\underline{x}_L = 0$ (last column of table 4). In table 5, we report the percentage change in the same variables of table 4 in the three cases. Regarding the

interaction with public sector employment, we do not find large interaction effects with aggregate output and unemployment. That is, either increasing m from $e_g = 0.124$ or $e_g = 0.136$ generates similar percentage changes in aggregate output, unemployment, and welfare (4.63% in 4.59 in each case).

On the other hand, increasing the minimum wage in an economy with zero productivity requirements in the public sector has a somewhat lower relative impact on welfare and a larger impact on unemployment (third column of 5) compared with the baseline economy. There are also large composition effects, as the increase in the minimum wage makes it a lot more binding in the private sector in the economy with $\underline{x}_L = 0$ than in the baseline economy. Since every low-skill unemployed worker meeting a public sector vacancy is hired in the economy with $\underline{x}_L = 0$, changing the minimum wage there has little impact on the fraction of workers earning m as it can be seen in the table. The private sector would instead become more responsive to the minimum wage in this case. Productivity does not fall that much because the inflow of low skilled workers into the public sector is not increased by much either. On the other hand the unemployment effects are larger because the capacity of the public sector to employ more unproductive workers is already very high when $\underline{x}_L = 0$ before the increase in m. Summarizing, the output effects are mitigated and the unemployment effects are larger when the minimum wage increases in an economy with no hiring rule in the public sector.

4.2   Counterfactual Experiments

We also analyze the effect of varying the parameters that determine the differences in sectorial characteristics. This experiment lets us analyze the effects of the existence of the public sector in the labor market. The results are presented in table 6. In the first column, we report the model results using the baseline estimated parameters. In the second column, we show the results for an economy where all the parameters in the public sector are the same as the estimates for the private sector. That is, we consider the effect of increasing expected productivity and removing any productivity, destruction rates, wages and hiring rules differences between the sectors. Combining the results reported in columns two and three it is observed that most of the results are driven by the large estimated productivity differences in the baseline model. Indeed, with an increase in expected productivity there is more vacancy creation in the private sector, unemployment falls, wages increase and the minimum wage becomes much less binding in both sectors. Indeed, in the simulated economy of column three, 9.6% and .5% of the low skilled workers earn the minimum wage in the private and public sectors, respectively. Given the large estimated productivity differences

27

between the public and private sector in the baseline, to a large extent, the existence of the public sector explains the large bite of the minimum wage in the Chilean labor market. Moreover, comparing the baseline with the results of column two, we find that aggregate welfare is 10.6% lower because of the existence of the public sector.

Finally, in the last column of table 6 we report the results for an economy where the wage setting rule in the public sector is equal to the one in the private sector, so we set $\lambda_H = \lambda_L = 1$, which are reduced from the estimated values of $\lambda_H = 1.275$ and $\lambda_L = 1.024$. We find that removing the pure public sector wage premium has a null effect on unemployment and total output. Total welfare reduces by 1.2%, mainly explained by the large fall in welfare for H type workers, as expected.. Eliminating the government wage premium makes reservation productivities in both sectors the same. For high-skill workers, the much lower expected wages in the public sector (with respect to the baseline) reduces their reservation productivity for taking jobs in the private sector and increases it in the public sector. For low skilled workers the range of productivities for which workers earn the minimum wage in the public sector increases moderately, therefore the fraction of minimum wage earners increases. On the other hand, the mass of private sector minimum wage earners decreases slightly. The combined results of the counterfactual experiments confirm that it is the productivity differences between the sectors and not the different wage setting rules what mainly determines the impact of the minimum wage in the labour market.

5 Concluding Remarks

This paper develops a search and matching model with a public and a private sector and a mandatory minimum wage. The model is estimated to match recent data for Chile, an economy with a large fraction of public sector workers and a binding minimum wage. The model economy has two types of workers, high and low skilled, who are heterogeneous in their ex-post productivity. Estimation results suggest that match specific productivity is larger for high skilled workers in expected terms in both sectors. In addition, both types of workers are more productive in the private sector than in the public sector, with the sectorial productivity gap being much larger for low skilled workers. Counterfactual experiments show that increasing productivity levels in the public sector to those in the private sector has a large distributive impact, reducing the fraction of minimum wage earners in the private sector almost by half. We also find that, given that the hiring rule and employment level in the public sector are assumed constant, increasing the minimum wage shifts the composition of employment in the public sector from high to low skilled workers, reducing aggregate public sector productivity. Our results highlight a previously unexplored margin by which

the minimum wage affects the labor market and total output.

References

Acemoglu, Daron, "Good jobs versus bad jobs," *Journal of Labor Economics*, 2001, *19* (1), 1–20.

Alaniz, Enrique, T.H. Gindling, and Katherine Terrell, "The impact of minimum wages on wages, work, and poverty in Nicaragua," *Labour Economics*, 2011, *18* (S1), S45–S59.

Albrecht, James, Monica Robayo-Abril, and Susan Vroman, "Public-Sector Employment in an Equilibrium Search and Matching Model," *Economic Journal*, 2017, *forthcoming.*

Boeri, Tito, "Setting the minimum wage," *Labour Economics*, 2012, *19* (3), 281–290.

_ , **Brooke Helppie, and Mario Macis**, "Labor regulations in developing countries: A review of the evidence and directions for future research," *Social Protection Discussion Papers, 46306. The World Bank*, 2008.

Bover, Olympia and Ramon Gomez, "Another look at unemployment duration: exit to a permanent vs. a temporary job," *Investigaciones Economicas*, May 2004, *28* (2), 285–314.

Bradley, Jake, Fabien Postel-Vinay, and Helene Turon, "Public Sector Wage Policy and Labour Market Equilibrium: A Structural Model," Technical Report 2014.

Burdett, Kenneth, "Towards a Theory of the Labor Market with a Public Sector," *Labour Economics*, 2011, *19* (1), 68–75.

_ **and Dale Mortensen**, "Wage Differentials, Employer Size and Unemployment," *International Economic Review*, 1998, *39*, 257–273.

Dolado, Juan, Marcel Jansen, and Serrano Juan Jimeno, "A Positive Analysis of Targeted Employment Protection Legislation," *The B.E. Journal of Macroeconomics*, July 2007, *7* (1), 1–30.

Eckstein, Zvi and Gerard J. van den Berg, "Empirical labor search: A survey," *Journal of Econometrics*, February 2007, *136* (2), 531–564.

Flinn, C. and J. Heckman, "New methods for analyzing structural models of labor force dynamics," *Journal of Econometrics*, January 1982, *18* (1), 115–168.

Flinn, Christopher J., "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates," *Econometrica*, 07 2006, *74* (4), 1013–1062.

＿ , *The Minimum Wage and Labor Market Outcomes*, The MIT Press, 2011.

Gindling, T.H. and Katherine Terrell, "The effects of multiple minimum wages throughout the labor market: the case of Costa Rica," *Labour Economics*, 2007, *14* (3), 485–511.

＿ **and** ＿ , "Minimum wages, wages and employment in various sectors in Honduras," *Labour Economics*, 2009, *16* (3), 291–303.

Gomes, Pedro, "Optimal public sector wages," *forthcoming, The Economic Journal*, 2015a.

＿ , "Heterogeneity and the Public Sector Wage Policy," 2015b. Mimeo.

Gourieroux, Christian and Alain Monfort, *Simulation-Based Econometric Methods*, Oxford University Press, 2002.

Harasztosi, Peter and Attila Lindner, "Who Pays for the Minimum Wage?," 2015. Mimeo University of California at Berkeley.

Lamo, Ana, Javier Pérez, and Ludger Schuknecht, "Public or private sector wage leadership? An international perspective," *The Scandinavian Journal of Economics*, 2012, *114* (1), 228–244.

Lemos, Sara, "The effects of the minimum wage in the private and public sectors in Brazil," *Journal of Development Studies*, 2007, *43* (4), 700–722.

Maloney, William and Jairo Mendez, "Measuring the impact of minimum wages. Evidence from Latin America. In Law and Employment: Lessons from Latin America and the Caribbean," *NBER Chapters, pages 109-130, National Bureau of Economic Research*, 2004.

Meer, Jonathan and Jeremy West, "Effects of the Minimum Wage on Employment Dynamics," *forthcoming, Journal of Human Resources*, 2015.

Meghir, Costas, Renata Narita, and Jean-Marc Robin, "Wages and Informality in Developing Countries," *American Economic Review*, 2015, *105* (4), 1509–1546.

Michaillat, Pascal, "A Theory of Countercyclical Government Multiplier," *American Economic Journal: Macroeconomics*, 2014, *6* (1), 190–217.

Mizala, Alejandra, Pilar Romaguera, and Sebastian Gallegos, "Public-Private Wage Gap In Latin America (1999-2007): A Matching Approach," *Labour Economics*, 2011, *18*, S115–S131.

Mortensen, Dale T and Christopher A Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, July 1994, *61* (3), 397–415.

Neumark, David and William Wascher, *Minimum Wages*, The MIT Press, 2008.

OECD, "Minimum wages relative to median wages," 2018.

Petrongolo, Barbara and Christopher A. Pissarides, "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, June 2001, *39* (2), 390–431.

Pissarides, Christopher A., *Equilibrium Unemployment Theory*, The MIT Press, 2000.

Quadrini, Vincenzo and Antonella Trigari, "Public Employment and the Business Cycle," *Scandinavian Journal of Economics*, 2007, *109*, 723–742.

Silva, Carolina, "An Empirical Analysis of the Minimum Wage in Chile," 2007.

Telegdy, Álmos, "Public Wage Spillovers. Identification from a Large and Unexpected Public Wage Increase," 2014. Mimeo.

Table 1: Descriptive Statistics

	High Skilled	Low Skilled
Hourly Wage - Private Sector (US$/hour)		
Mean	10.119	3.251
Standard Deviation	7.551	1.915
Minimum	2.408	1.798
Hourly Wage - Public Sector (US$/hour)		
Mean	9.730	3.680
Standard Deviation	6.510	2.154
Minimum	3.034	1.798
Ratio of Average Wages	1.040	0.883
Unemployment Duration (Months)		
Mean	3.009	2.263
Unemployment Rate	0.007	0.066
Employment in the Private Sector	0.109	0.693
Employment in the Public Sector	0.042	0.083
Proportion of Transitions $u \to e_p$	0.672	0.900
Proportion of Transitions $u \to e_g$	0.328	0.100
Proportion of Workers with $w_p = m$	-	0.210
Proportion of Workers with $w_g = m$	-	0.147
Proportion of Workers	0.158	0.842

Note: Data extracted from CASEN 2013. Wage distributions are trimmed at the top and bottom 1 percentile by sector and skill level and are reported in US Dollars of December 2009 (Exchange Rate = 559.67 Pesos/US$).

Table 2: Estimated Parameters

	Parameter	Standard Errors
Supply Side Parameters		
$\mu_{H,p}$	2.615	(0.044)
$\sigma_{H,p}$	0.729	(0.030)
$\mu_{H,g}$	0.844	(0.035)
$\sigma_{H,g}$	0.883	(0.014)
$\mu_{L,p}$	2.339	(0.162)
$\sigma_{L,p}$	0.669	(0.054)
$\mu_{L,g}$	-0.325	(0.116)
$\sigma_{L,g}$	1.262	(0.031)
δ_p	0.045	(0.003)
δ_g	0.022	(0.006)
λ_H	1.275	(0.171)
λ_L	1.024	(0.030)
$\underline{x}$	1.960	(0.102)
Demand Side Parameters		
ϕ	0.855	(0.038)
θ	0.751	(0.103)
η	0.091	(0.003)
c	17.604	(2.367)
v_g	0.008	(0.003)
Non Estimated Parameters		
m	1.798	
β	0.500	
ρ	0.067	
γ	0.500	
$Loss$	192.09	

Note: Bootstrap standard errors (in parenthesis) are

based on 500 replications.

Table 3: Model Predictions

	Model	Data
Productivity		
$\bar{x}_{H,p}$	17.825	-
$\bar{x}_{H,g}$	12.967	-
$\bar{x}_{L,p}$	3.434	-
$\bar{x}_{L,g}$	1.601	-
$x^*_{H,p}$	2.408	-
$x^*_{H,g}$	1.369	-
$\tilde{x}_{L,p}$	2.308	-
$\tilde{x}_{L,g}$	2.224	-
Labor Market States		
u	0.075	0.074
η	0.091	0.100
$e_{H,p}$	0.112	0.109
$e_{H,g}$	0.040	0.042
$e_{L,p}$	0.690	0.693
$e_{L,g}$	0.085	0.083
Labor Market Dynamics		
α_p	0.741	-
α_g	0.126	-
$\Pr[u \rightarrow e_p]$	0.854	0.672
$\Pr[u \rightarrow e_g]$	0.944	0.900
$\bar{t}_H$	1.163	3.009
$\bar{t}_L$	2.074	2.263
Wages		
$\bar{w}_{H,p}$	10.175	10.119
$\bar{w}_{H,g}$	9.806	9.730
$\bar{w}_{L,p}$	3.124	3.251
$\bar{w}_{L,g}$	3.271	3.680
$\sigma_{w_{H,p}}$	7.423	7.551
$\sigma_{w_{H,g}}$	6.189	6.510
$\sigma_{w_{L,p}}$	2.021	1.915
$\sigma_{w_{L,g}}$	2.768	2.154
$\bar{w}_{H,p}/\bar{w}_{H,g}$	1.038	1.040
$\bar{w}_{L,p}/\bar{w}_{L,g}$	0.956	0.883
$\Pr[w_{L,p} = m]$	0.181	0.210
$\Pr[w_{L,g} = m]$	0.130	0.147

Table 4: Policy Experiments

	Baseline	Minimum Wage $1.1 \times m$	Public Sect. Employment $1.1 \times e_g$	Public Sect. Hiring Rule $0 \times \underline{x}$
ϕ	0.855	0.861	0.840	0.954
η	0.091	0.087	0.090	0.112
θ	0.751	0.724	0.747	0.812
α_p	0.741	0.732	0.726	0.860
α_g	0.126	0.119	0.138	0.042
$x^*_{H,p}$	2.408	2.160	2.416	2.648
$x^*_{H,g}$	1.369	1.229	1.374	1.506
$\tilde{x}_{L,p}$	2.308	2.704	2.332	2.068
$\tilde{x}_{L,g}$	2.224	2.612	2.248	1.985
u	0.075	0.080	0.075	0.064
u_H	0.007	0.007	0.007	0.007
$e_{H,p}$	0.112	0.113	0.108	0.137
$e_{H,g}$	0.040	0.038	0.043	0.014
u_L	0.068	0.073	0.068	0.057
$e_{L,p}$	0.690	0.683	0.681	0.675
$e_{L,g}$	0.085	0.086	0.094	0.110
e_p	0.802	0.796	0.789	0.812
e_g	0.124	0.124	0.136	0.124
$\bar{t}_H$	1.163	1.181	1.166	1.123
$\bar{t}_L$	2.074	2.249	2.101	1.755
v_g	0.008	0.008	0.009	0.002
$\bar{w}_{H,p}$	10.175	10.051	10.203	10.346
$\bar{w}_{H,g}$	9.806	9.658	9.825	9.978
$\bar{w}_{L,p}$	3.124	3.244	3.119	3.231
$\bar{w}_{L,g}$	3.271	3.295	3.262	2.138
$\bar{w}_{H,p}/\bar{w}_{H,g}$	1.038	1.041	1.039	1.037
$\bar{w}_{L,p}/\bar{w}_{L,g}$	0.956	0.985	0.956	1.512
$\Pr[w_{L,p} = m]$	0.181	0.245	0.189	0.101
$\Pr[w_{L,g} = m]$	0.130	0.281	0.142	0.788
Output	4.685	4.615	4.646	4.828
$Welfare_H$	16.649	16.213	16.718	16.626
$Welfare_L$	17.092	16.238	16.839	19.651
$Welfare$	17.022	16.234	16.820	19.174

Table 5: Policy Interactions with a 10% Increase in the Minimum Wage

	% Change in Variable from:		
	Baseline	Public Sect. Employment $1.1 \times e_g$	Public Sect. Hiring Rule $0 \times \underline{x}$
ϕ	0.63	0.71	0.25
η	-4.71	-4.57	-5.35
θ	-3.56	-3.56	-3.76
α_p	-1.17	-1.10	-1.65
α_g	-5.50	-5.44	-6.97
$x^*_{H,p}$	-10.28	-10.33	-9.26
$x^*_{H,g}$	-10.27	-10.33	-9.26
$\tilde{x}_{L,p}$	17.17	16.96	18.59
$\tilde{x}_{L,g}$	17.45	17.22	18.95
u	6.98	7.09	7.47
u_H	1.47	2.99	1.39
$e_{H,p}$	1.07	1.29	0.44
$e_{H,g}$	-3.54	-3.26	-5.76
u_L	7.53	7.49	8.23
$e_{L,p}$	-0.94	-0.97	-0.80
$e_{L,g}$	1.66	1.50	0.73
e_p	-0.65	-0.67	-0.59
e_g	0.00	0.00	0.00
$\bar{t}_H$	1.56	1.57	1.58
$\bar{t}_L$	8.42	8.32	8.99
v_g	-1.23	0.00	0.00
$\bar{w}_{H,p}$	-1.44	-1.45	-1.46
$\bar{w}_{H,g}$	-1.65	-1.66	-1.62
$\bar{w}_{L,p}$	3.69	3.74	3.47
$\bar{w}_{L,g}$	0.68	0.75	6.68
$\bar{w}_{H,p}/\bar{w}_{H,g}$	0.21	0.21	0.16
$\bar{w}_{L,p}/\bar{w}_{L,g}$	2.99	2.97	-3.00
$\Pr[w_{L,p} = m]$	35.73	33.74	67.99
$\Pr[w_{L,g} = m]$	115.00	104.10	4.77
Output	-2.68	-2.65	-2.49
$Welfare_H$	-2.62	-2.64	-2.47
$Welfare_L$	-5.00	-4.96	-4.24
$Welfare$	-4.63	-4.59	-4.00

Table 6: Counterfactual Experiments

	Baseline	Identical Sectors $G_g = G_p$ $\lambda_g = \lambda_p$ $\delta_g = \delta_p$ $\underline{x} = m$	Equal Productivity $G_g = G_p$	Equal Wages $\lambda_g = \lambda_p$
ϕ	0.855	0.867	0.928	0.855
η	0.091	0.107	0.107	0.091
θ	0.751	0.794	0.768	0.765
α_p	0.741	0.772	0.813	0.748
α_g	0.126	0.119	0.063	0.127
$x^*_{H,p}$	2.408	2.384	2.820	1.927
$x^*_{H,g}$	1.369	2.384	1.604	1.927
$\tilde{x}_{L,p}$	2.308	2.067	2.055	2.292
$\tilde{x}_{L,g}$	2.224	2.067	1.971	2.292
u	0.075	0.071	0.068	0.074
u_H	0.007	0.008	0.007	0.007
$e_{H,p}$	0.112	0.130	0.130	0.112
$e_{H,g}$	0.040	0.020	0.021	0.039
u_L	0.068	0.064	0.061	0.067
$e_{L,p}$	0.690	0.675	0.679	0.690
$e_{L,g}$	0.085	0.104	0.103	0.085
e_p	0.802	0.805	0.808	0.802
e_g	0.124	0.124	0.124	0.124
$\bar{t}_H$	1.163	1.132	1.158	1.148
$\bar{t}_L$	2.074	1.827	1.866	2.054
v_g	0.008	0.008	0.004	0.008
$\bar{w}_{H,p}$	10.175	10.183	10.455	9.918
$\bar{w}_{H,g}$	9.806	10.182	13.191	7.489
$\bar{w}_{L,p}$	3.124	3.232	3.237	3.135
$\bar{w}_{L,g}$	3.271	3.232	3.410	3.210
$\bar{w}_{H,p}/\bar{w}_{H,g}$	1.038	1.000	0.793	1.324
$\bar{w}_{L,p}/\bar{w}_{L,g}$	0.956	1.000	0.950	0.977
$\Pr[w_{L,p} = m]$	0.181	0.100	0.096	0.176
$\Pr[w_{L,g} = m]$	0.130	0.100	0.005	0.160
$Output$	4.685	5.028	5.037	4.685
$Welfare_H$	16.649	15.974	17.747	14.945
$Welfare_L$	17.092	19.625	19.900	17.162
$Welfare$	17.022	19.048	19.560	16.812

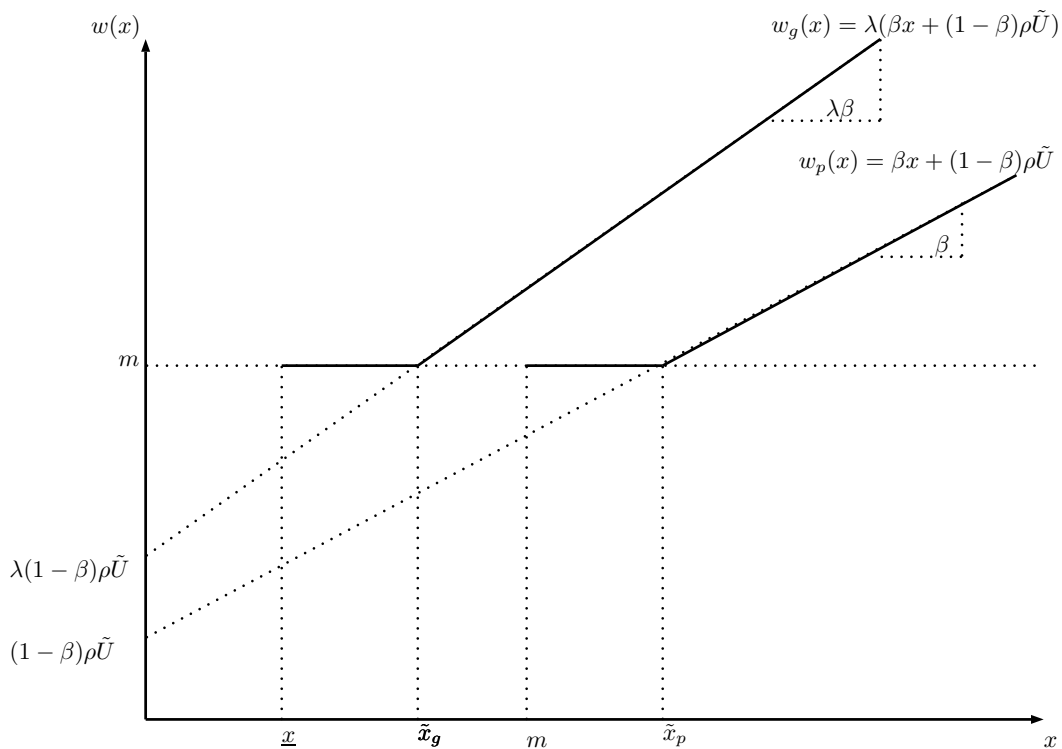Figure 1: Cut-off Point Determination with Binding Minimum Wage

Figure 2: Density of Hourly Wages by Sector

A Additional Material on The Model

A.1 Additional Tables

Table A.1: Summary of the Model Environment

State	Value Function	Measure	Shock	Flow Utility	Policy Instrument
Workers ($y = H, L$)					
Unemployed	U_y	u_y	α_w^p, α_w^g	z_y	$-$
Private Sector Employee	$N_{y,p}(x)$	$e_{y,p}$	δ_p	$w_{y,p}(x)$	$-$
Public Sector Employee	$N_{y,g}(x)$	$e_{y,g}$	δ_g	$w_{y,g}(x)$	$-$
Private Sector					
Filled Job	$J_{y,p}(x)$	$e_{y,p}$	δ_p	$x - w_{y,p}(x)$	$-$
Vacant Job	V_p	v_p	α_e^H, α_e^L	$-c$	$-$
Public Sector					
Filled Job	$-$	$e_{y,g}$	δ_g	$x - w_{y,g}(x)$	λ_y, m
Vacant Job	$-$	v_g	$-$	$-$	$\underline{x}_y$

A.2 Model Extension

In this extension, we assume that the public sector produces an output h which is consumed and valued by all the population. The quantity of the public good h is equal to the aggregate output produced by public sector workers,

$$h = \sum_{y=L,H} \int x \, dG_g^y(x) e_{y,g} \quad y = H, L$$

where $e_{y,g}$ represents the number of type y workers employed in the public sector. Like in the basic model, the numbers of workers of each type and their productivity levels in the government are endogenous variables of the model. We also assume that a fraction D of the public sector wage bill is financed by labor income taxes, and that the payroll tax τ is such that the following budget constraint holds,

$$\tau \left[\sum_{y=H,L} \int w_{y,p}(x) dG_p^y(x) e_{y,p} \right] = (D - \tau) \left[\sum_{y=H,L} \int w_{y,g}(x) dG_g^y(x) e_{y,g} \right]$$

This government budget constraint could be relaxed to consider a fixed tax revenue constraint or a requirement to fund a certain fraction of the government wage bill under the assumption that lump sum transfers fund the remaining government expenditures. On the

workers side, the utility flows consider the contribution of the public good and the payroll tax. Indeed, while unemployed workers receive an income flow of:

$$z_y + v(h) \quad y = H, L$$

where the $v(h)$ is the utility from consumption h of the public good. On the other hand, an employed worker in any sector receives a flow income of:

$$(1 - \tau)w_{y,s}(x) + v(h) \quad y = H, L \quad s = p, g$$

where $(1 - \tau)w_{y,s}(x)$ is the after tax wage. Finally, as before wages are determined by Nash bargaining and the resulting wage schedule for the private sectors is:

$$w_{y,p}(x) = \begin{cases} m & if \ \widetilde{x}_{y,p} \leq m \\ \beta x + \frac{(1-\beta)}{(1-\tau)}\left[\rho U_y - v(h)\right] & otherwise \end{cases}$$

The rest of the model remains the same. In simulations, we assumed that $v(h) = h^{1/2}$ and that the payroll tax τ is determined endogenously to fund $D = 0.3$ of government wages. The comparative statics results are very similar to those obtained with the model outlined in the paper. However, we do not attempt to estimate this model because, without any information on the valuation side and the effective payroll tax, the identification of the remaining parameters of the model becomes trickier.

B   Solution Algorithm

The solution algorithm involves the following three steps.

1. Guess η and ϕ.

   (a) Guess θ

   (b) Compute α_w^p and α_w^g.

   (c) Find the value of unemployment:

      - Find ρU_H and ρU_L iterating equations in (10) if case 1.
      - Find ρU_H and $\rho \tilde{U}_L$ iterating equations (10) and (12) if case 2.
      - Find $\rho \tilde{U}_H$ and $\rho \tilde{U}_L$ iterating equations in (14) if case 3.

   (d) Find the labour market tightness

      - Compute α_e^H and α_e^L and find θ solving equation (11) if case 1.
      - Compute α_e^H and α_e^L and find θ solving equation (13) if case 2.
      - Compute α_e^H and α_e^L and find θ solving equation (15) if case 3.

   (e) Iterate over θ.

2. Find η and ϕ using the steady state conditions in (16) and equations (17) and (18).

3. Iterate over η and ϕ.

Two choose between models (cases 1, 2 or 3), solve case 1 first and compare ρU_H and ρU_L with m.

- If $\rho U_L \leq m < \rho U_H$ then solve case 2.

- If $\rho U_L < \rho U_H \leq m$ then solve case 3.

- Otherwise, keep the solution of case 1.