Copyright © 2002 IFAC
15th Triennial World Congress, Barcelona, Spain

ELSEVIER
IFAC
PUBLICATIONS
www.elsevier.com/locate/ifac

DYNAMICAL ADAPTIVE FUZZY SYSTEMS: AN APPLICATION ON SYSTEM IDENTIFICATION

J. Aguilar ** **M. Cerrada** *** **E. Colina** **** **A. Titli** ‡

Universidad de Los Andes, Facultad de Ingeniería
** *Departamento de Computación*
e-mail: aguilar@ing.ula.ve
*** *Departamento de Sistemas de Control*
e-mail: cerradam@ing.ula.ve
**** *Postgrado en Automatizacióny Contr ol*
e-mail: ecolina@ing.ula.ve
A venida Tulio Febr es Cordero.
T elf: 58-274-2402847 Fax: 58-274-2402846
Mérida 5101. VENEZUELA
‡ *Laboratoire d'Analyse et d'Architecture de Systèmes*
(LAAS-CNRS)
Gr oup Automatique Symbolique, e-mail: titli@laas.fr
7, avenue du Colonel Roche-31077.
Phone: 33-5-61336200 F ax: 33-5-61553577
Toulouse, Cedex. FRANCE

Abstract: In order to control the processes, it is required to have a model which describes the process behaviour. Iden tificationtechniques provide a representation of a real system based on a linguistic or mathematical expression, or an algorithm. In this w ork,w euse a dynamical adaptive fuzzy model in order to propose fuzzy iden tifiers.Dynamical adaptive fuzzy models on iden tificationproblems permit an adaptive approach through the continuous adaptation of the membership functions on their universes of discourse, taking into account the process behaviour. *Copyright © 2002 IFAC*

Keywords: Identifiers, Identification Algorithms, Fuzzy Modeling, Fuzzy Systems, F uzzy Models.

1. INTRODUCTION

The processes control tasks require a model that describes the process behaviour. Iden tification techniques provide a representation of the real system through linguistic or mathematical expressions. Thus, an identification system can estimate the real behaviour through an appropriate model (Ljung, 1997). Input-Output Iden tification propose iden tificationmodels which only ha vethe input and output variables of the processes.

Different formal methods hav e been used in system iden tification,how ev er,computational techniques are being developed in order to ev aluate and quantify the processes. P articularly if there is not a formal knowledge about the process dynamic, linguistic models based on Fuzzy Logic (FL) hav e been used. In this case, IF-THEN fuzzy rules are proposed based on the expert knowledge and the resulting fuzzy model approximates the process behaviour. However, if the expert kno wledge is not good, then, the fuzzy model will hav e a wrong performance. On the other hand, fuzzy models generally are static models, that is, once they ha vebeen designed they only can approximate the real process on a particular interval of their variables values. In order to avoid these problem, adaptive fuzzy models with learning algorithms for the parameters adjustment have been considered. However, classical adaptive fuzzy models are off-line tuned resulting a static model. In (Cerrada *et al.*, 2002), a dynamical adaptive fuzzy model is proposed incorporating the dynamical behaviour of the process variables into the membership functions. This way, their membership functions can dynamically change on the time.

On system iden tification,if the process dynamic (or the domain of its variables) suddenly changes, static fuzzy models can be not adequate, and the process control will be not good. In this work, we present an application of the dynamical adaptive fuzzy model proposed in (Cerrada *et al.*, 2002) on the input-output identification. Three application examples are presented: first, a highly non-linear discrete system; second, a non-linear system of tw ointerconnected tanks and then, a non-linear time-varying reaction process that is carried out into a reactor tank continually stirring. In each case, the resulting model is a generic aproximator of the process behaviour, through the continuous adaptation of the membership functions on their universe of discourse, which can be used in control task.

2. THE DYNAMICAL ADAPTIVE FUZZY MODEL

A Dynamical Adaptive Fuzzy Model (DAFM) is a fuzzy model whose membership functions change dynamically on the time (Cerrada *et al.*, 2002). The idea behind of the dynamical membership functions is illustrated in the figure 1. Let a fuzzy

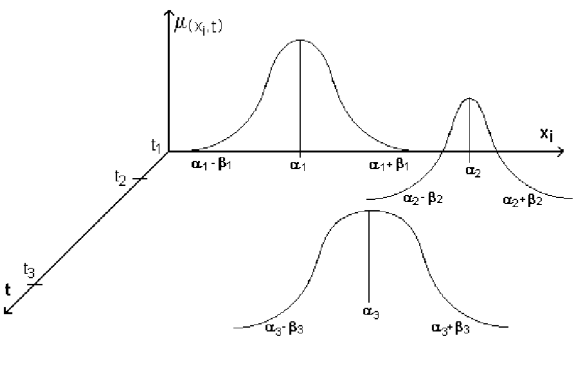

Fig. 1. Dynamical membership functions for a variable x_i

logic model be described by a set of M fuzzy rules:

$$R^{(l)}: IF\ x_1\ is\ F_1^l\ \ AND ... AND\ \ x_n\ is\ F_n^l \quad THEN\ \ y\ \ is\ G^l \tag{1}$$

where $\mathbf{X} = (x_1\ \ x_2\ \ ...\ x_n)^T$ is a vector of the input fuzzy variables x_i defined on an universe of discourse U_i, $(i = 1, ..., n)$; y is the output fuzzy variable defined on an universe of discourse V; F_i^l are fuzzy sets on U_i; G^l is a fuzzy set on V; $(i = 1, ..., n)$, $(l = 1, ..., M)$.

The analytic expression which defines a DAFM is given b y the equation (2):

$$y(\mathbf{X}, t) = \frac{\sum_{l=1}^{M} \left(\gamma^l(u^l, t) * f(t)\right)}{\sum_{l=1}^{M} f(t)} \tag{2}$$

where:

$$f(t) = \prod_{i=1}^{n} exp\left[-\frac{\left(x_i\ -\ \alpha_i^l(v_i^l, t)\right)^2}{\beta_i^l(\mathbf{w_i^l}, t)}\right] \tag{3}$$

where t is the time; u^l is the parameter of the function γ^l; v_i^l is the parameter of the function α_i^l; $\mathbf{w_i^l}$ is a vector of R parameters w_{ir}^l of the function β_i^l, $r = 1, ..., R$.

This analytic expression is obtained from the base of rules (2), based on an inference mechanism that uses the fuzzification method of ordinary sets, the average-center desfuzzification method and the gaussian membership function for the fuzzy sets associated to the input variables.

In this w ork,the generic structures of the functions $\alpha_i^l(v_i^l, t_j)$, $\beta_i^l(\underline{w_i^l}, t_j)$ and $\gamma^l(\underline{u^l}, t_j)$, are given by the following equations:

$$\alpha_i^l(v_i^l, t_j) = v_i^l * \overline{x}_i(t_j) \tag{4}$$

$$\beta_i^l(\mathbf{w_i^l}, t_j) = w_{i1}^l * (\sigma_i^2(t_j) + w_{i2}^l) \tag{5}$$

$$\gamma^l(u^l, t_j) = u^l * y(t_{(j-1)}) \tag{6}$$

where:

$$\overline{x}_i(t_j) = \frac{\sum_{k=1}^{j}(x_i(t_k))}{j} \tag{7}$$

$$\sigma_i^2(t_j) = \frac{\sum_{k=1}^{j}(x_i(t_k) - \overline{x}_i(t_j))^2}{j-1} \tag{8}$$

A supervised gradient-descent learning algorithm is used for the parameters tuning. The learning algorithm uses a collection of historical patterns $(\mathbf{X}(t_j), y(t_j))$ of the real system and the quadratic error given b y the equation (9):

$$e = \frac{1}{2}(y_e(t_j) - y(t_j))^2 \tag{9}$$

where $y(t_j)$ is the real output at time t_j and $y_e(t_j)$ is the estimated output given b y the fuzzy model. The adaptation laws of each parameter, using the equation (9), are generically given in the following equations and they are detailed in (Cerrada *et al.*, 2002):

$$\left.\frac{\partial E}{\partial u_p^l}\right|_K = \left.\frac{\partial E}{\partial \gamma^l(u^l,t_j)}\frac{\partial \gamma^l(u^l,t_j)}{\partial u_p^l}\right|_K \quad (10)$$

$$\left.\frac{\partial E}{\partial v_i^l}\right|_K = \left.\frac{\partial E}{\partial \alpha_i^l(v_i^l,t_j)}\frac{\partial \alpha_i^l(v_i^l,t_j)}{\partial v_i^l}\right|_K \quad (11)$$

$$\left.\frac{\partial E}{\partial w_{ir}^l}\right|_K = \left.\frac{\partial E}{\partial \beta_i^l(\mathbf{w_i^l},t_j)}\frac{\partial \beta_i^l(\mathbf{w_i^l},t_j)}{\partial w_{ir}^l}\right|_K \quad (12)$$

3. SYSTEM IDENTIFICATION

In order to design con trollaws for a process, it must to dev elopa model that describes its dynamic. The system identification process develops a valid mathematical model for a system which approximates its essential properties, taking into account its static and dynamic behaviour on a given interv al of time (Ljung, 1997). An identification model can be used in order to improve a real system: control tasks, fault tolerance, or any other tasks that require to know the system dynamic.

The input-output iden tification models are defined as a nonlinear function of the current input and previous inputs and outputs of the system. F or instance, for a SISO system, this is:

$$y_e(t) = f(u(t), u(t-1), ..., u(t-m); \quad y(t-1), y(t-2), ..., y(t-n); \mathbf{W}) \quad (13)$$

where $y_e(t)$ is the estimated output; $u(t-a), a = 0, .., m$ is the input signal to the real system at time t and at m previous times; $y(t-b), b = 1, .., n$ is the output signal of the real system at time $t-1$ and at n previous times; $\mathbf{W}$ is a vector of parameters.

Figure 2 shows the classical input-output iden tification model. For an unknown system, with an input signal and its output signal, the model of the system receives the current input and the previous inputs and outputs. Based on this information, the parameters of the iden tification model are tuned according to the error betw een the current output of the real system and the output of the model. There are different methods of systems identification, from those based on the classical theory (Ljung, 1997), until the new techniques based on the computational in telligence (Takagi and Sugeno, 1985), (P. Sastry ,1995), (Angeline and Fogel, 1998), (Aguilar and Cerrada, 2001).

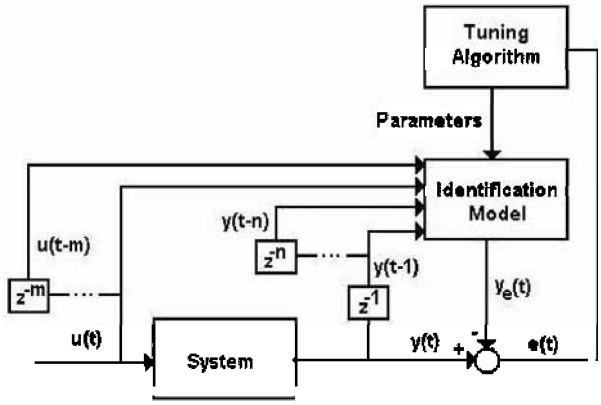

Fig. 2. Input-output identification model

4. D AFM AND INPUT-OUTPUT FUZZY IDENTIFICATION

The DAFM can be used to develop identification models. On input-output identification models, the input vector $\mathbf{X}$ to the fuzzy model is composed by the current value of the input $u(t)$ ($u(t_j)$) to the real system and the previous values of the input and output $y(t)$ at time t_j ($u(t_{j-1}), u(t_{j-2}), ..., y(t_{j-1}), y(t_{j-2}), ...$). The output $y_e(t_j)$ is estimated by the fuzzy model (2) at time t_j. In the next section it is presented three application examples by using a DAFM.

4.1 *Example 1*

In this example, the system has been described by the following difference equation:

$$y(k+1) = g[y(k), y(k-1), y(k-2), \quad u(k), u(k-1)] \quad (14)$$

where: $g[.] = \frac{y(k)y(k-1)y(k-2)u(k-1)(y(k-2)-1)+u(k)}{1+y(k-2)^2+y(k-1)^2}$

The estimated function $y_e(k+1)$ is:

$$y_e(k+1) = g_e[y(k), y(k-1), y(k-2), \quad u(k), u(k-1)] \quad (15)$$

where $g_e[.]$ is estimated by the DAFM given in (2).

Experiences

The input variables to the fuzzy model are $x_1(k) = y(k)$, $x_2(k) = y(k-1)$, $x_3(k) = y(k-2)$, $x_4(k) = u(k)$ y $x_5(k) = u(k-1)$ and in the equation (6), we put $y(t_{(j-1)}) = g[u(k-1)]$. In the training phase, 1000 random patterns uniformly distributed over [-1,1] have been used, making one training cycle for each pattern (1000 training cycles). Different experiments have been made and the resulting models with $M = 10, 20, 30$

and $\rho_i = 0,1$ ha vebeen tested with the input signal $u(k) = \sin(2\pi k/250)$. In the figure 3, the identification errors of $y(k+1)$ have been shown. The first model ($M = 10$) has the solid line, the second model ($M = 20$) has the dashed line and third model ($M = 30$) has the pointed line. The low error has been gathered with $M = 10$. In

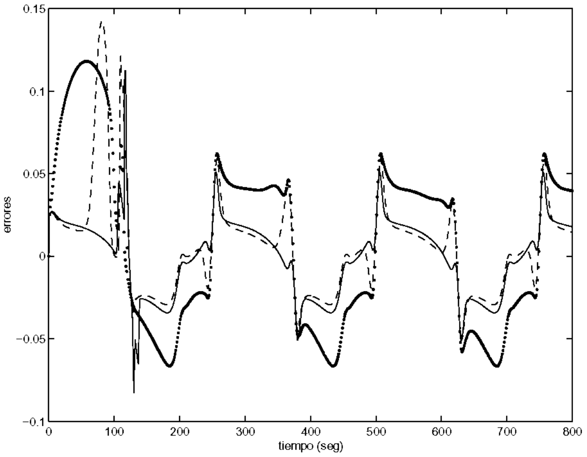

Fig. 3. Identification errors for $u(k)$

the figure 4, the performance of the model with $M = 10$ and the input $u(k)$ is shown.

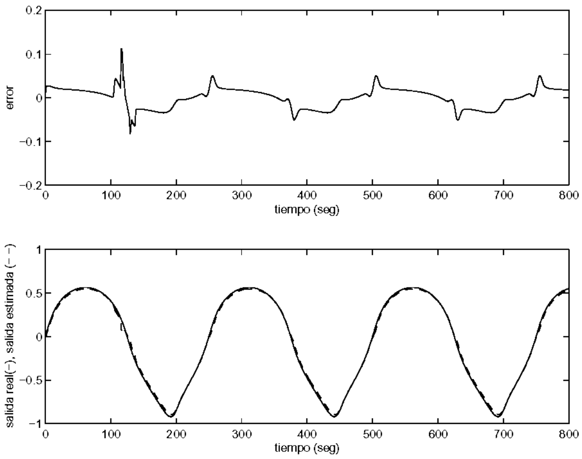

Fig. 4. Real output and estimated output (dashed line) with $u(k)$

Then, using the input $u_1(k)$ the performance of this model has been tested. In the figure 5 the performance of the model is shown.

$$u_1(t) = \begin{cases} \sin(2\pi k/250) & if \quad 1 < k < 500 \\ u_a + u_b & if \quad k > 500 \end{cases} \qquad (16)$$

where:
$u_a = 0.8\sin(2\pi k/250)$
$u_b = 0.2\sin(2\pi k/25)$

Analysis of Results

In this example, the dynamical fuzzy model has a good performance with only $M = 10$ (110

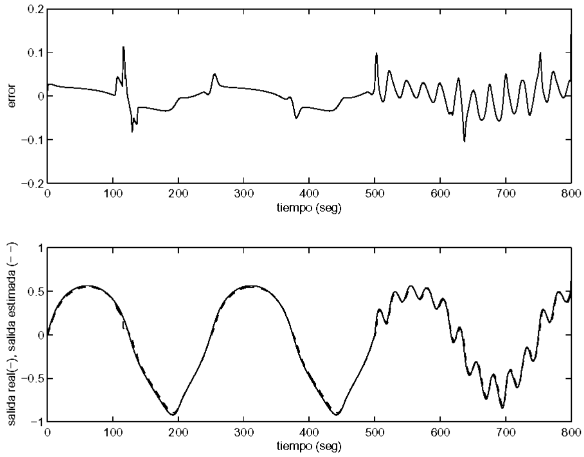

Fig. 5. Real output and estimated output (dashed line) with $u_2(k)$

adjusted parameters) from a initial randomly selection of the parameters.

This example has been developed in (Wang, 1994), using a model with $M = 40$ (440 adjusted parameters) and 5000 training cycles from a adequate selection of the initial values of the parameters.
In (Narendra and Parthasarathy, 1990) the neural iden tifier proposedfor this case shows an identification error in the operational phase after 10^5 training cycles.

4.2 *Example 2.*

In this example, w e treat a non-linear time-varying reaction process (Wu and Chou, 1999). Let us consider a reactor tank continually stirring with a constant volume V_r. The reactants feed at the reactor tank with a feeding rate F. The products are deposited in the bottom of the tank. In presence of a place of separate reaction on the catalyst, the kinetic of those places is usually different and time-varying. The dynamic of this system is described by the following equations:

$$\frac{dz_1}{dt} = 1 - D_{a_1} z_1 + D_{a_2} z_2^2$$
$$\frac{dz_2}{dt} = z_2 - D_{a_1} z_1 - D_{a_2} z_2^2 - D_{a_3} d_2(t) z_2^2 + u$$
$$\frac{dz_3}{dt} = -z_3 - D_{a_3} d_2(t) z_2^2$$
$$y = z_3 \qquad (17)$$

where $D_{a_1} = \frac{k_1 d_1 V_r}{F} = 3$; $D_{a_2} = \frac{k_2 d_1 V_r}{F} = 0.5$; $D_{a_3} = \frac{k_3 V_r}{F} = 1$; F is the volumetric feeding rate; d_1 is the first reaction place on the catalyst; $d_2(t)$ is the second reaction place on the catalyst, $d_2(t) = 1 + 0.1\sin t$; k_1 is the first order constant reaction rate; k_2 and k_3 are the second order constant reaction rates; V_r is the volume of the reactor; z_1 is the con versionof the reactant A; z_2 is the conversion of the reactant B; z_3 is the con version of the product C;u is the control signal (input to the system); y is the output signal.

Experiences

The input variables to the DAFM are $x_1 = U(t_j)$ and $x_2 = U(t_{(j-1)})$. The training patterns are composed by different input signals $U_i(t)$ selected on different times. The training data has been taken with a sampling rate of 0.1 sec., on 250 sec. The initial values of the parameters have been randomly selected on the interval $[0, 1]$. The parameter $w^l_{i2} = 0.1$, then this parameter is not adjusted in the training phase. The training phase was stopped by the error value of $5x10^{-4}$.
In the training phase, different experiments have been made with $M = 10, 12, 16$ and $\rho_1 = \rho_2 = \rho_3 = 0.001, 0.005, 0.01, 0.1, 0.2, 0.3$. In the operation phase, the best results, according to the behavior of the identification error signal, has been obtained with $M = 16$ and $\rho_j = 0.1$. The input signal $U(t)$ selected to prove the performance of the one model, is defined by the following equation:

$$U(t) = \begin{cases} 1 & if \quad t \le 300 \\ 0.5 * u(t) & if \quad 300 < t < 600 \\ 0.6 & if \quad t \ge 600 \end{cases} \qquad (18)$$

where $u(t) = \sin(0.05 * t) + 1$.

Figure 6 shows the performance of the identification model for the input signal $U(t)$. The evolution on the time of the functions α^l_i, β^l_i and γ^l, as well as the values of their parameters, have been shown with detail in (Cerrada, 2000).

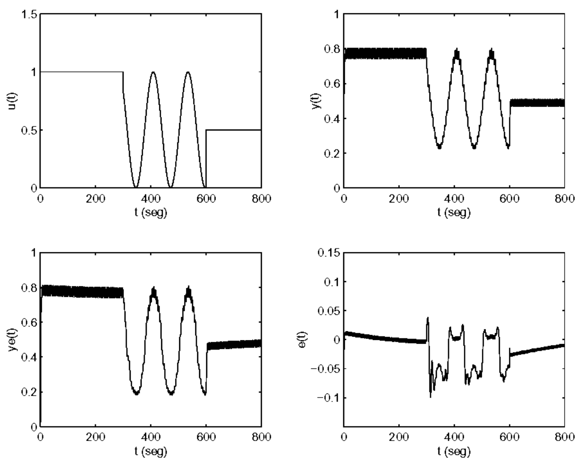

Fig. 6. Performance of the DAFM for the input $U(t)$ and $y(0) = 0$, for the reactor tank

Analysis of Results

The previous experiences show a good performance of the proposed model for the input-output identification of this system. The behaviour of the identification error signal is acceptable, considering that we have a time-varying nonlinear system.

4.3 *Example 3.*

In this example, a system of two identical interconnected tanks it is considered (Sira-Ramirez and Angulo, 1997). The output flow of one tank is the input to the other one. The dynamic of this system is described by the following equations:

$$\begin{aligned} \frac{dz_1}{dt} &= K_1 z_2 - K_2 z_1^2 \\ \frac{dz_2}{dt} &= K_3(K_4^{-(1-u)} - z_1) \\ \frac{dz_3}{dt} &= K_1 z_4 - K_2 z_3^2 \\ \frac{dz_4}{dt} &= K_3(z_1 - z_3) \\ y &= z_4 \end{aligned} \qquad (19)$$

where:
$K_1 = \frac{A_p g}{L}$
$K_2 = \frac{K_f}{\phi A_p^2}$
$K_3 = \frac{1}{A_t}$
$K_4 = F_{c_m ax}\alpha$
$F_{c_m ax}$ is the maximal value of the volumetric rate of the feeding to the first tank, g is the gravitational acceleration constant, L is the length of the pipes, K_f is the friction coefficient, ϕ is the liquid density, A_p is the traverse area of the pipes, A_t is the traverse area of the tanks, α is the valve parameter, z_1 and z_3 are the volumetric flow rates, z_2 and z_4 are the heights of the liquid in the tanks, u is the position of the valve (control signal on the interval $[0, 1]$) and y is the output variable.

Experiences

The input variables to the DAFM are $x_1 = U(t_j)$ and $x_2 = U(t_{(j-1)})$. The training phase has been made under the same considerations mentioned in the previous example. In this case, different experiments have been made with $M = 8, 10, 12$ and $\rho_1 = \rho_2 = \rho_3 = 0.001, 0.005, 0.01, 0.1, 0.2, 0.3$, with the stopped criteria based on the error value in $1x10^{-3}$. In the operation phase, the best results, according to the behavior of the identification error signal, were obtained with $M = 10$ and $\rho_j = 0.1$. The input signal $U(t)$ given in (18) has been used in order to test the performance of the one model. Figure 7 shows the fuzzy model performance for the input $U(t)$. The temporal evolution of the functions $\alpha^l_i(t_j)$, $\beta^l_i(t_j)$ and $\gamma^l(t_j)$ obtained in this case, as well as the values of their parameters, have been shown with detail in (Cerrada, 2000).

Analysis of Results

Like the previous example, the values of M and ρ_i has been obtained through trial and error. The performance of the fuzzy model obtained in this case is also satisfactory. The magnitude of the identification error is enough acceptable.

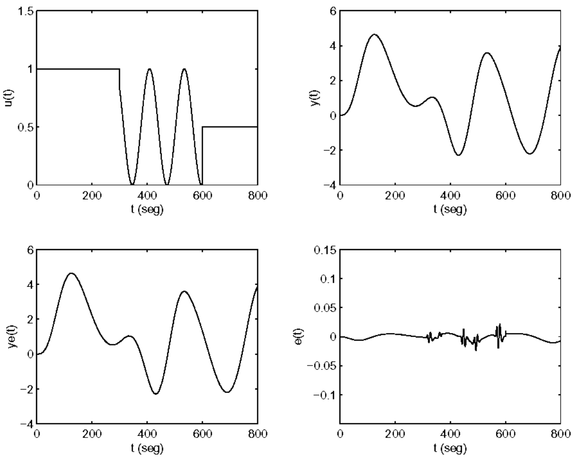

Fig. 7. Performance of the DAFM for the input $U(t)$ and $y(0) = 0$ for the system of tanks

5. CONCLUSIONS

In this work, a dynamical adaptive fuzzy model has been used in order to propose input-output fuzzy identifiers. Based on the work developed in (Cerrada *et al.*, 2002), the parameters of the model have been adjusted using an off-line gradient-descent supervised learning algorithm.

The resulting dynamical fuzzy identifiers have a good performance. In each case, the fuzzy model is a generic aproximator of the process behaviour because its dynamic has been incorporated through the functions α, β and γ. This way, the identification fuzzy model can notice suddenly changes of the process. This fact is an important characteristic of an adequate identification model on control tasks. Particularly, the fuzzy identifier in the example 1 shows an acceptable performance using few rules (value of M) and less number of training cycles than fuzzy and neural identifiers proposed in other works.

6. REFERENCES

Aguilar, J. and M. Cerrada (2001). Genetic programming-based approach for system identification. *Proc. of the WSES International Conference on Evolutionary Computation.*

Angeline, P. and D. Fogel (1998). An evolutionary program for the identification of dynamical system. Natural Selection.

Cerrada, M. (2000). Systèmes flous adaptatifs dynamiques pour l'identification des systèmes non-linéaires variants avec le temps. DEA Report. LAAS-CNRS. Toulouse-France.

Cerrada, M., J. Aguilar, E. Colina and A. Titli (2002). An approach for dynamical adaptive fuzzy modeling. *Accepted for publication in FUZZ-IEEE 2002 World Congress.*

Ljung, L. (1997). *System Identification. Theory for the User.* Prentice Hall. New York.

Narendra, K.S. and K. Parthasarathy (1990). Identification and control of dynamical systems using neural networks. *IEEE Trans. on Neural Networks* **1**, 4–27.

P. Sastry, G. Santharam, K. Unnikrishnan (1995). Memory neuron networks for identification and control of dynamical system. *IEEE Trans. On Neural Networks* **5**, 306–319.

Sira-Ramirez, H. and M.I. Angulo (1997). Passivity-based control of nonlinear chemical processes. *Int. Journal of Control* **68**, 971–996.

Takagi, T. and M. Sugeno (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst., Man, Cybern* **15**, 116–132.

Wang, L. (1994). *Adaptive Fuzzy Systems and Control. Design and Stability.* Prentice Hall. New Jersey.

Wu, W. and Y.S. Chou (1999). Adaptative feedforward and feedback control of nonlinear time-varying uncertain systems. *Int. Journal of Control* **72**, 1127–1138.