CS 534
Artificial Intelligence

Week 4: Search in Complex Environments

By

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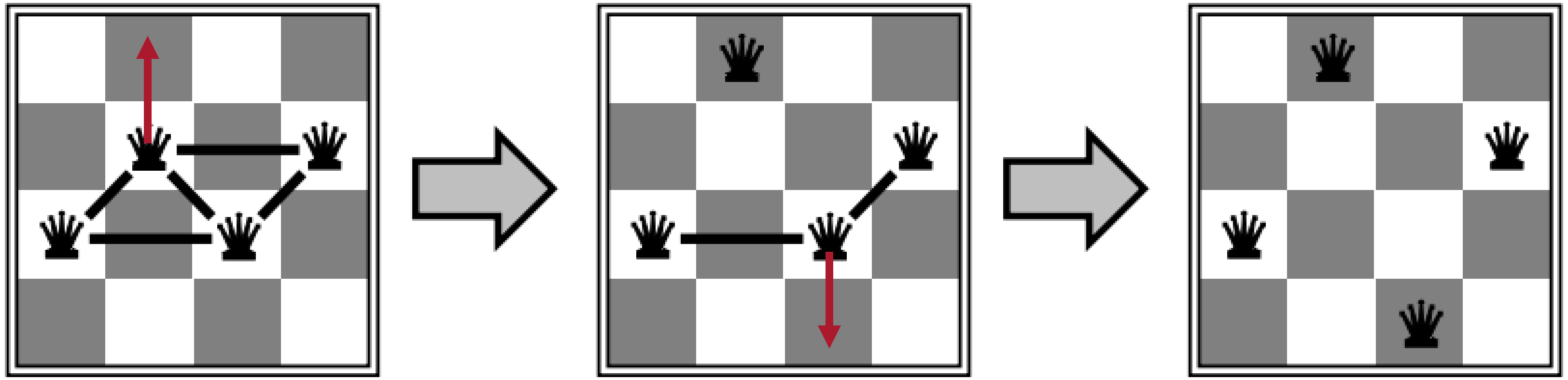
Local Search and Optimization Problems

- A **<u>local search</u>** algorithm operates by searching **from a start state to neighboring states, without keeping track of the paths**, i.e., **the path to the goal is irrelevant**, according to an objective function..
  - Use **iterative** improvement algorithms, as it keeps a single "current" state and try to improve it.
  - Use a single current node, **not a path**, and move to a neighbor of that node. It is a **heuristic search**.
  - The updated node value is closer to the goal than the previous node value. It is known as a **local search**.
  - <u>A local search algorithm is a type of heuristic search algorithm</u>.

- Local search algorithms can solve **optimization problems**, in which the aim is to find **the best state** according to an **objective function** and all the **constraints** being satisfied.

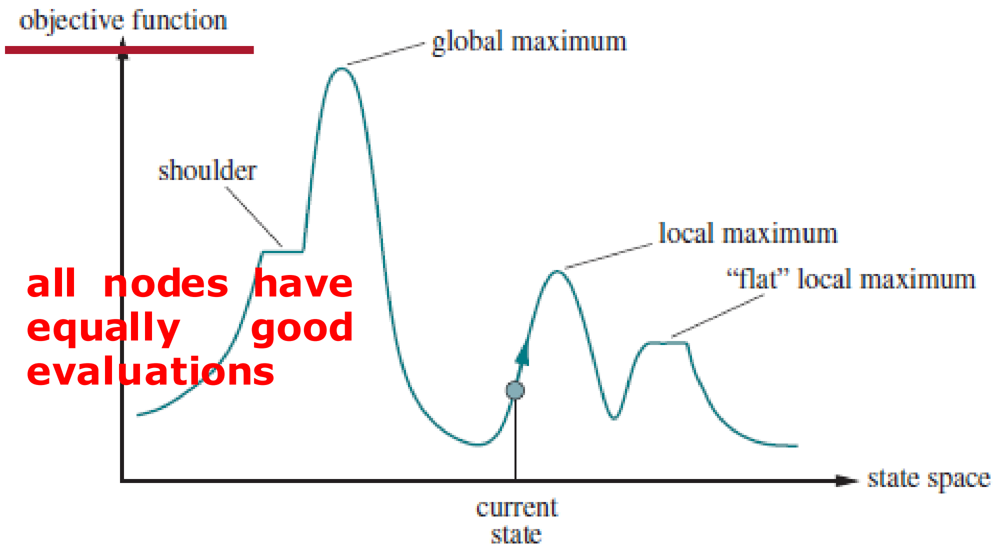
- Example: **n-Queens**
  - Put n queens on an n × n chessboard
  - No two queens on the same row, column, or diagonal that are the constraints.
    - Start with one queen in each column
    - Move a queen to reduce number of conflicts



Local Search and Optimization Problems

- Local search solve optimization problems
  - **Always find the next best state**, **the path to the goal is irrelevant**, according to an objective function.
  - Each point (state) in the state-space landscape has an "elevation", i.e., the value computed by the objective function.
    - If elevation corresponds to an objective function (e.g., **profit**), the aim is to find the highest peak – **a "global"/"local" maximum** → **A Hill Climbing Process**
    - If elevation corresponds to a **cost**, the aim is to find the lowest valley – **a "global"/"local" minimum** → **A Gradient Descent Process**.

- Local Search Algorithms can be:
  - Hill-climbing Search
  - Simulated Annealing
  - Local Beam Search
  - Genetic Algorithms



State Space Diagram for Hill Climbing

Hill-Climbing Search

- Keeps track of one current state and one each iteration moves to the neighboring state with the highest value of the objective function:
    - ❑ Start **an initial state (e.g., a solution)**
    - ❑ A greedy local search: select **the nearest** neighbor state/solution without thinking ahead about where to go next.
    - ❑ Repeat: move to **the best** neighboring state/solution in the direction of increasing value (uphill)
    - ❑ Terminate when it reaches a **"peak"** where no neighbor has a higher value than the current one.
- The **hill_climbing(problem)** function in search.py.
- **Adv: Low computation power and Lesser time**
- **Problem:** Depending on the initial state, it can get stuck for any of the following reasons.

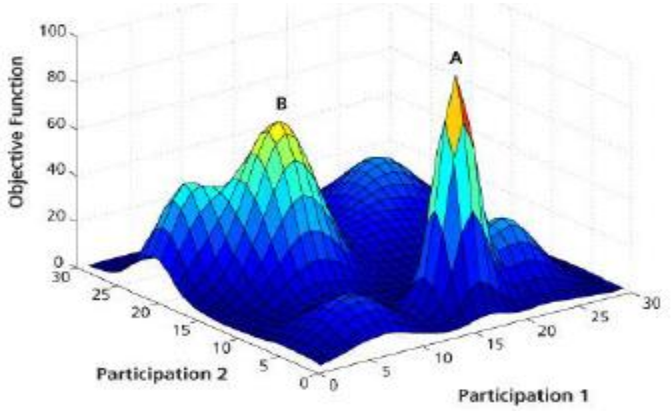
function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current*
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

```python
def hill_climbing(problem):
    """
    [Figure 4.2]
    From the initial node, keep choosing the neighbor with highest value,
    stopping when no neighbor is better.
    """
    current = Node(problem.initial)
    while True:
        neighbors = current.expand(problem)
        if not neighbors:
            break
        neighbor = argmax_random_tie(neighbors, key=lambda node: problem.value(node.state))
        if problem.value(neighbor.state) <= problem.value(current.state):
            break
        current = neighbor
    return current.state
```

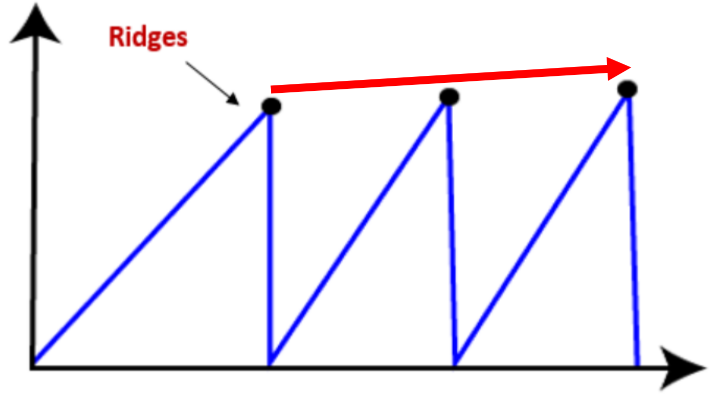
"Return an element with highest fn(seq[i]) score; break ties at random."

Problems of Hill-Climbing Search

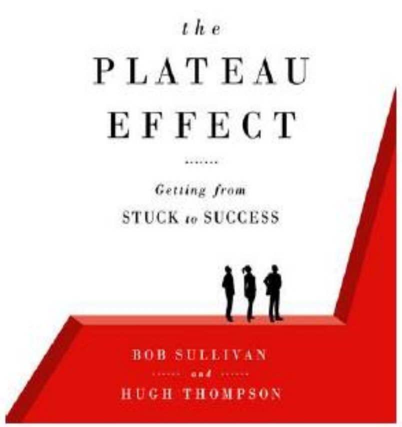
- **Local Maxima and Minima**: They are the peak that is better than each of its neighboring states but worse than the global maxima and global minima.



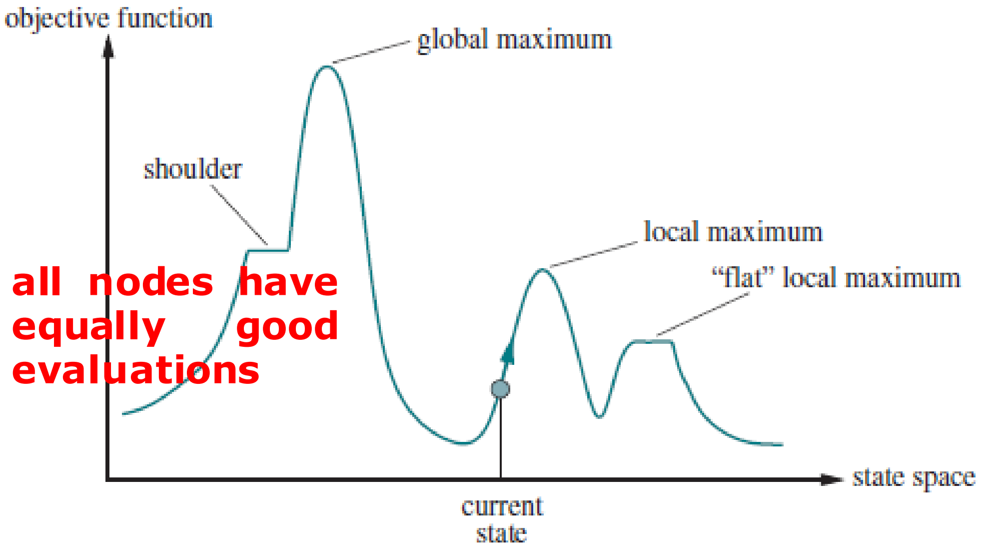
- **Ridges**: It is like a knife edge that gives a false sense of top of the hill, as no slope change appears.



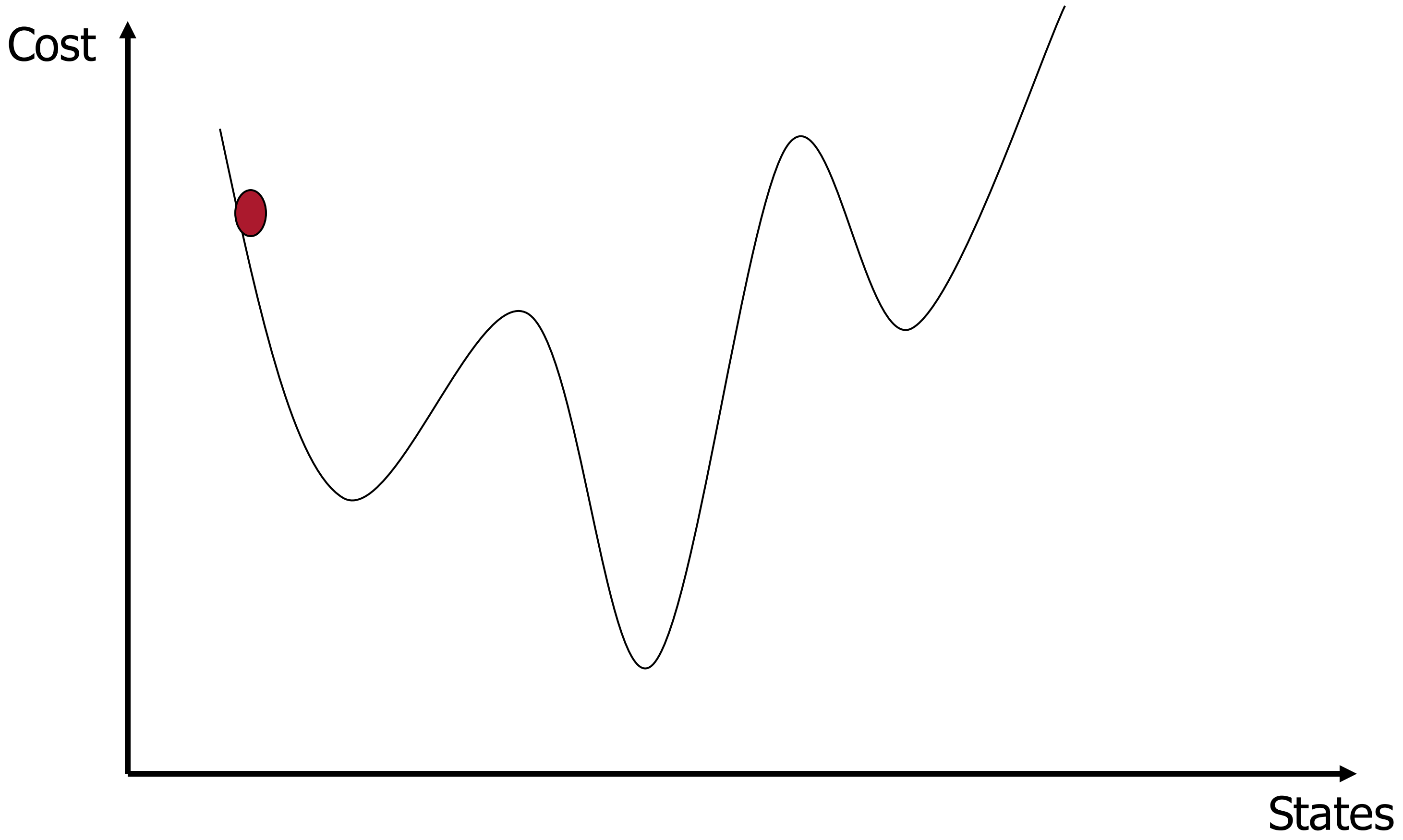
- **Plateaus**: All nodes have equally good evaluations, called a shoulder.



Less optimal solution and the solution is not guaranteed



Hill-Climbing Search



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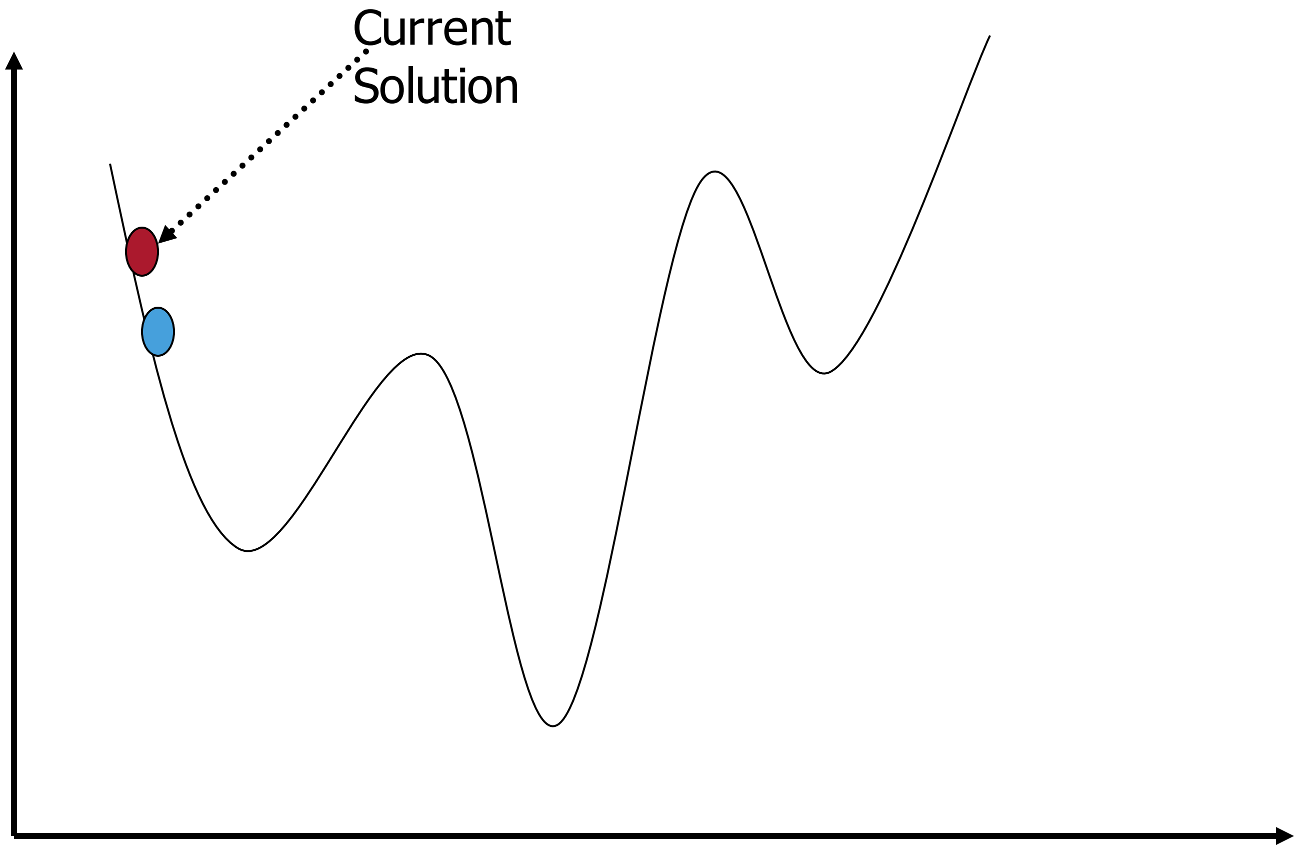

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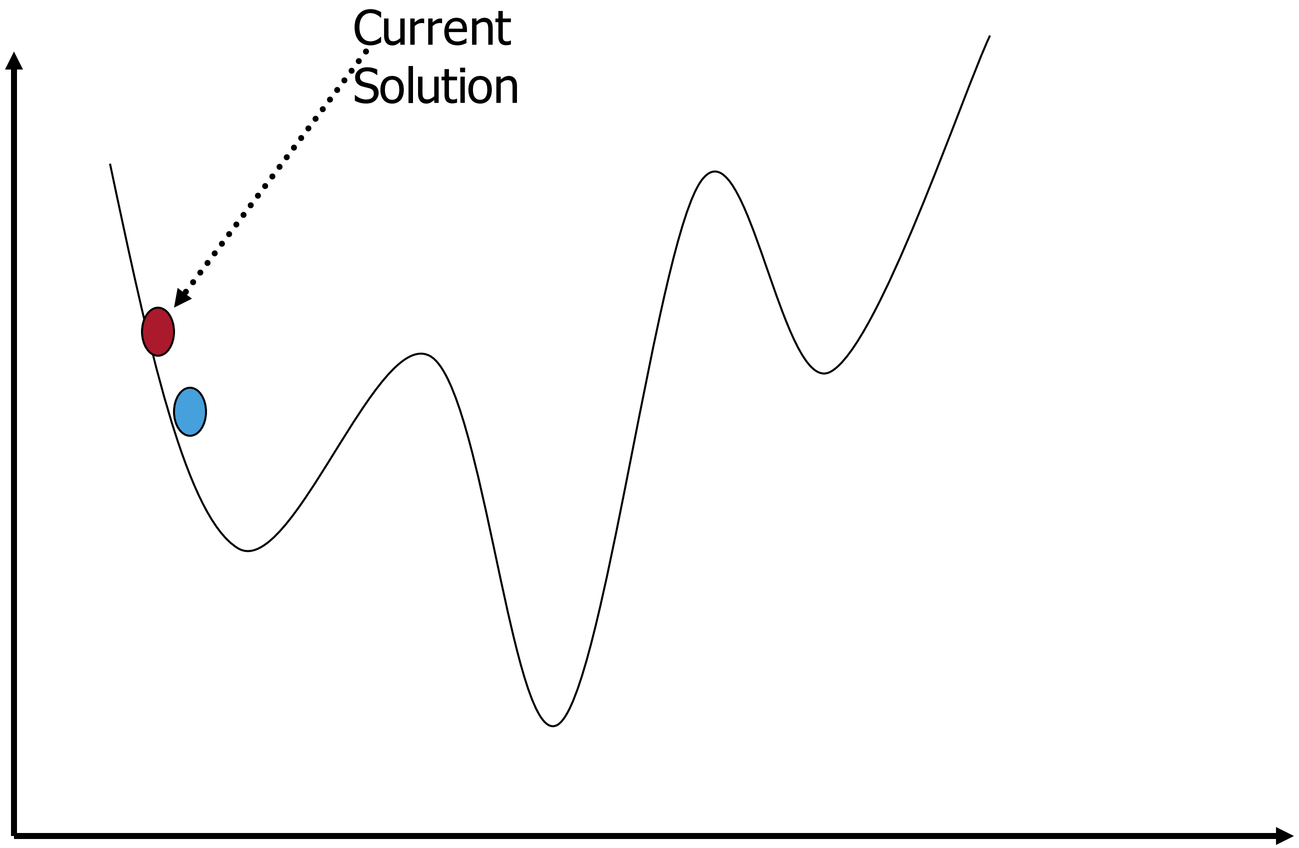

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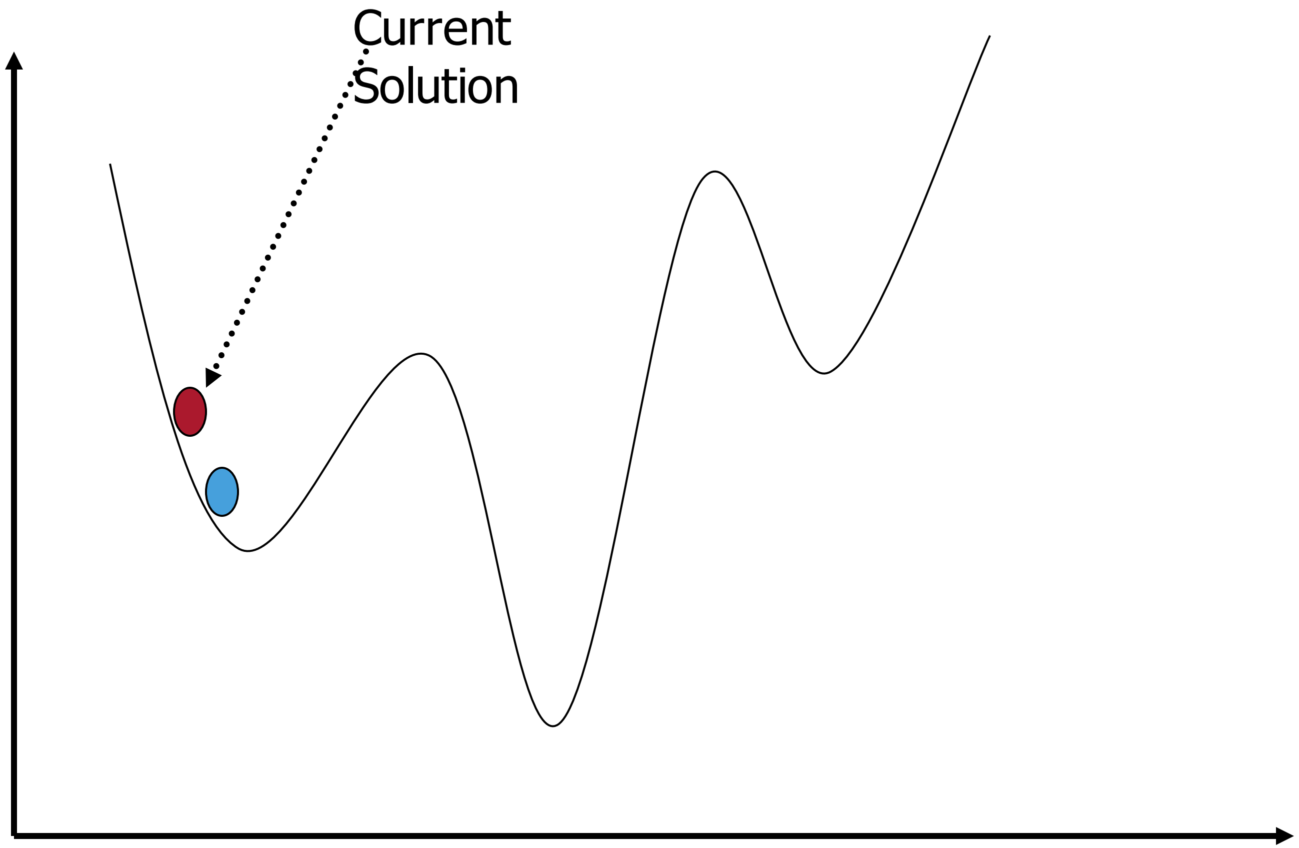

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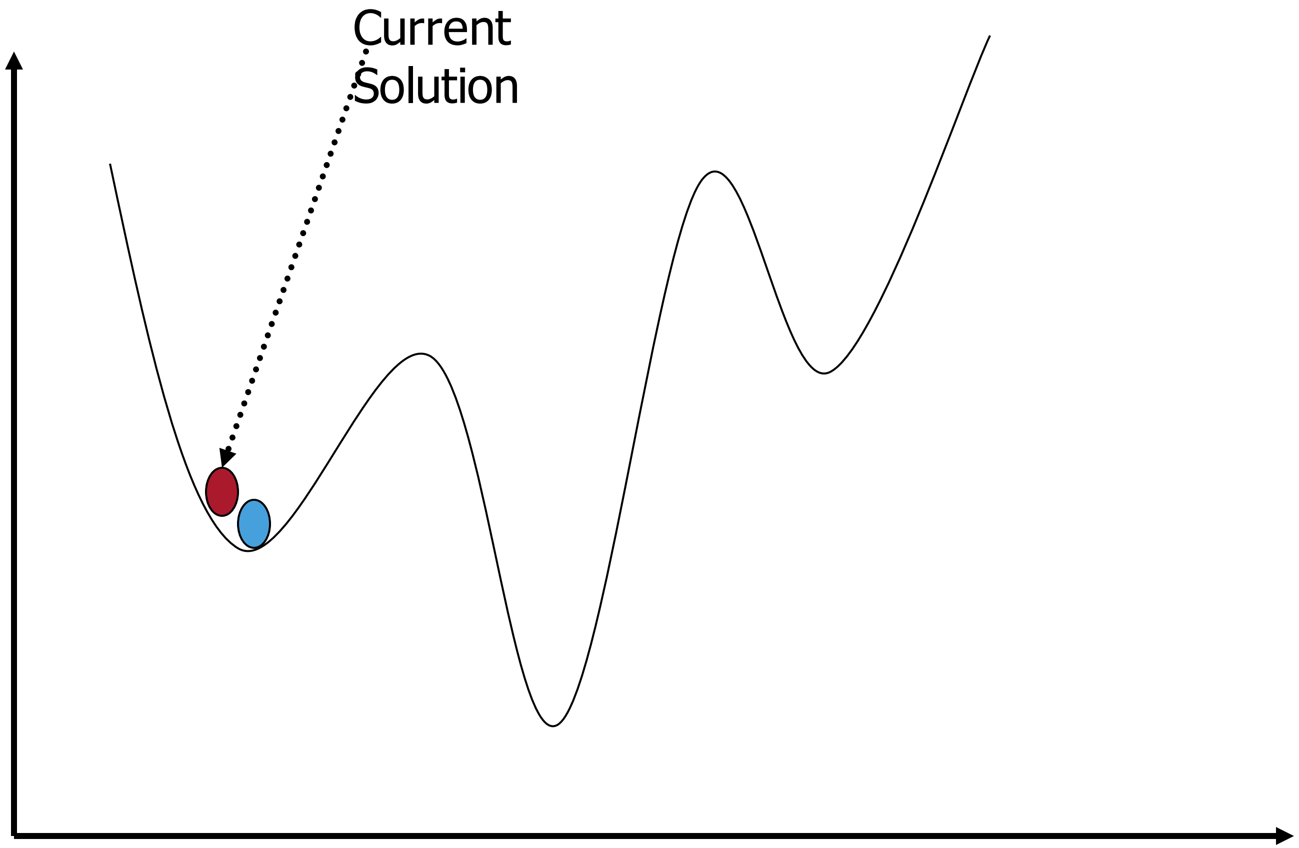

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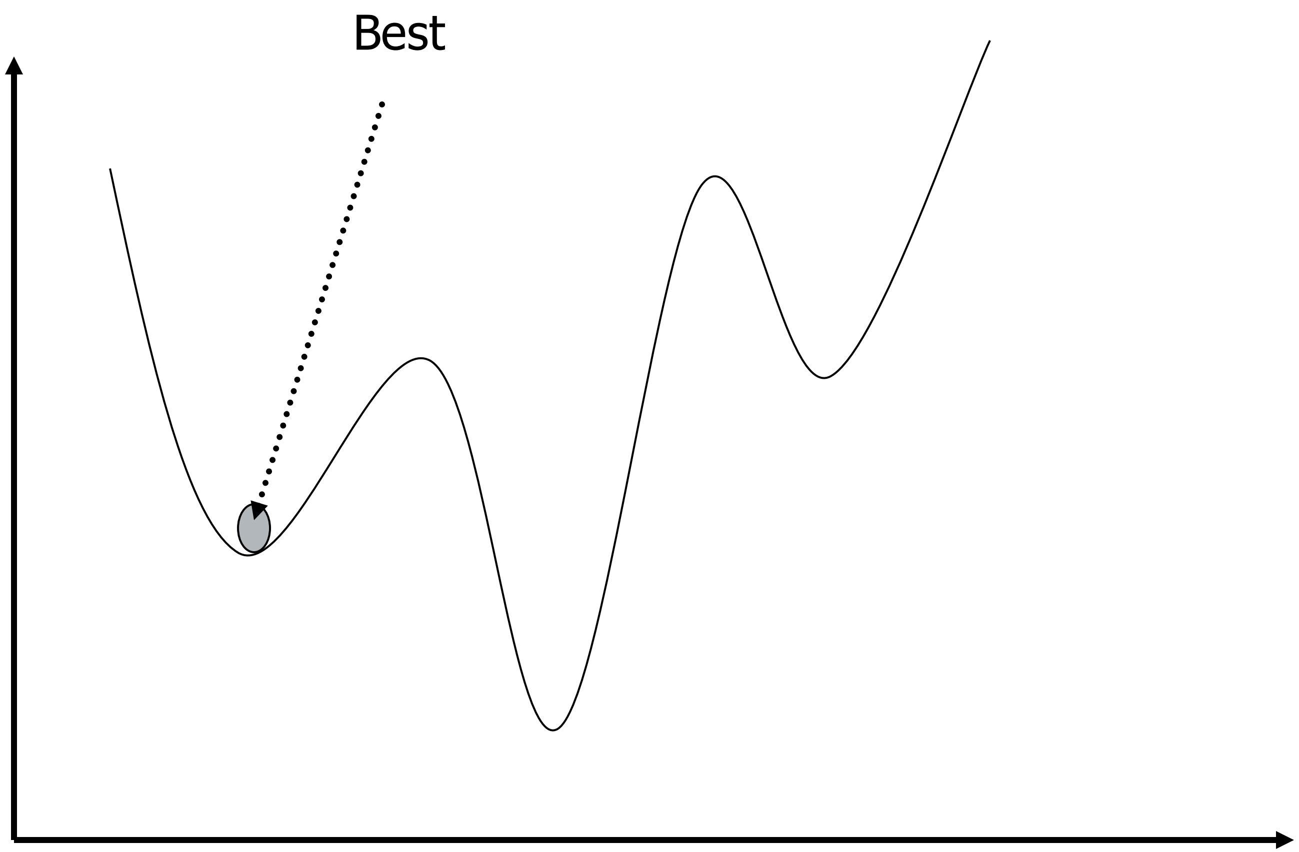


Hill-Climbing on the 8-Queens Problem

- Hill Climbing start with **an initial state** (i.e., a random configuration of the board) and chooses among the best successors. **Any move of a green is one of the next possible states.**

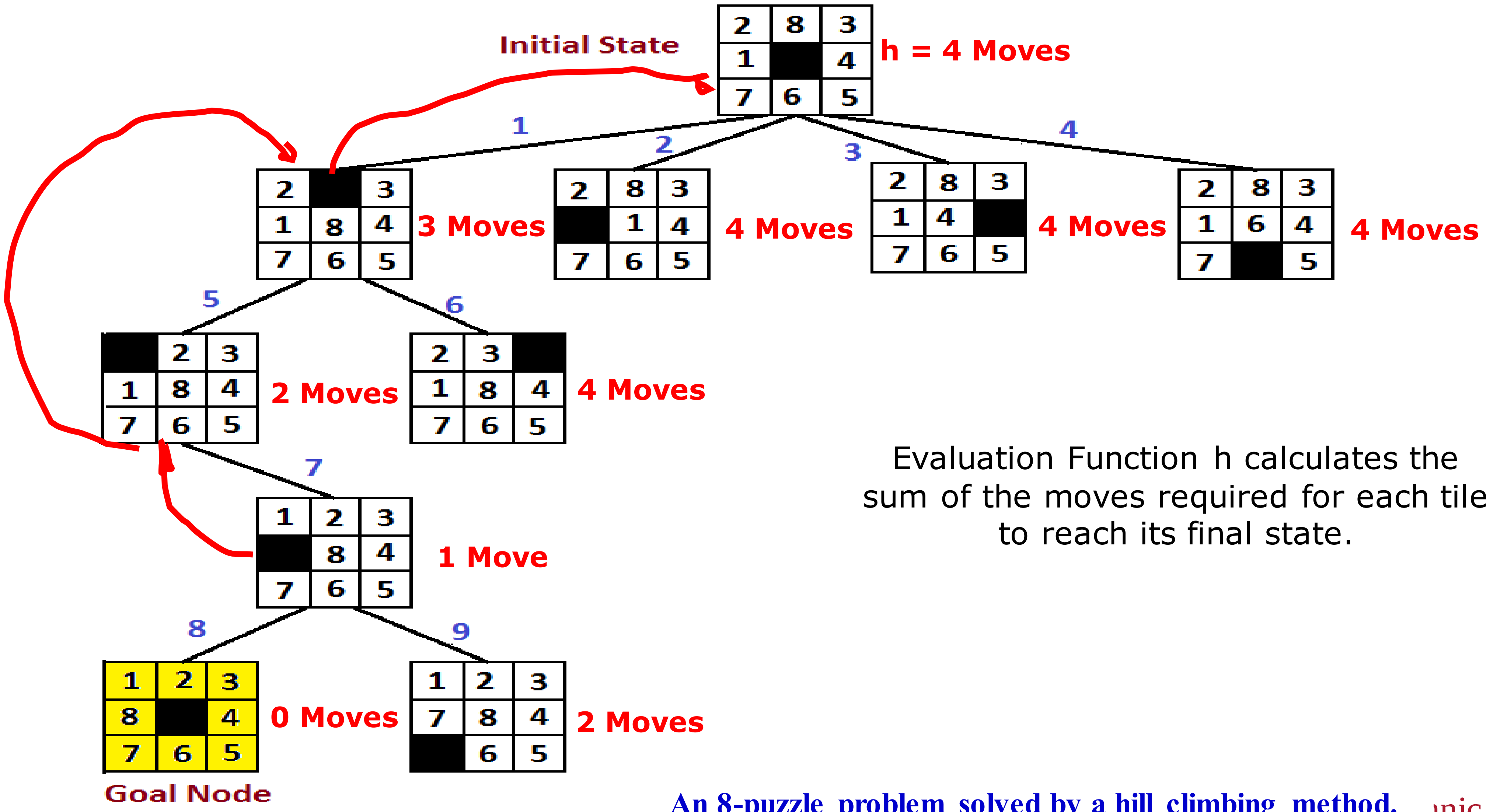
- The **objective cost function h** is the number of pairs of queens that are attacking each other.

  - It counts as an attack if two states are in the same line, even if there is an intervening piece between them.
  - Objective Function = -h, i.e., the global maximum is **zero** at the perfect solution.
  - Objective Function = h, i.e., the global minimum is **zero** at the perfect solution.

- An 8-queens state with the current heuristic cost estimate **h=17**.

- The board shows the value of h for each possible successor obtained by moving a queen within its column.

Hill-Climbing on the 8-Puzzle Problem

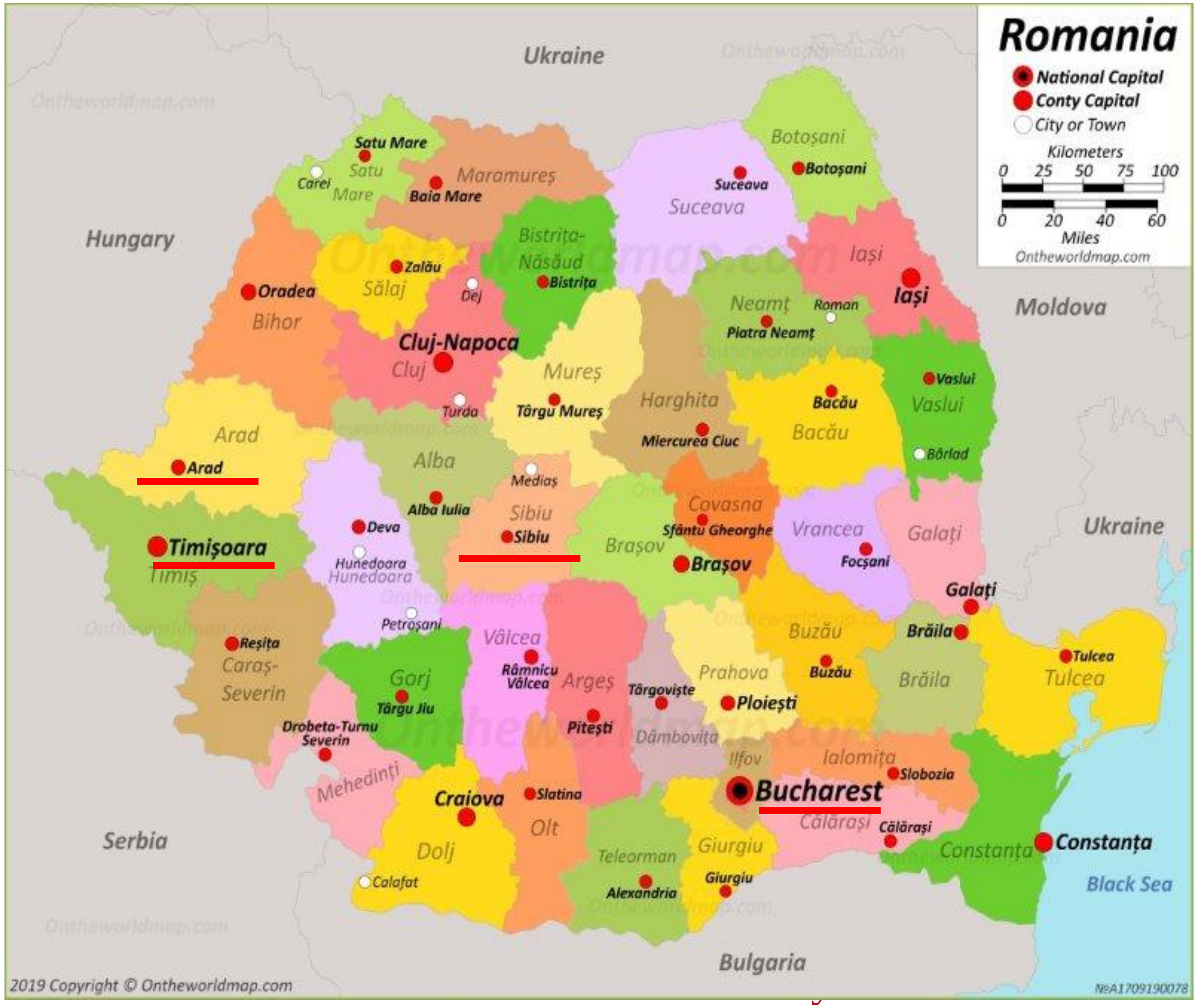


Evaluation Function h calculates the sum of the moves required for each tile to reach its final state.

An 8-puzzle problem solved by a hill climbing method.

Hill-Climbing on the TSP Problem

- **TSP – Traveling Salesman Problem**
- The salesman must travel to all cities once before returning home.
- The distance between each city is given and is assumed to be the same in both directions.
- The objective is to **minimize** the total distance to be travelled.
- Actual Map is slightly different than the map, as an example, provided by the reference textbook.



Hill-Climbing on the TSP Problem

Return random integers from low (inclusive) to high (exclusive)

```python
from search import *
import numpy as np

distances = {}
all_cities = []

class TSP_problem(Problem):

    """ subclass of Problem to define various functions """

    def two_opt(self, state):
        """ Neighbour generating function for Traveling Salesman Problem """
        neighbour_state = state[:]
        left = random.randint(0, len(neighbour_state) - 1)
        right = random.randint(0, len(neighbour_state) - 1)
        if left > right:
            left, right = right, left
        neighbour_state[left: right + 1] = reversed(neighbour_state[left: right + 1])
        return neighbour_state

    def actions(self, state):
        """ action that can be excuted in given state """
        return [self.two_opt]

    def result(self, state, action):
        """  result after applying the given action on the given state """
        return action(state)

    def path_cost(self, c, state1, action, state2):
        """ total distance for the Traveling Salesman to be covered if in state2  """
        cost = 0
        for i in range(len(state2) - 1):
            cost += distances[state2[i]][state2[i + 1]]
        cost += distances[state2[0]][state2[-1]]
        return cost

    def value(self, state):
        """ value of path cost given negative for the given state """
        return -1 * self.path_cost(None, None, None, state)
```

```python
def hill_climbing(problem):
    """From the initial node, keep choosing the neighbor with highest value,
    stopping when no neighbor is better. [Figure 4.2]"""

    def find_neighbors(state, number_of_neighbors=100):
        """ finds neighbors using two_opt method """

        neighbors = []

        for i in range(number_of_neighbors):
            new_state = problem.two_opt(state)
            neighbors.append(Node(new_state))
            state = new_state

        return neighbors

    # as this is a stochastic algorithm, we will set a cap on the number of iterations
    iterations = 10000

    current = Node(problem.initial)

    while iterations:
        neighbors = find_neighbors(current.state)
        if not neighbors:
            break
        neighbor = argmax_random_tie(neighbors,
                                     key=lambda node: problem.value(node.state))
        if problem.value(neighbor.state) > problem.value(current.state):
            current.state = neighbor.state
        iterations -= 1

    return current.state
```

Hill-Climbing on the TSP Problem

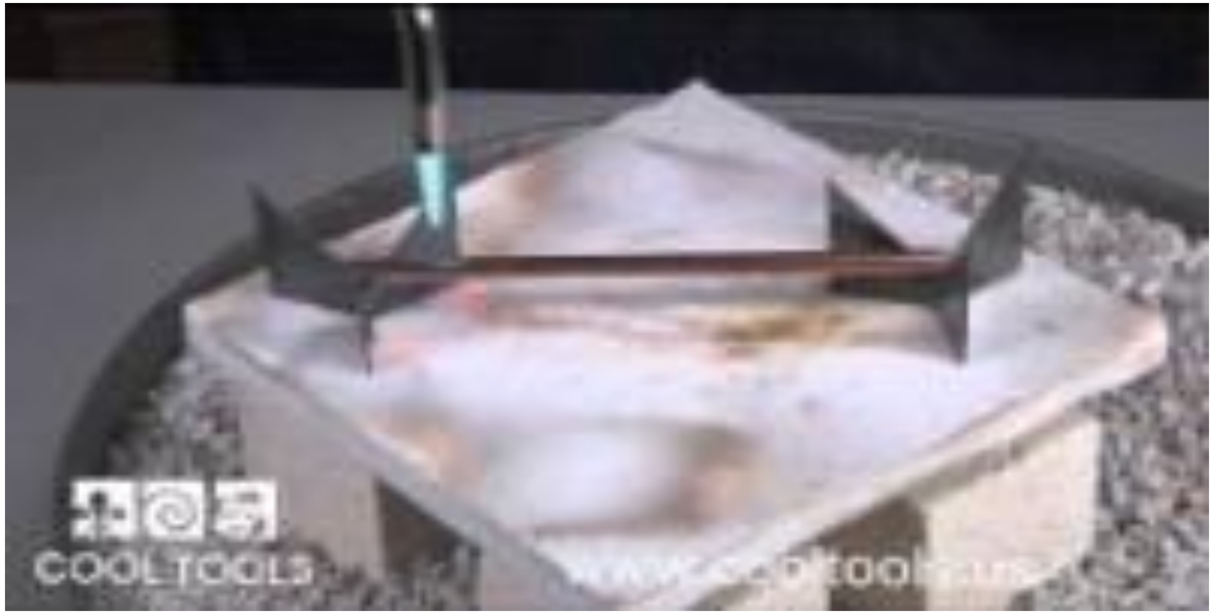
```python
def main():
    for city in romania_map.locations.keys():
        distances[city] = {}
        all_cities.append(city)

    all_cities.sort()
    print("All the sorted cities in Romania:")
    print(all_cities)
    print()
```

We populate the individual lists inside the dictionary with the Manhattan distance between the cities.

```python
    for name_1, coordinates_1 in romania_map.locations.items():
            for name_2, coordinates_2 in romania_map.locations.items():
                distances[name_1][name_2] = np.linalg.norm(
                    [coordinates_1[0] - coordinates_2[0], coordinates_1[1] - coordinates_2[1]])
                distances[name_2][name_1] = np.linalg.norm(
                    [coordinates_1[0] - coordinates_2[0], coordinates_1[1] - coordinates_2[1]])

    tsp = TSP_problem(all_cities)

    print("One shortest possible route that visits each city exactly once and returns to the origin city:")
    print(hill_climbing(tsp))

if __name__ == "__main__":
    main()
```

Simulated Annealing $T = \frac{T_0}{1 + log(t)}$

- Due to the **three** problems of Hill Climbing, Simulated Annealing can help us avoid getting **stuck in a Local Maxima/Minima, Ridges, and Plateaus**.
- Annealing is the metallurgical process of heating up a solid and then cooling it slowly until it crystallizes.
- Simulated Annealing (SA) is to mimic the annealing process, where a function E(S) needs to be minimized.
  - This function is analogous to the internal energy in that state S.
  - The goal is to start from **an initial state** to a state having minimum possible energy.

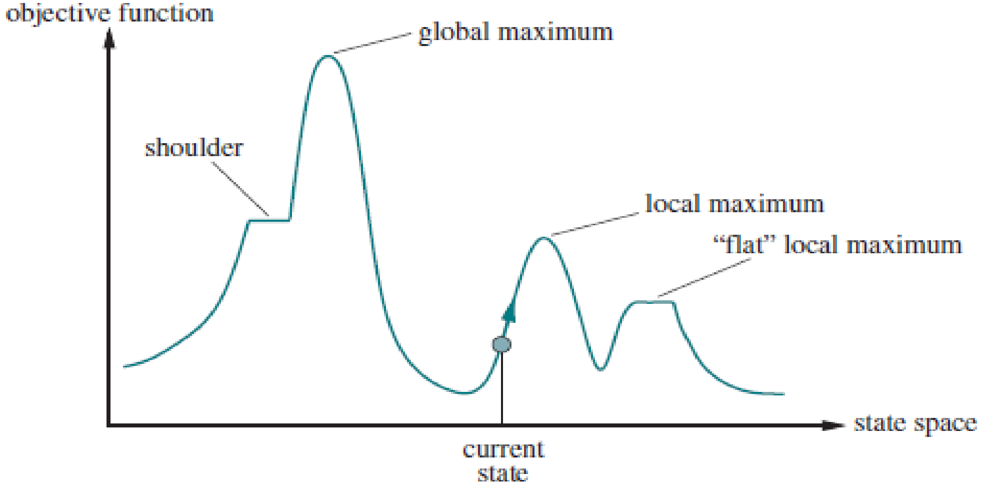


function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  current ← *problem*.INITIAL
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE(*current*) – VALUE(*next*)
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{-\Delta E/T}$

Temperature T ≥ 0 in Kelvin absolute temperature scale.

T = 0, i.e., Freezing Point

Figure 4.4 The simulated annealing algorithm, a version of stochastic hill climbing where some downhill moves are allowed. The *schedule* input determines the value of the "temperature" T as a function of time.



Simulated Annealing

$$T = \frac{T_0}{1 + log(t)}$$

- Basic idea:
  - Allow "random" moves occasionally, depending on "high temperature"
  - High temperature ➔ more random moves allowed, **shake the process out of its local minimum or local maximum**
  - schedule, i.e., annealing schedule, that leads to a better solution.
  - If the new state is **not better**, we make it the current state with **a certain predefined probability** by using a random number generator and deciding based on a threshold. If it is **above the threshold**, we set the current state to the next state.
- The **simulated_annealing(problem, schedule=exp_schedule())** function in search.py.

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Temperature T ≥ 0 in Kelvin absolute temperature scale.

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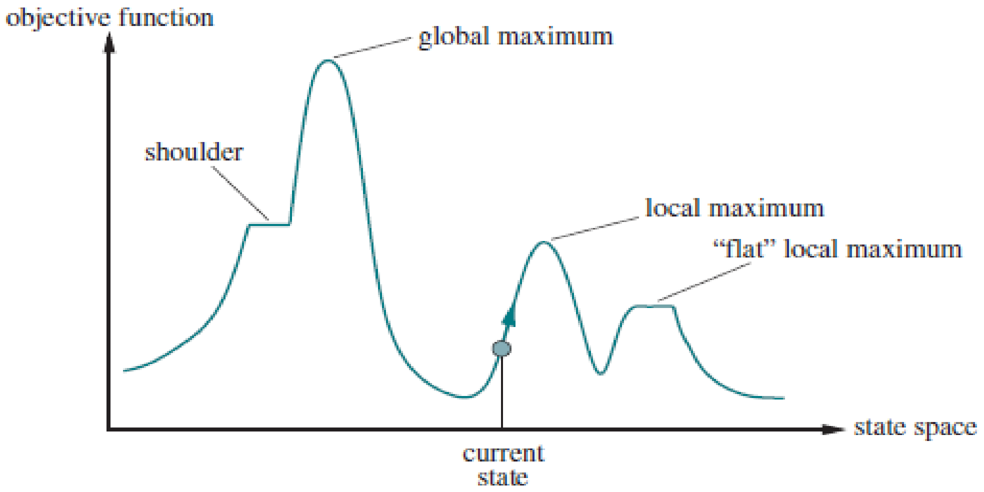

Simulated Annealing

$$T = \frac{T_0}{1 + log(t)}$$

$$20 \ * \ e^{-0.005*t}$$

```python
def exp_schedule(k=20, lam=0.005, limit=100):
    """One possible schedule function for simulated annealing"""
    return lambda t: (k * np.exp(-lam * t) if t < limit else 0)

def simulated_annealing(problem, schedule=exp_schedule()):
    """[Figure 4.5] CAUTION: This differs from the pseudocode as it
    returns a state instead of a Node."""
    current = Node(problem.initial)
    for t in range(sys.maxsize):
        T = schedule(t)
        if T == 0:
            return current.state
        neighbors = current.expand(problem)
        if not neighbors:
            return current.state
        next_choice = random.choice(neighbors)
        #delta_e = problem.value(next_choice.state) - problem.value(current.state)
        delta_e = problem.value(current.state) - problem.value(next_choice.state)
        if delta_e > 0 or probability(np.exp(-delta_e / T)):
            current = next_choice
```

utils.py

```python
def probability(p):
    """Return true with probability p."""
    return p > random.uniform(0.0, 1.0)
```

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  current ← *problem*.INITIAL
  for t = 1 **to** ∞ **do**
    T ← *schedule*(t)
    if T = 0 **then return** *current*
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    if ΔE > 0 **then** *current* ← *next*
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Temperature T ≥ 0 in Kelvin absolute temperature scale.

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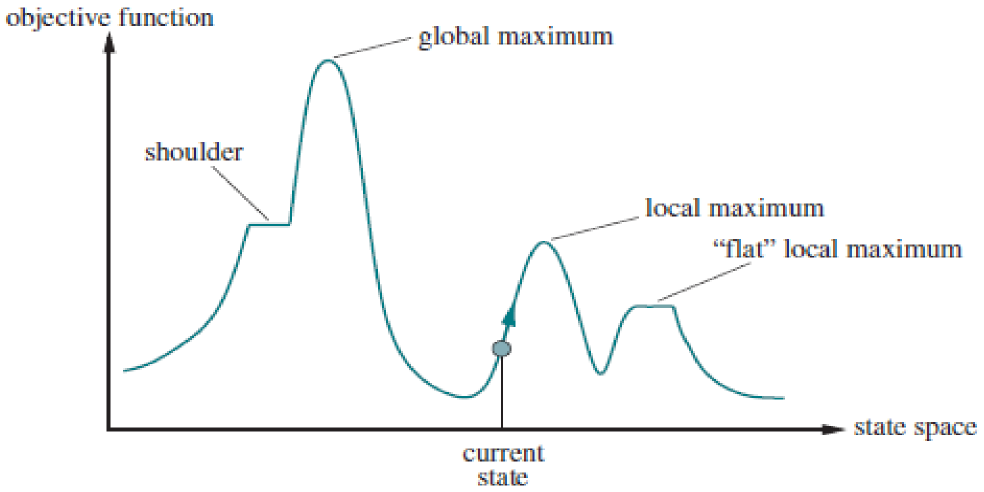
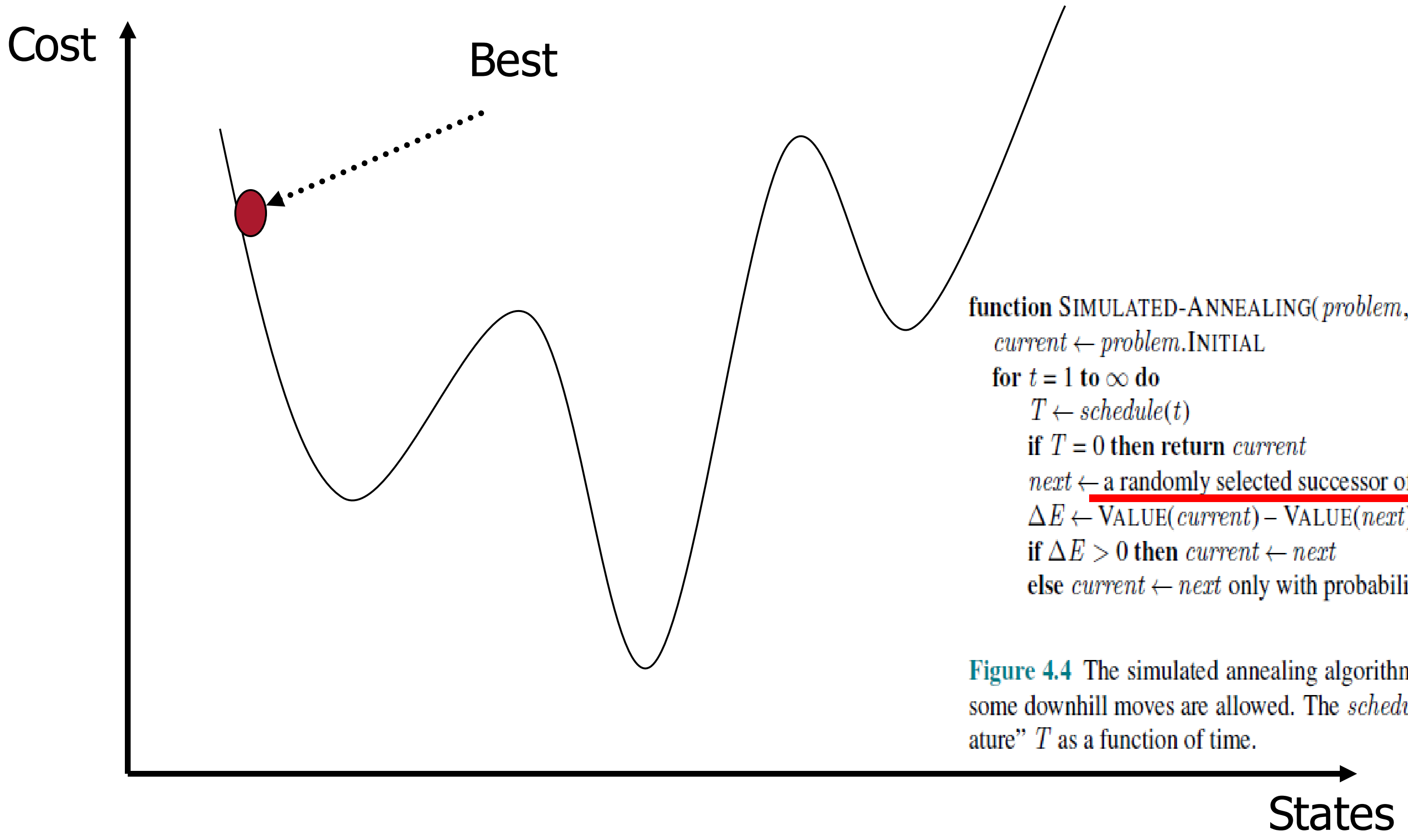
Figure 4.4 The simulated annealing algorithm, a version of stochastic hill climbing where some downhill moves are allowed. The *schedule* input determines the value of the "temperature" T as a function of time.



Simulated Annealing



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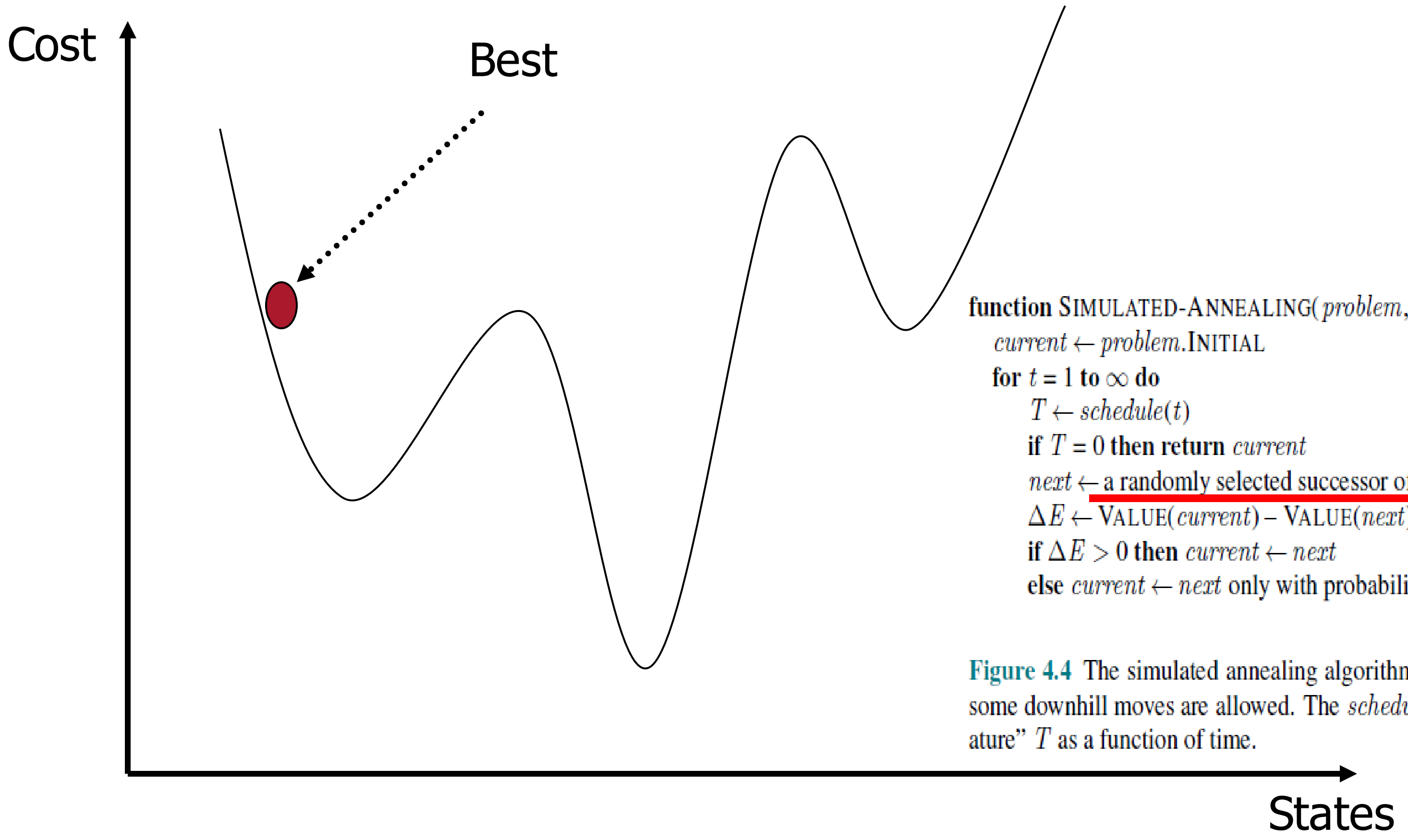
Simulated Annealing




```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  current ← problem.INITIAL
  for t = 1 to ∞ do
     T ← schedule(t)
     if T = 0 then return current
     next ← a randomly selected successor of current
     ΔE ← VALUE(current) – VALUE(next)
     if ΔE > 0 then current ← next
     else current ← next only with probability e^(−ΔE/T)
```

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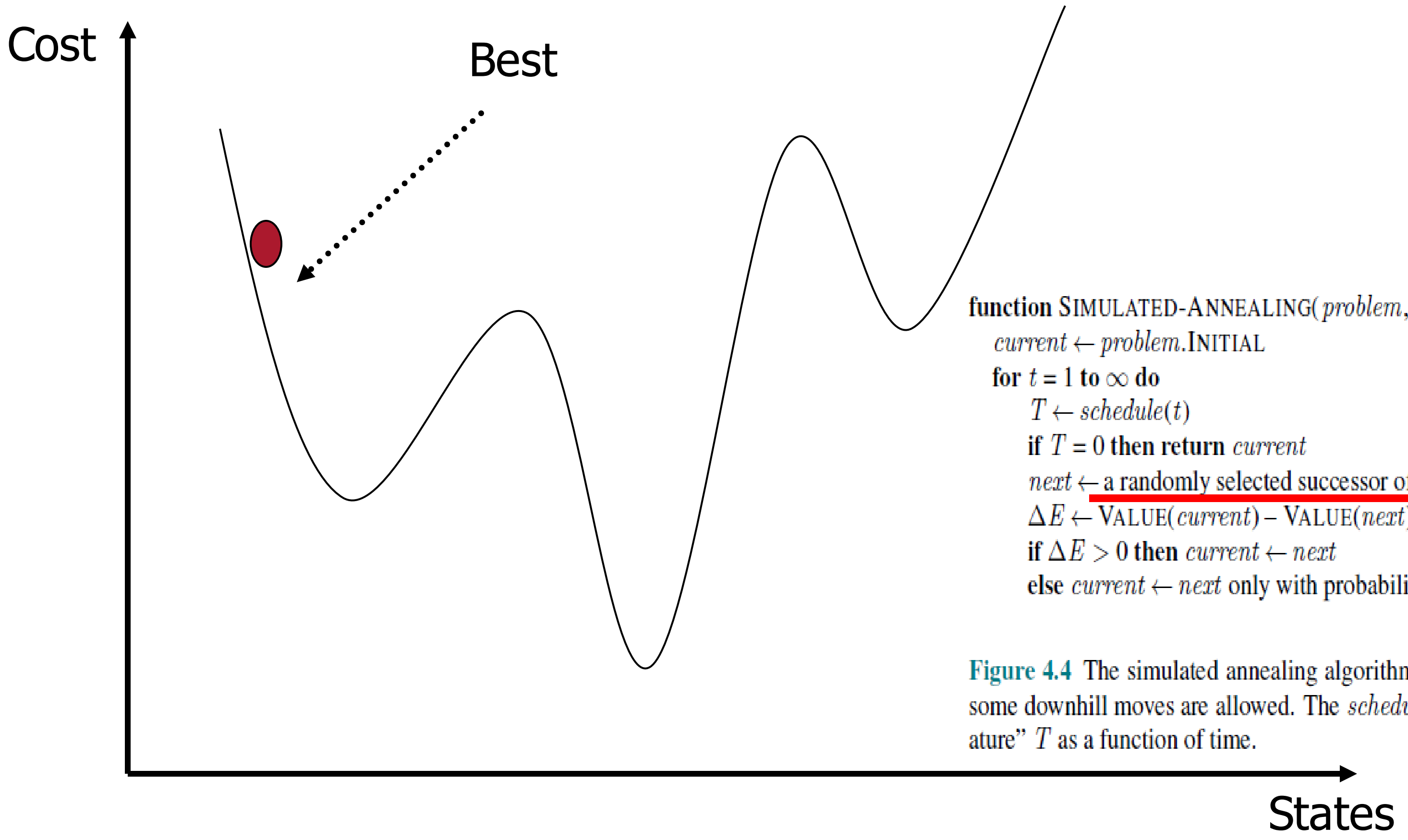
Simulated Annealing



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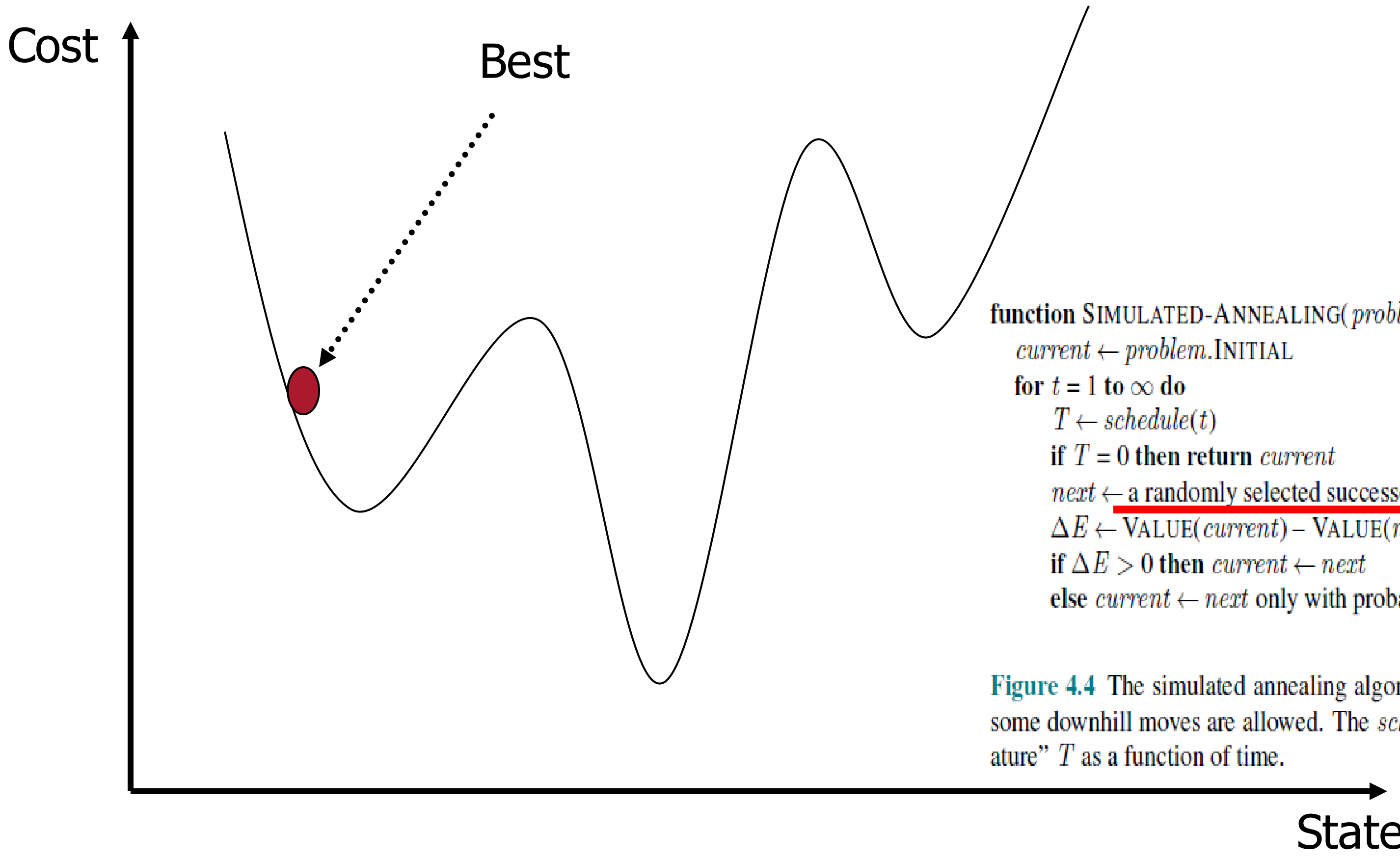

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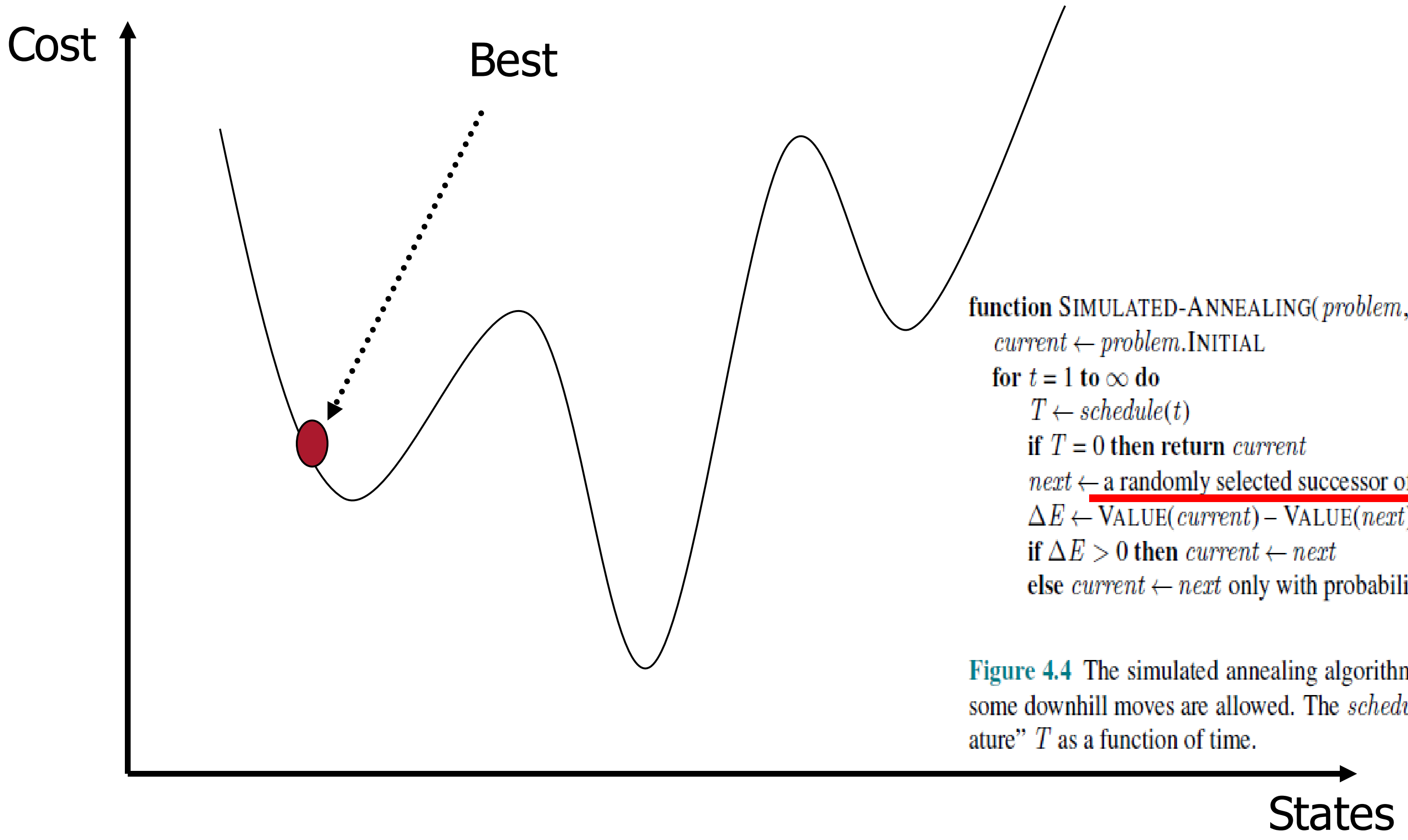

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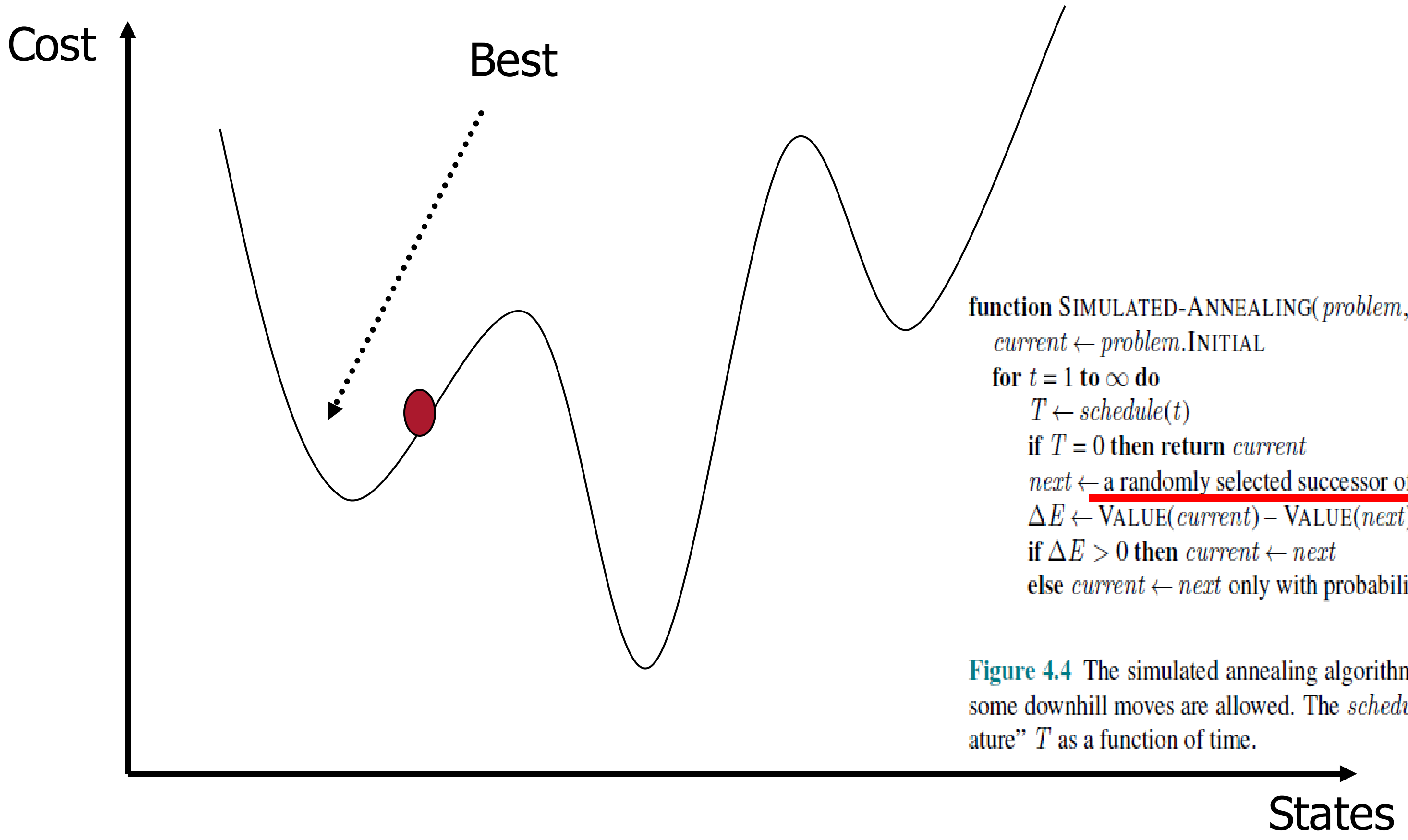

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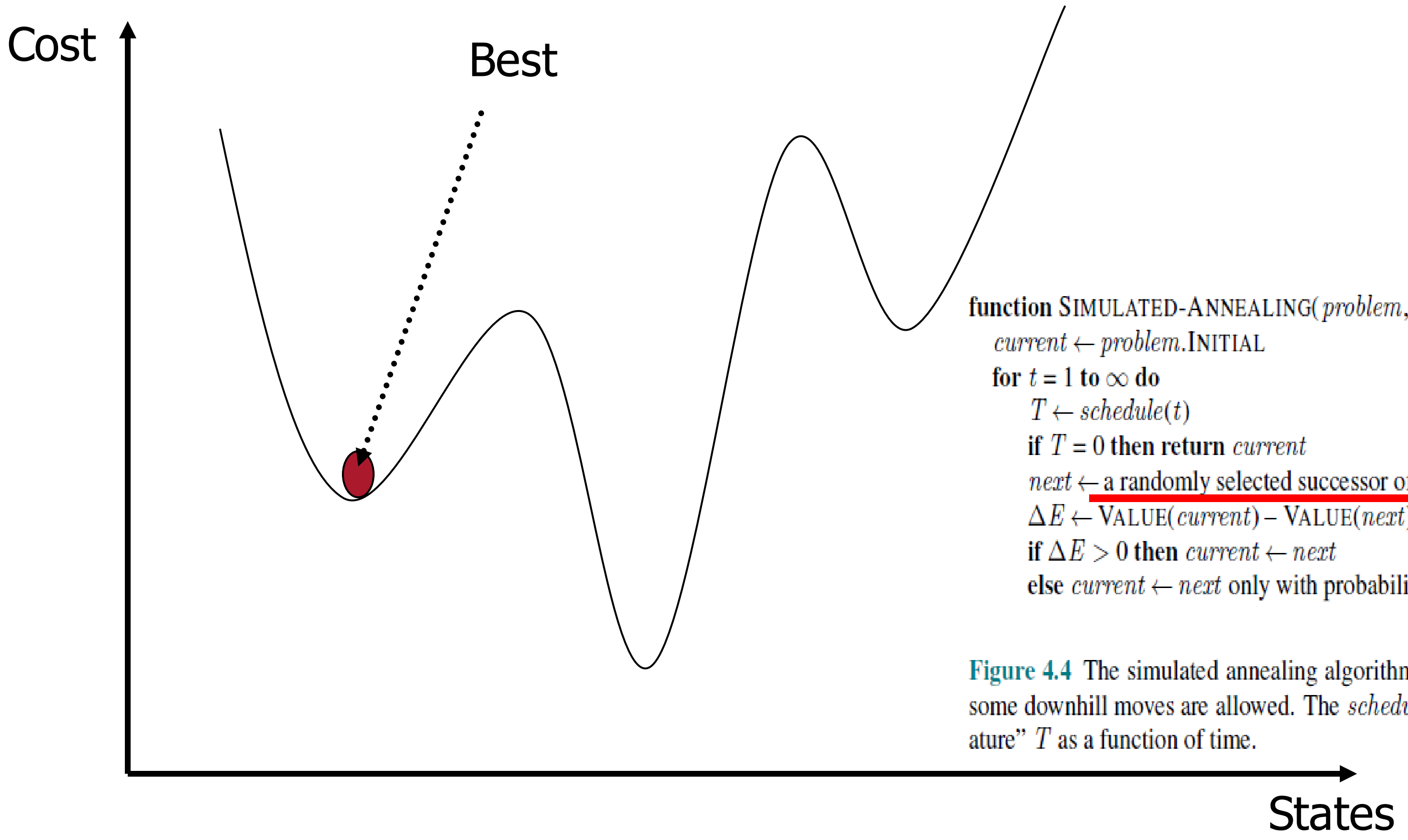

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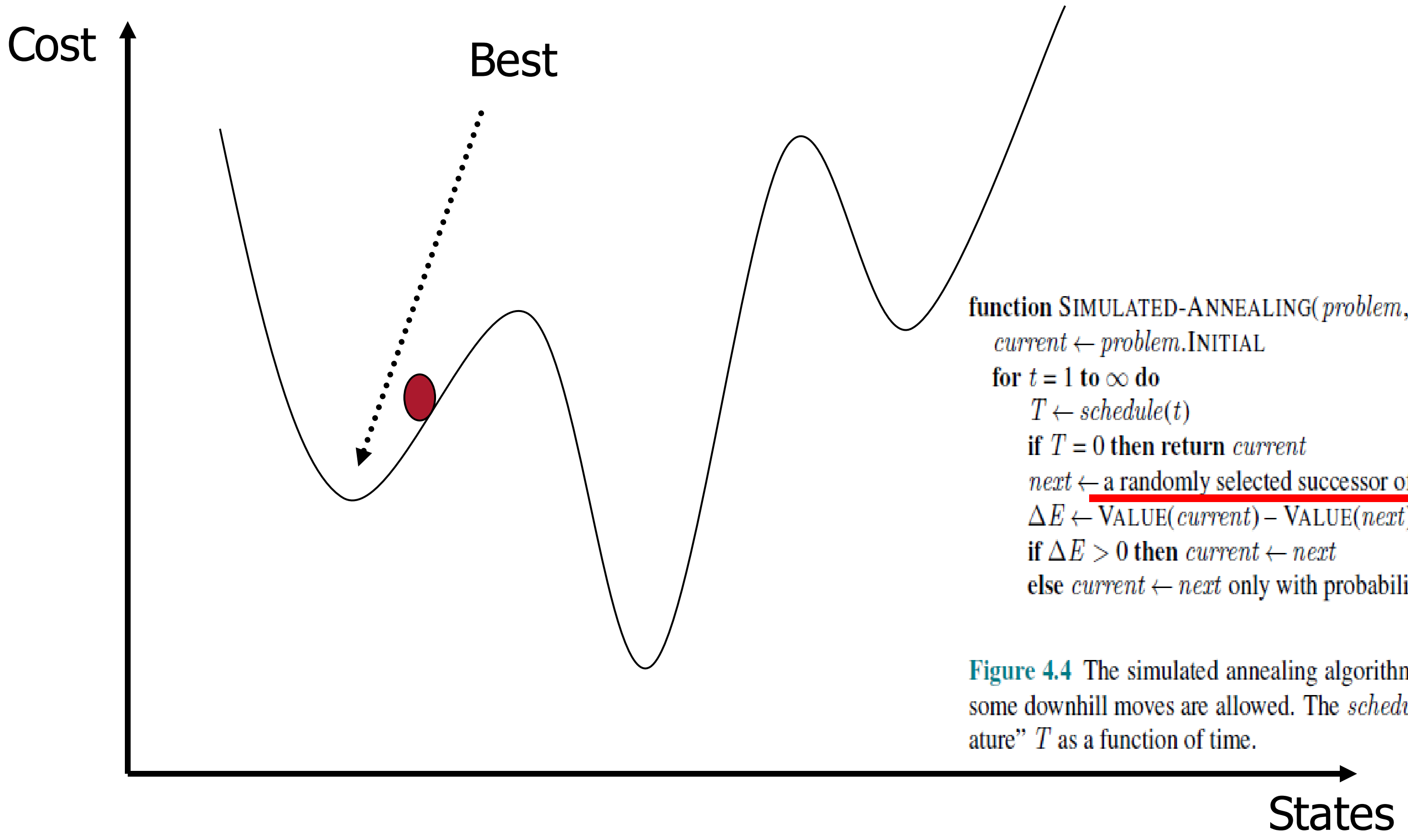

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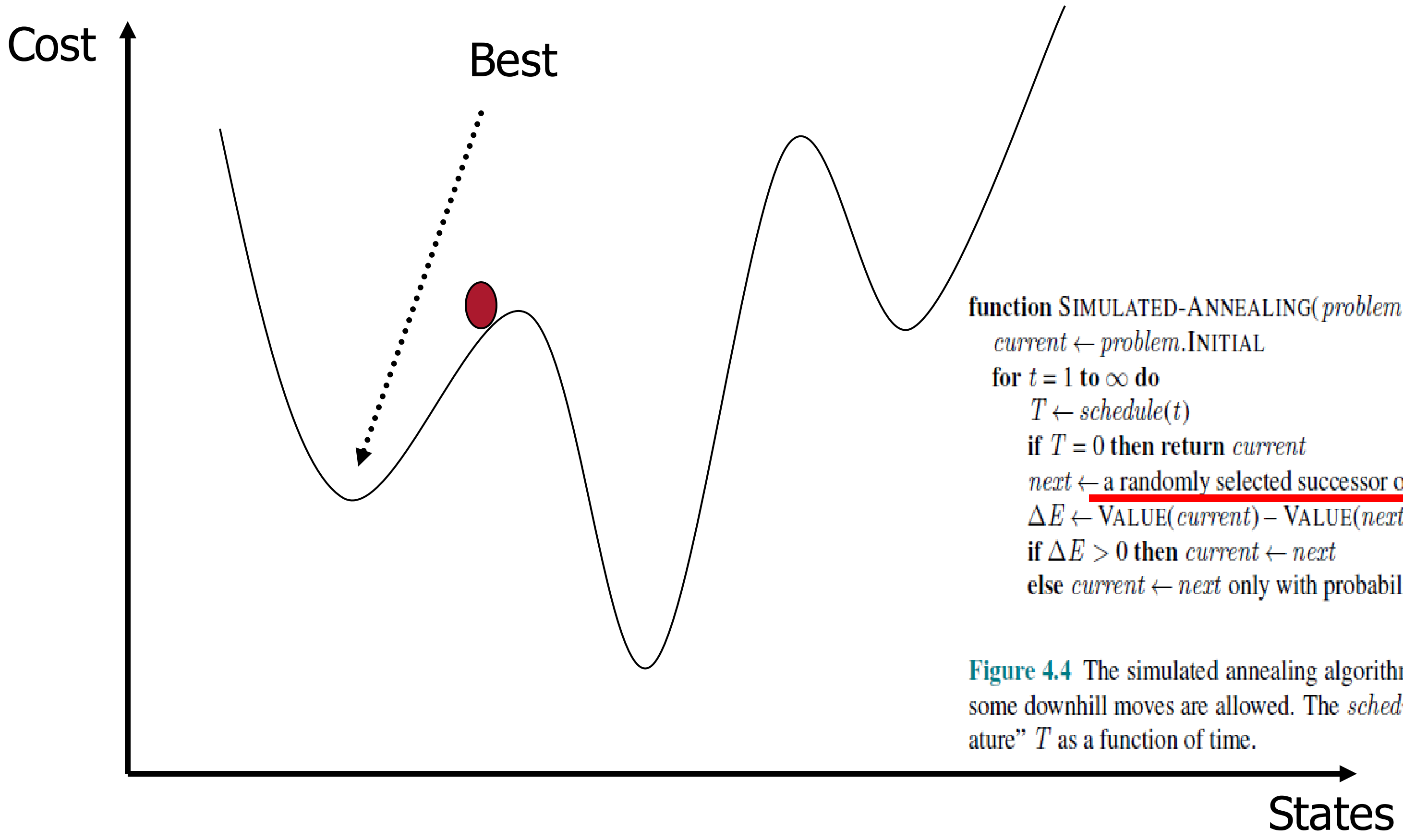
    else *current* ← *next* only with probability $e^{-\Delta E/T}$

Figure 4.4 The simulated annealing algorithm, a version of stochastic hill climbing where some downhill moves are allowed. The *schedule* input determines the value of the "temperature" T as a function of time.

Simulated Annealing

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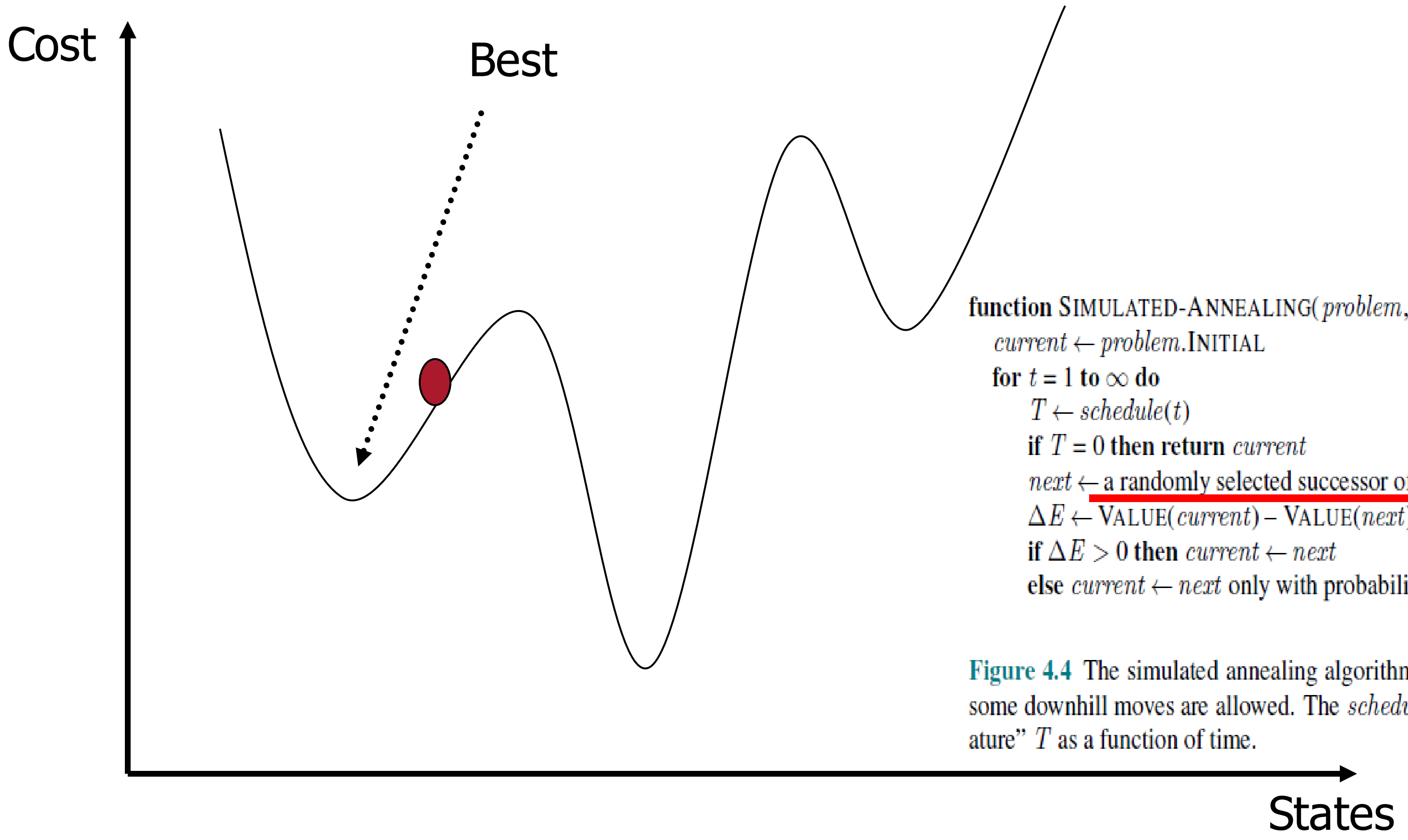

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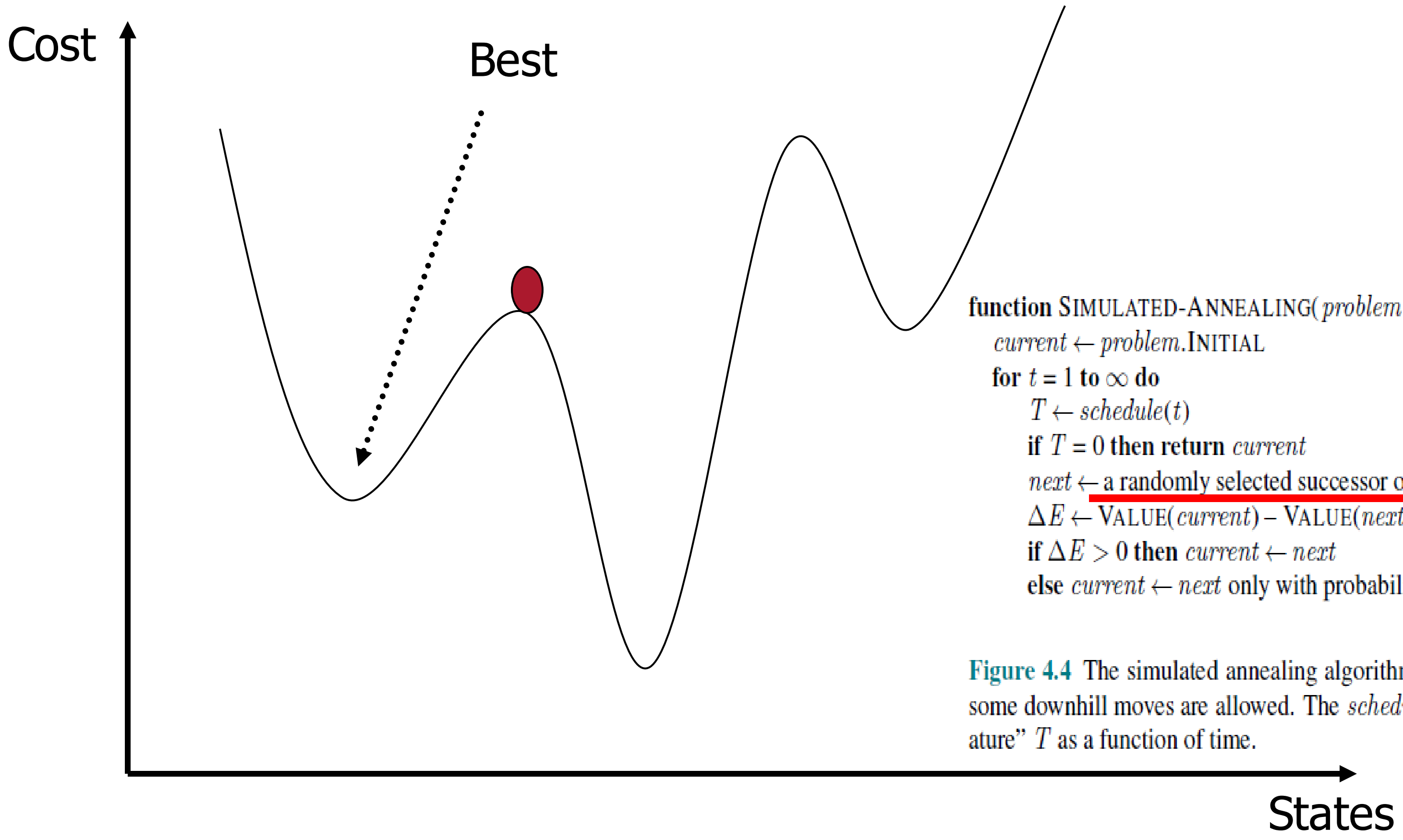

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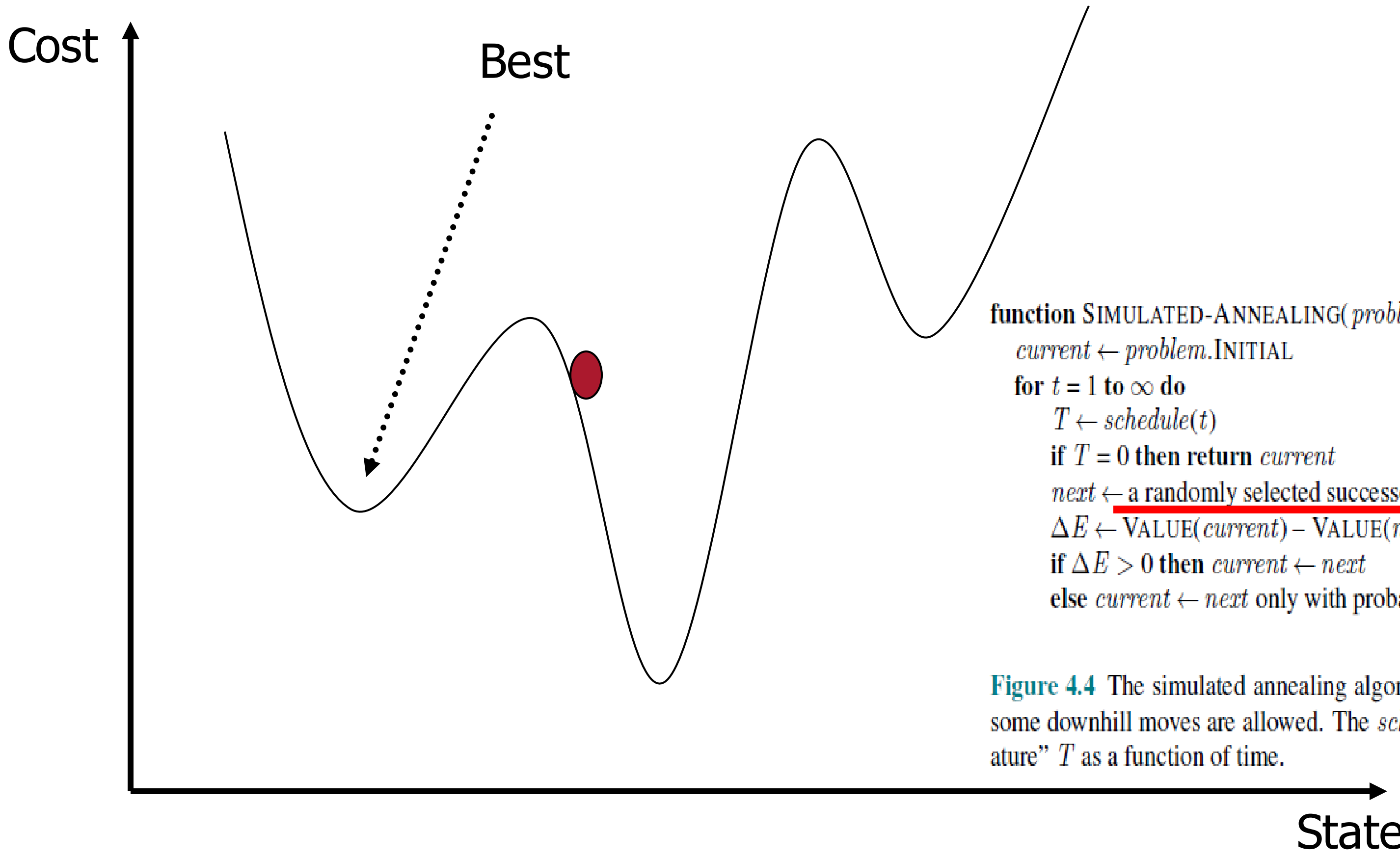

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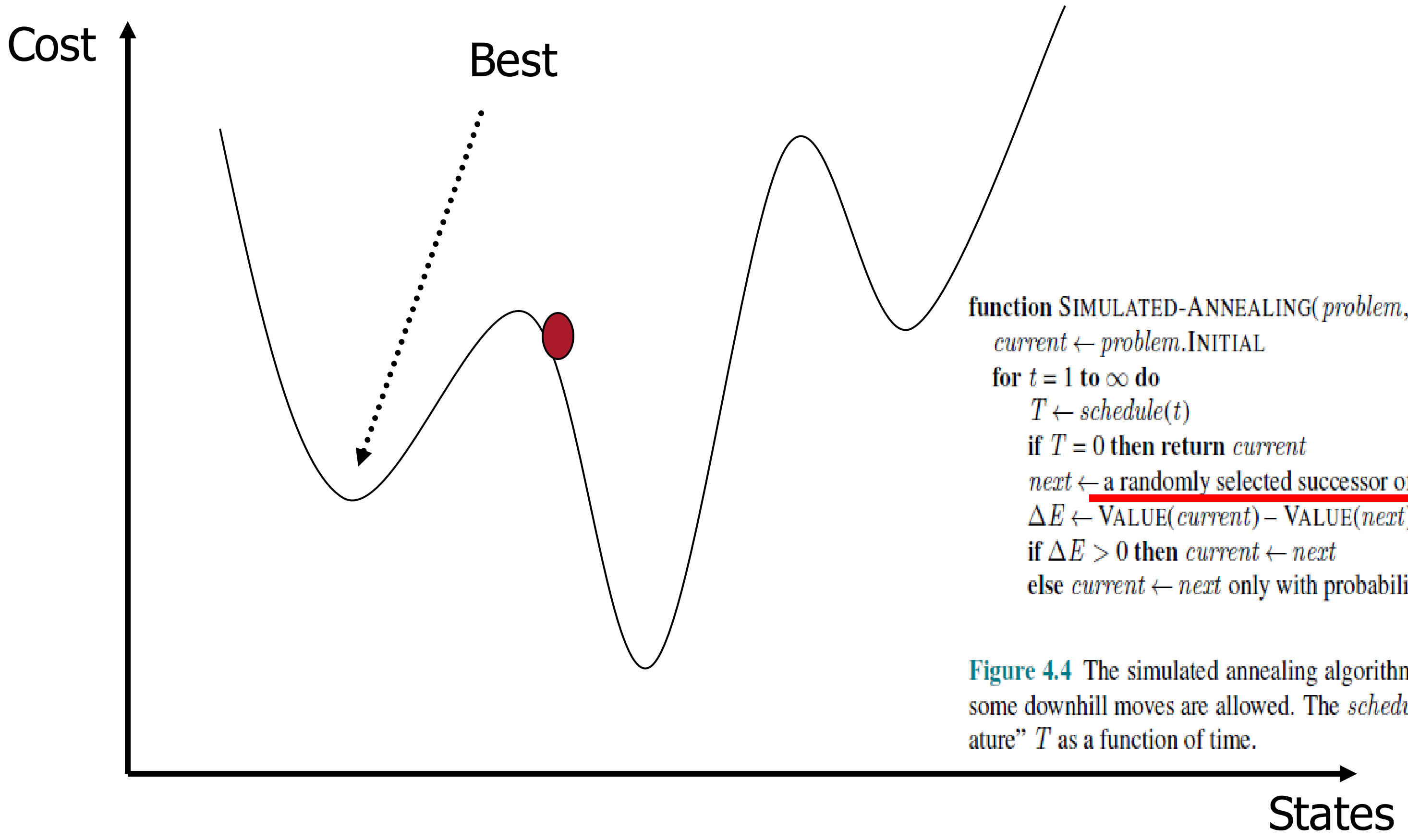

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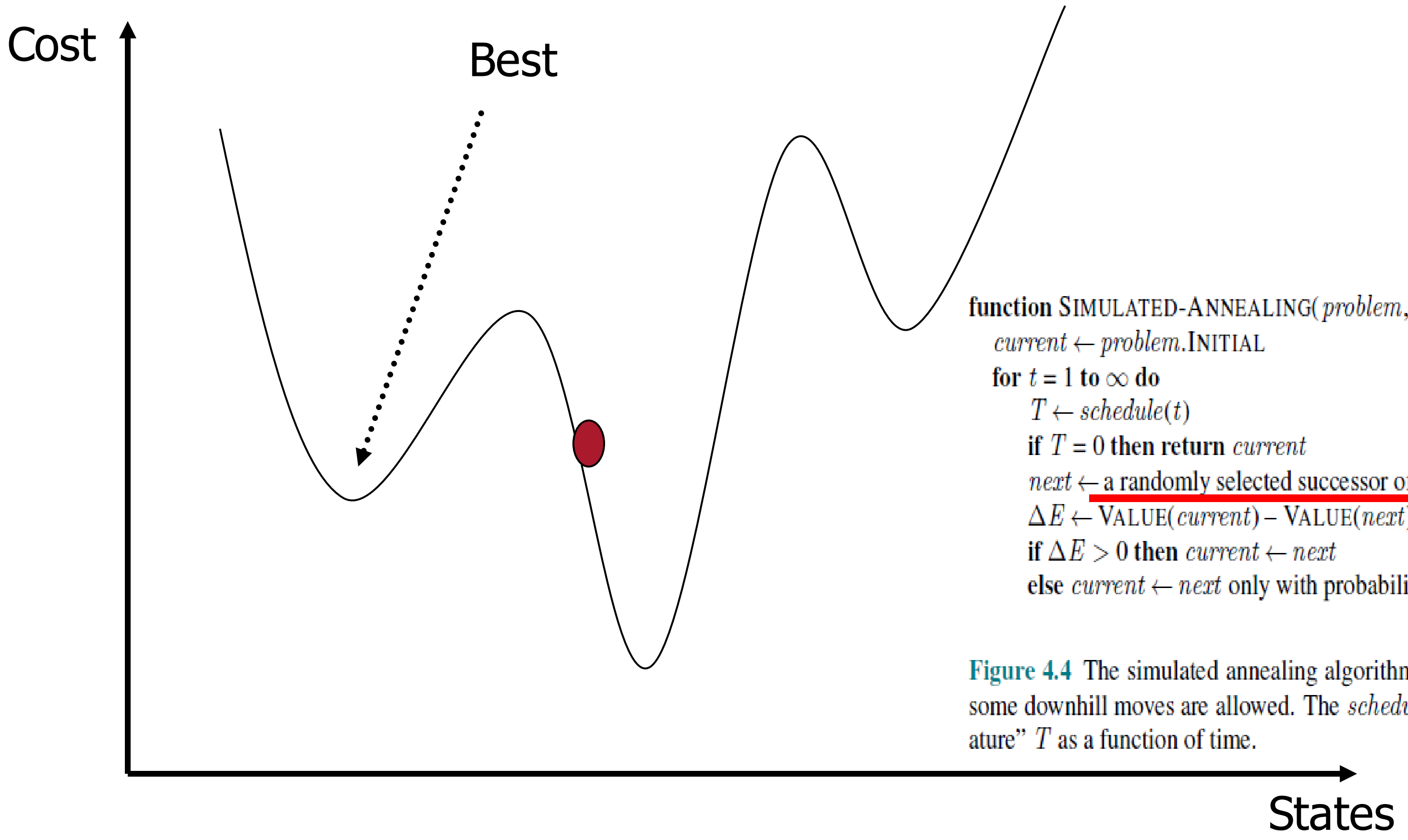

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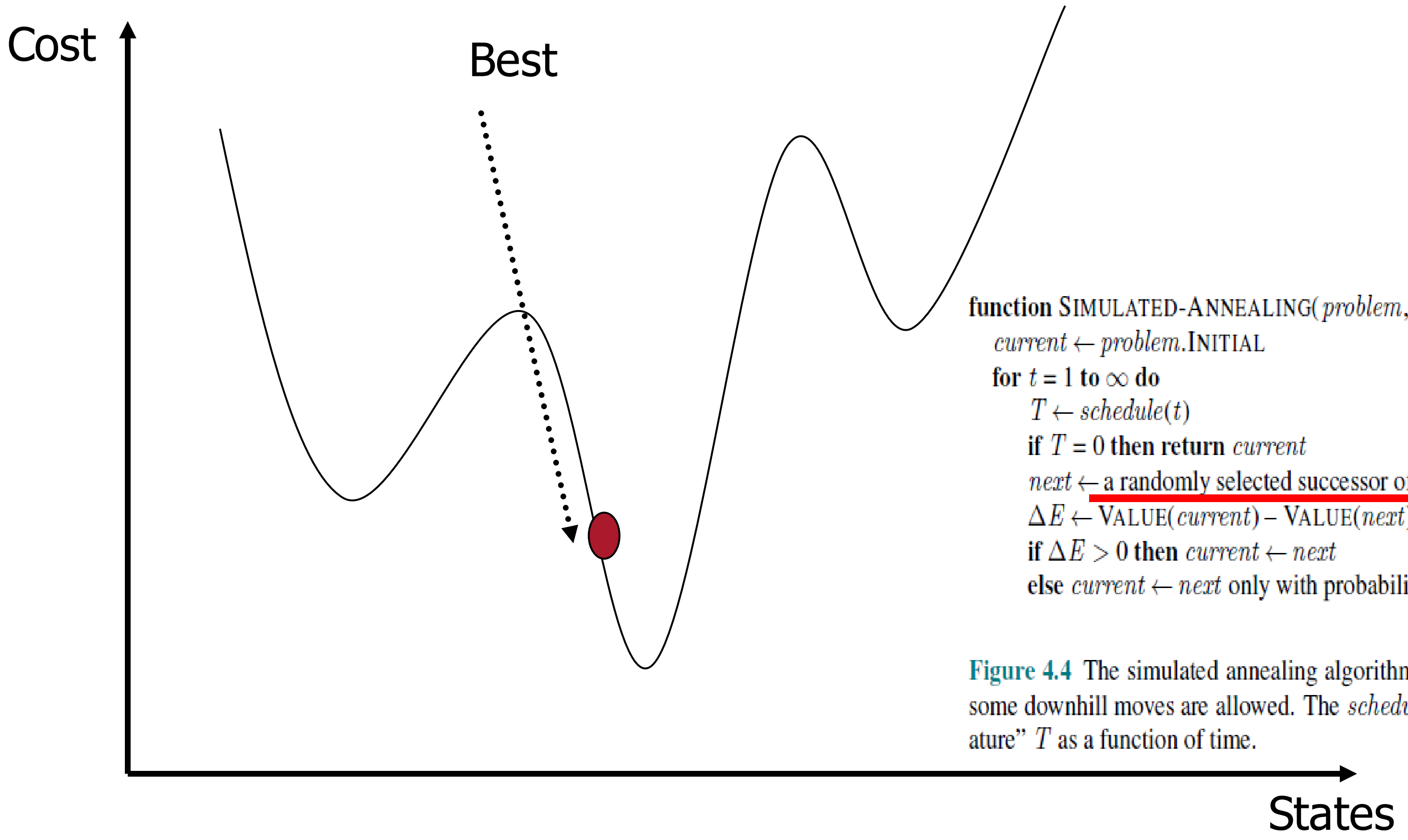

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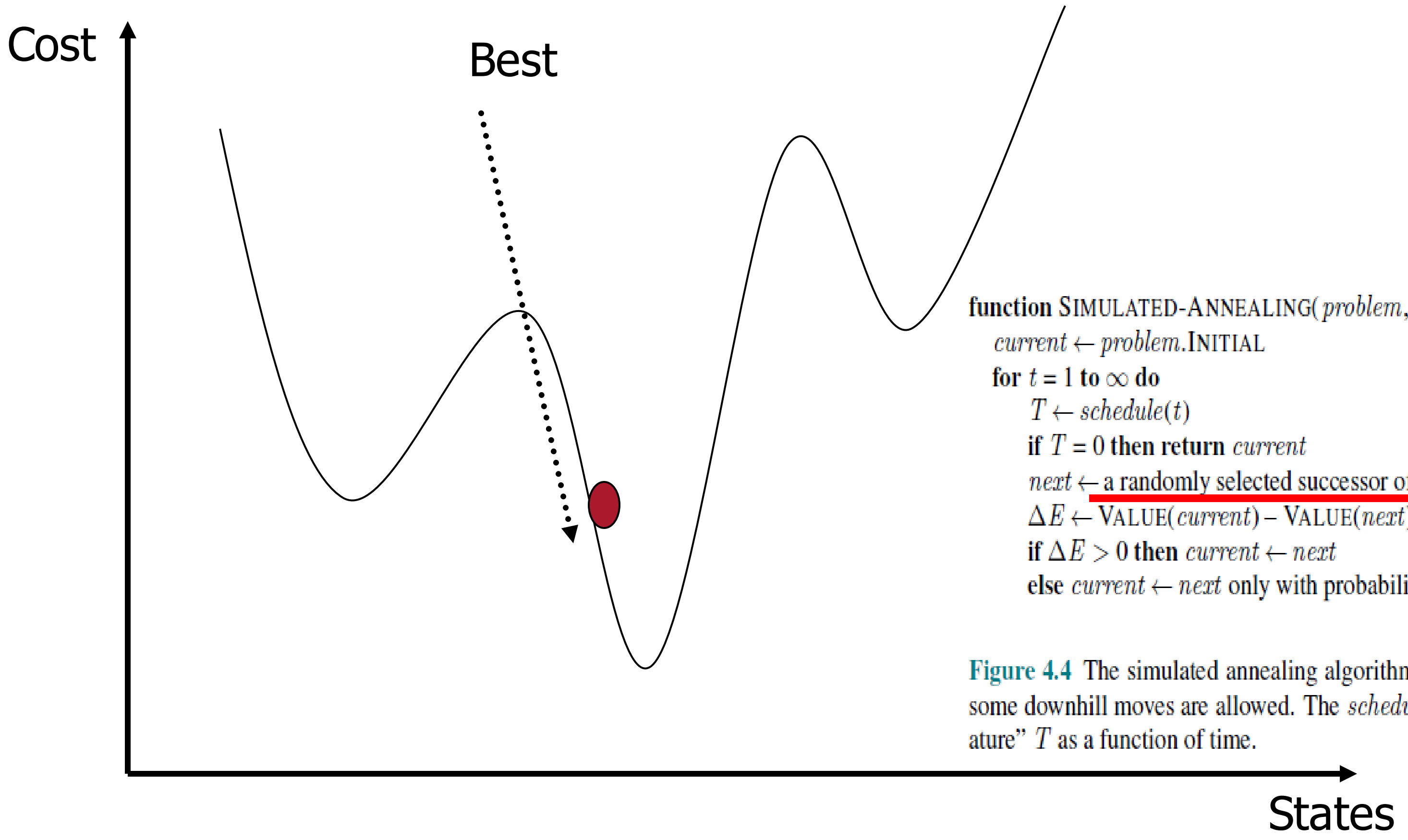

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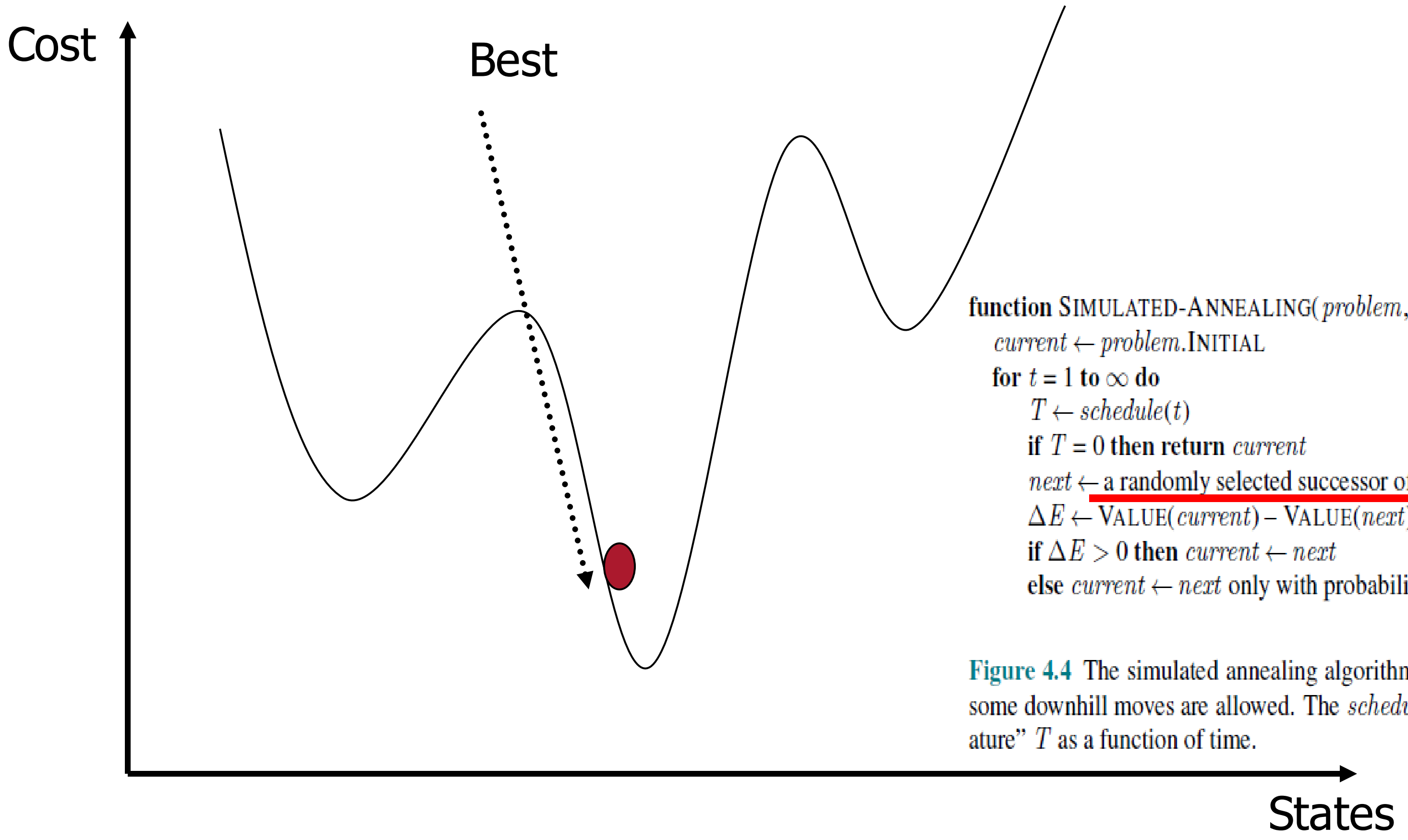


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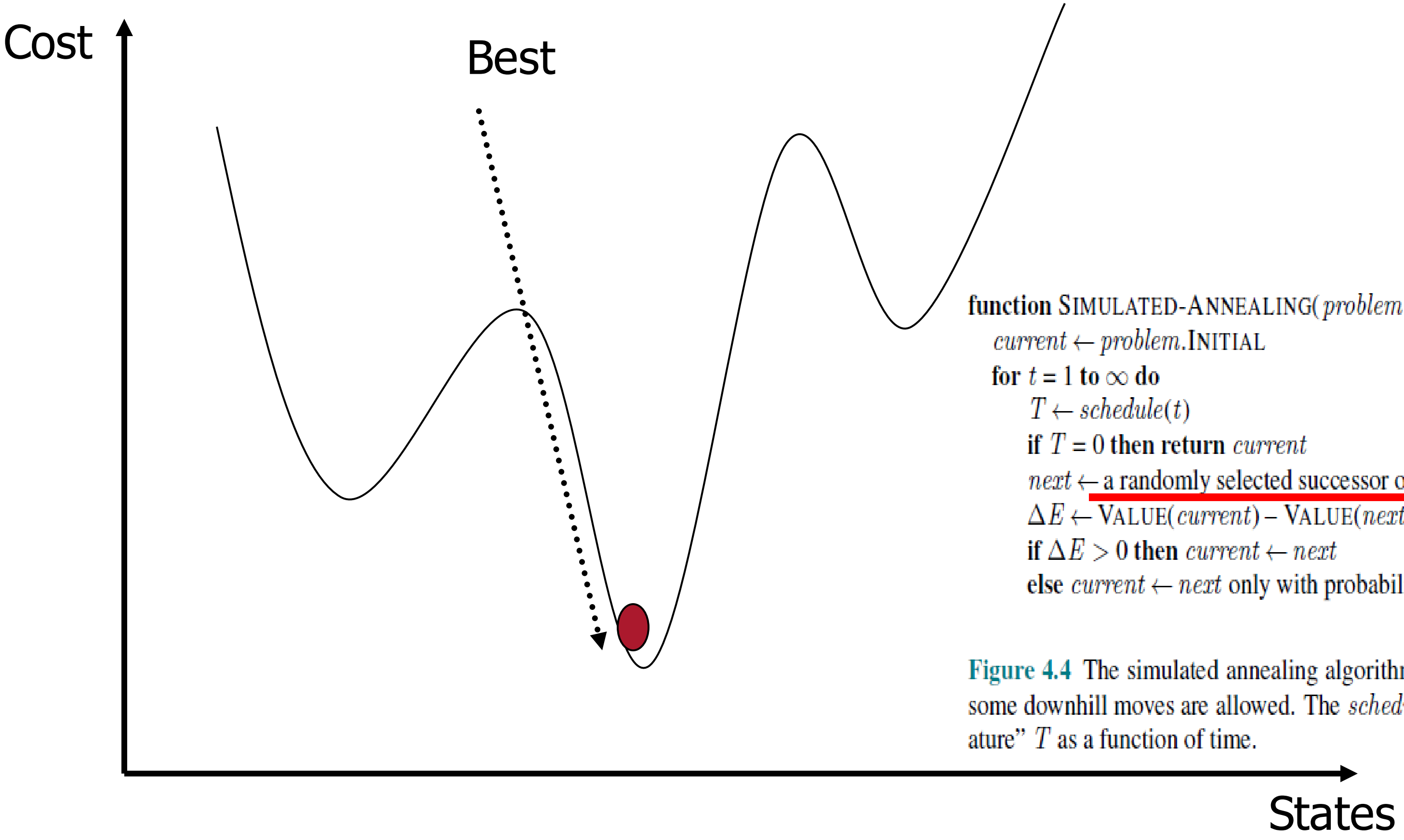

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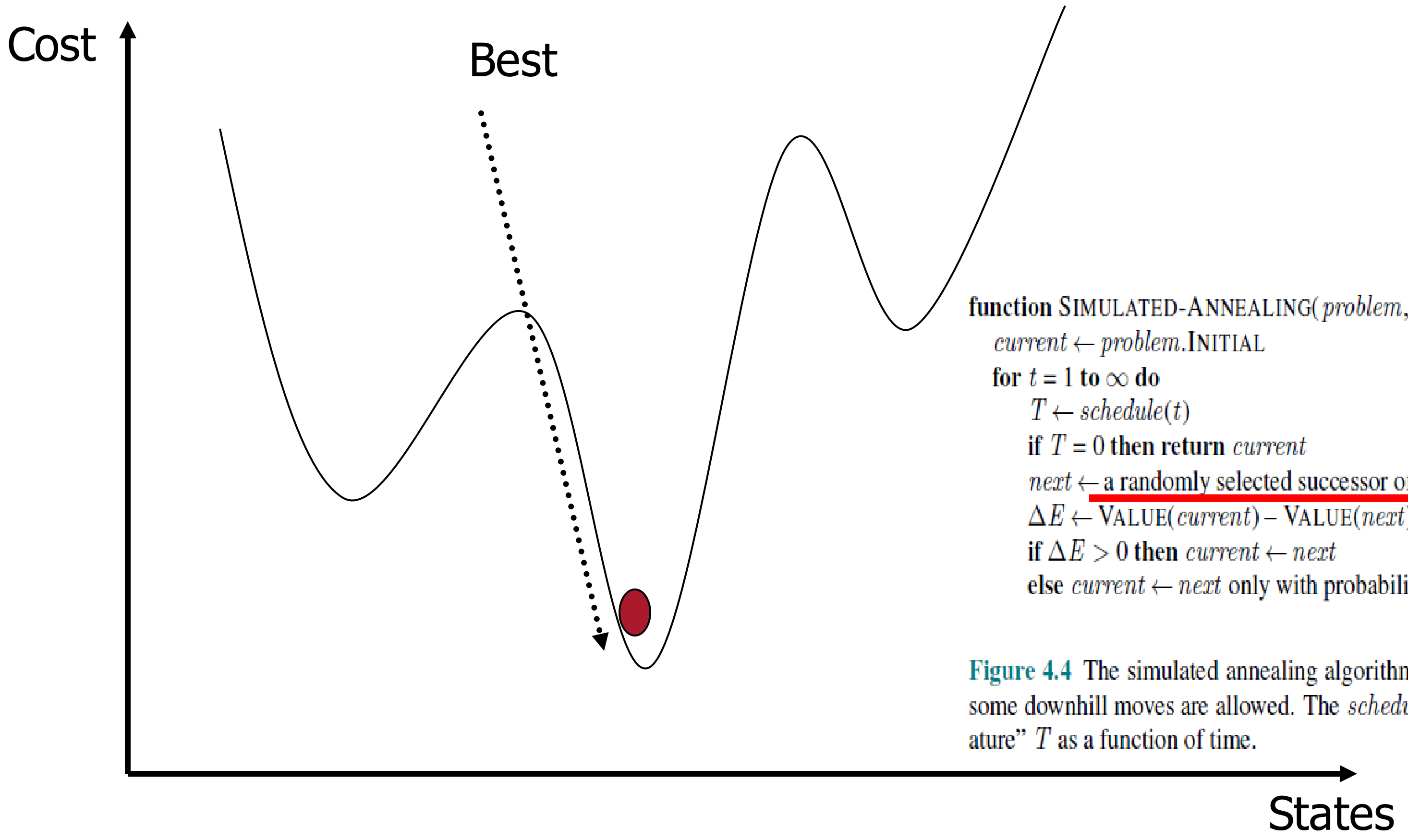

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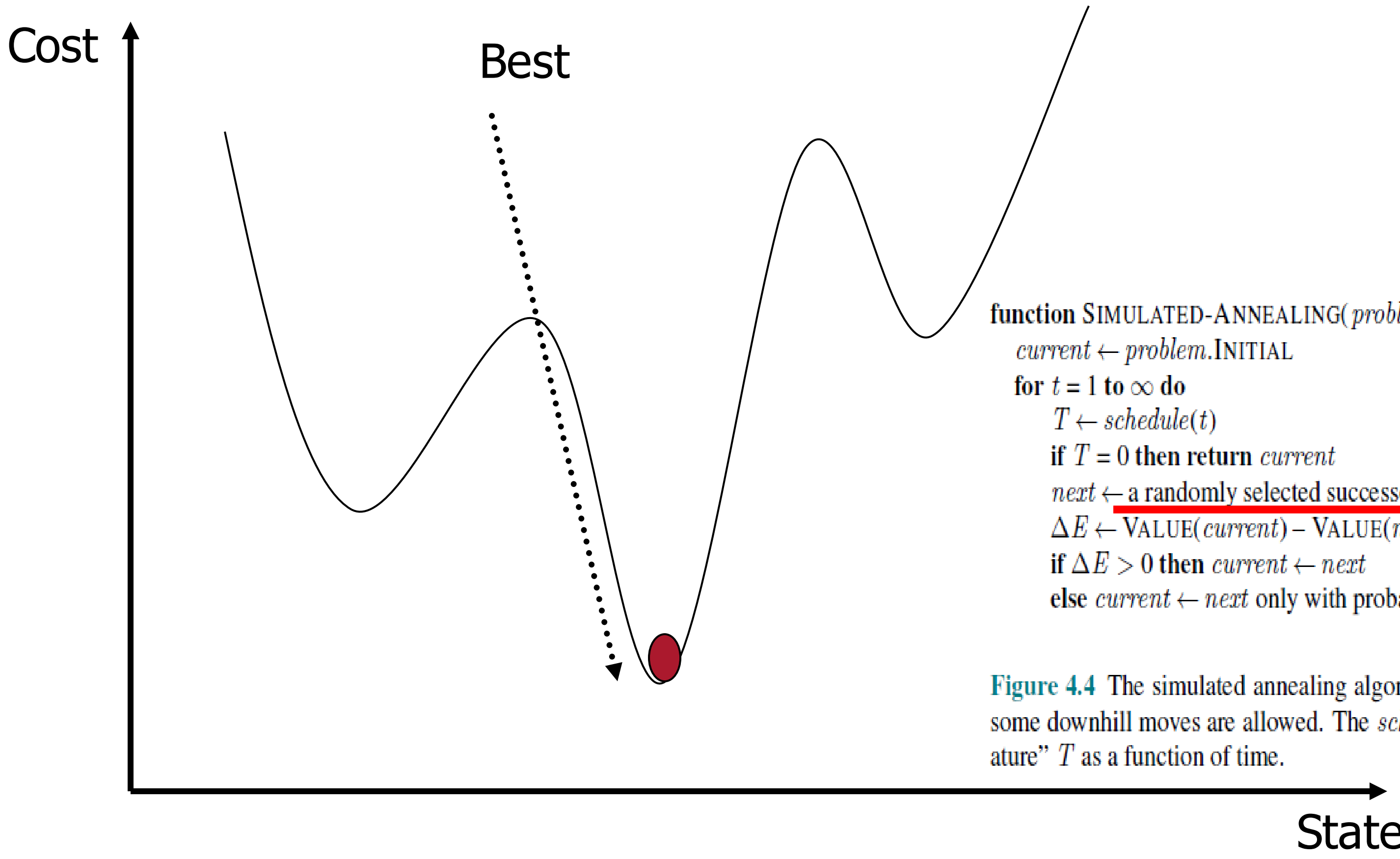

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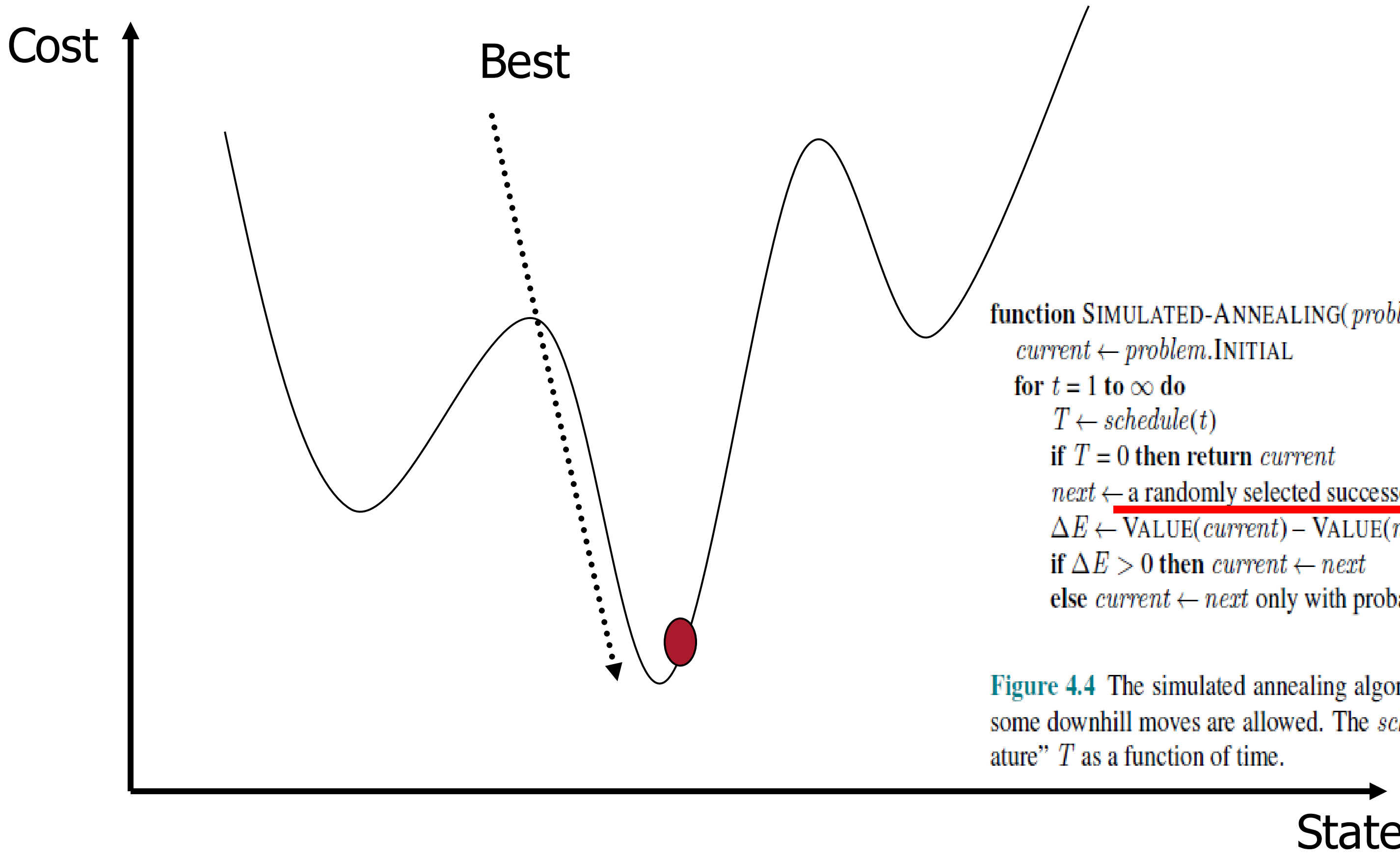

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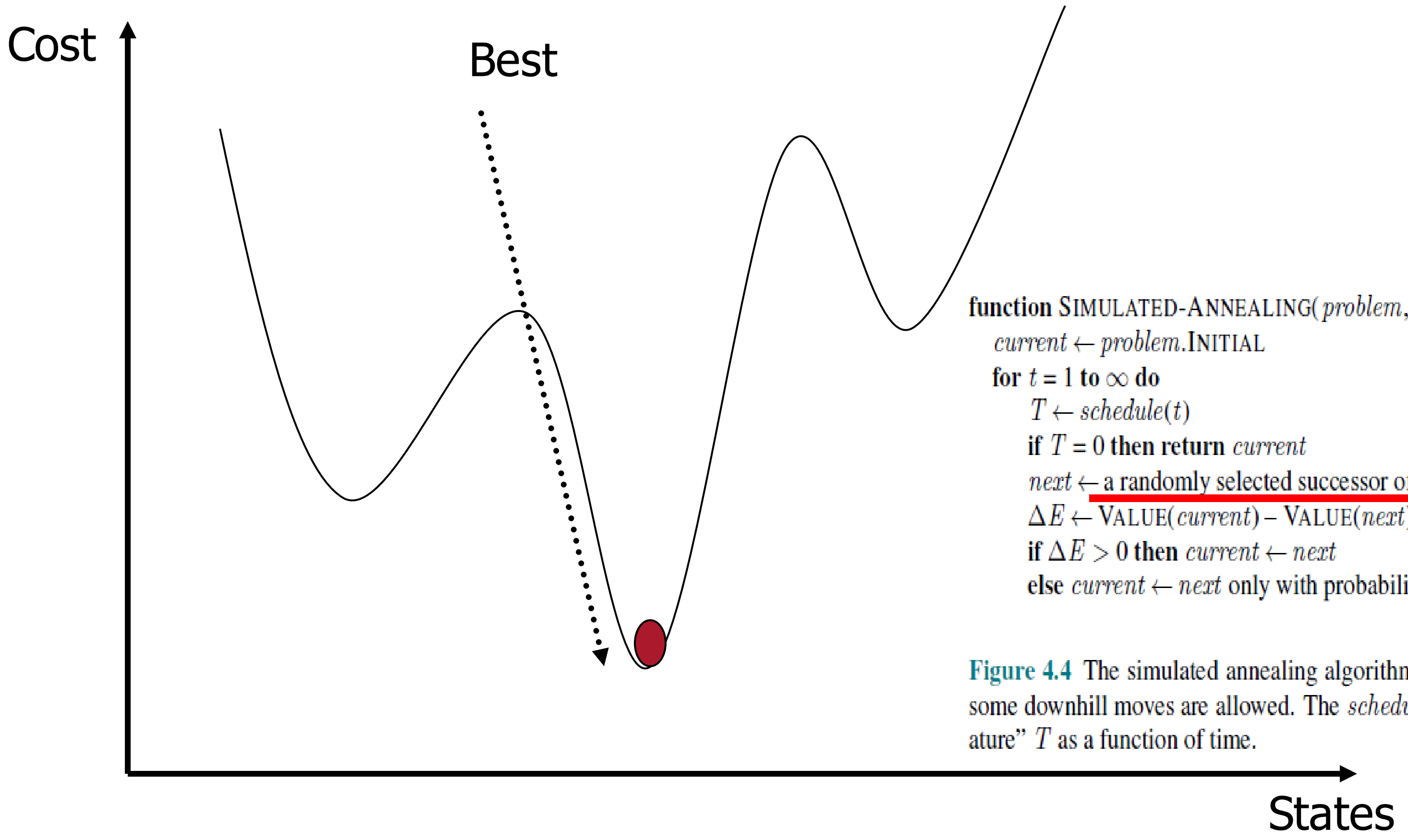

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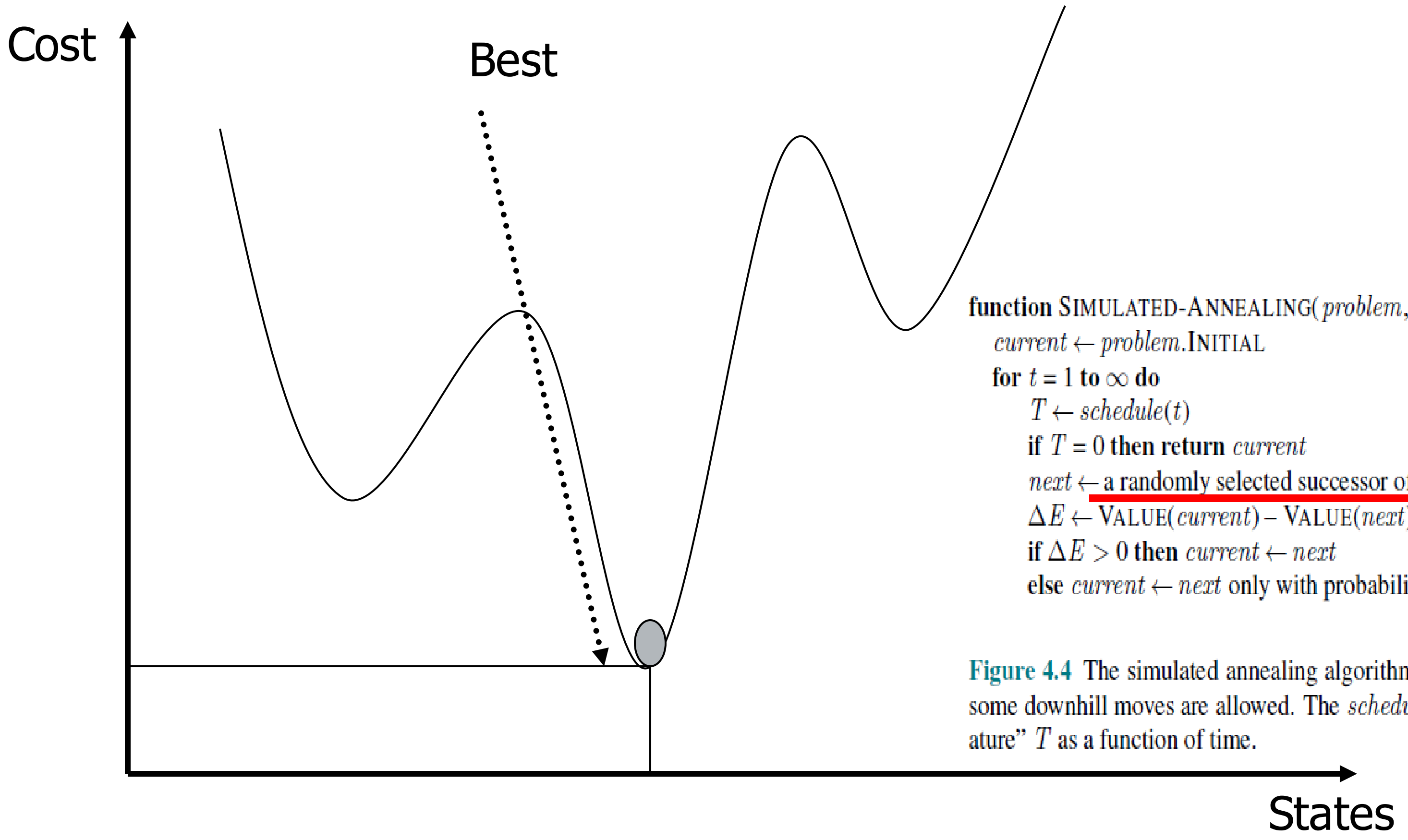

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Simulated Annealing on the Peak Finding Problem

Given a 2D Array/Matrix, the task is to find the Peak element.

An element is a peak element if it is **greater than or equal to** its four neighbors, left, right, top and bottom.

- A Diagonal adjacent is not considered a neighbor.
- A peak element is not necessarily the maximal element.
- More than one such element can exist.
- There is always a peak element.
- For corner elements, missing neighbors are considered of **negative infinite value**.

Test if a condition returns True:
#if condition returns **True**, then nothing happens

#if condition returns **False**, AssertionError is raised.

```python
# Pre-defined actions for PeakFindingProblem
directions4 = {'W': (-1, 0), 'N': (0, 1), 'E': (1, 0), 'S': (0, -1)}
directions8 = dict(directions4)
directions8.update({'NW': (-1, 1), 'NE': (1, 1), 'SE': (1, -1), 'SW': (-1, -1)})
```

```python
class PeakFindingProblem(Problem):
    """Problem of finding the highest peak in a limited grid"""

    def __init__(self, initial, grid, defined_actions=directions4):
        """The grid is a 2 dimensional array/list whose state is specified by tuple of indices"""
        super().__init__(initial)
        self.grid = grid
        self.defined_actions = defined_actions
        self.n = len(grid)
        assert self.n > 0
        self.m = len(grid[0])
        assert self.m > 0

    def actions(self, state):
        """Returns the list of actions which are allowed to be taken from the given state"""
        allowed_actions = []
        for action in self.defined_actions:
            next_state = vector_add(state, self.defined_actions[action])
            if 0 <= next_state[0] <= self.n - 1 and 0 <= next_state[1] <= self.m - 1:
                allowed_actions.append(action)

        return allowed_actions

    def result(self, state, action):
        """Moves in the direction specified by action"""
        return vector_add(state, self.defined_actions[action])

    def value(self, state):
        """Value of a state is the value it is the index to"""
        x, y = state
        assert 0 <= x < self.n
        assert 0 <= y < self.m
        return self.grid[x][y]
```

utils.py

```python
def vector_add(a, b):
    """Component-wise addition of two vectors."""
    return tuple(map(operator.add, a, b))
```

Simulated Annealing on the Peak Finding Problem

Given a 2D Array/Matrix, the task is to find the Peak element.

An element is a peak element if it is greater than or equal to its four neighbors, left, right, top and bottom.

- A Diagonal adjacent is not considered a neighbor.
- A peak element is not necessarily the maximal element.
- More than one such element can exist.
- There is always a peak element.
- For corner elements, missing neighbors are considered of **negative infinite value**.

```
from search import *

# directions4 = {'W': (-1, 0), 'N': (0, 1), 'E': (1, 0), 'S': (0, -1)}
# directions8 = dict(directions4)
# directions8.update({'NW': (-1, 1), 'NE': (1, 1), 'SE': (1, -1), 'SW': (-1, -1)})

def main():
    initial = (0, 0)
    grid = [[10, 20, 15], [21, 30, 14], [7, 16, 32]]

    problem = PeakFindingProblem(initial, grid, directions4)

    solutions = [problem.value(simulated_annealing(problem)) for i in range(5)] #Change to 100
    print(solutions)

    solutions = set(solutions)
    print(solutions)

    print(max(solutions))

if __name__ == "__main__":
    main()
```

Local Beam Search

- Basic idea:
  - *K* states instead of one of a local search algorithm, initialized **k randomly generated states**.
  - For each iteration
    - Generate ALL successors from *K* current states
    - If any one reaches a goal state, the algorithm stops.
    - Or else, the algorithm select the **best *K* successors** from the above complete list to be the new current *K* states.
  - This process repeats until it finds a successor reaches a goal, the algorithm stops.

Or, ***K*** chosen randomly with a bias towards good ones

```
function BEAM-SEARCH(problem, k) returns a solution state
    start with k randomly generated states
    loop
        generate all successors of all k states
        if any of them is a solution then return it
        else select the k best successors
```

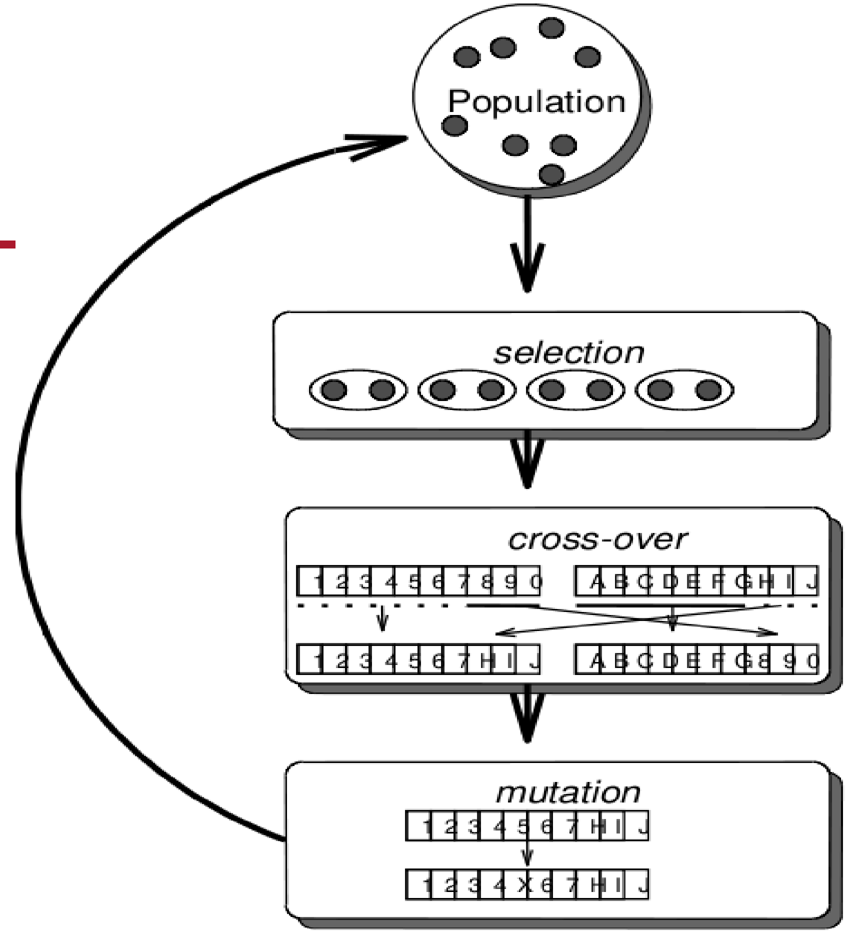
Genetic Algorithms

- A population of individual solutions (states), in which the fittest (highest value) individuals **produce offspring** (successor states) that populate the next generation, a process called **recombination**.

- Each state or individual is represented as a string over a finite alphabet. It is also called chromosome which contains genes.

- Genetic Algorithms are one of these algorithms.
  - Start with ***k* randomly generated solutions (states)** (population)
  - An individual solution (state) is a sequence of real numbers or a computer program or a CNN architecture, ….
  - Evaluation function (fitness function). Higher values for better solution (state).
  - Produce the next generation of solutions (states) by **selection**, **crossover**, and **mutation**

- The **genetic_algorithm()** function in search.py.



Algorithm 9.5 Genetic Algorithm(Input: Initial Population, fitness function, percent for mutation, selection threshhold)

```
Initialize the population with random candidate solutions
Apply fitness function to Evaluate each candidate's fitness value
repeat
    Select parents based on fitness value
    Recombine pairs of parents (crossover)
    Mutate resulting offspring
    Apply fitness function to Evaluate new candidates' fitness value
until termination condition/goal is reached
```

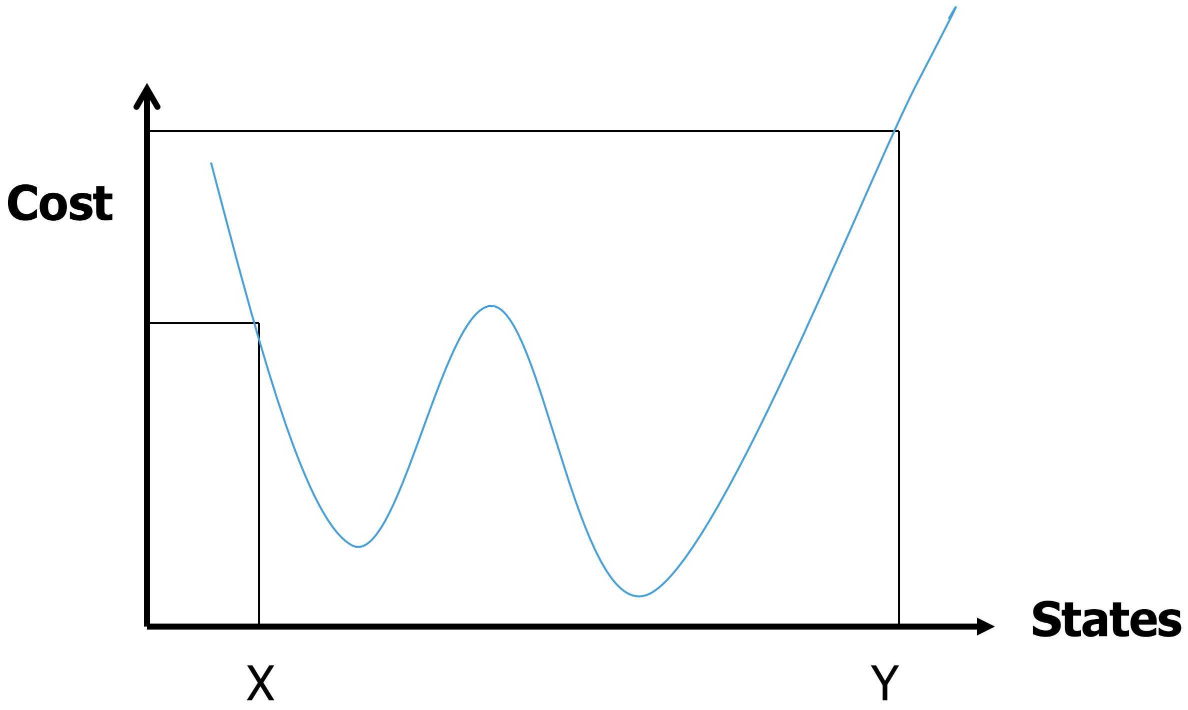
termination_condition = the max number of generations reached

Genetic Algorithms

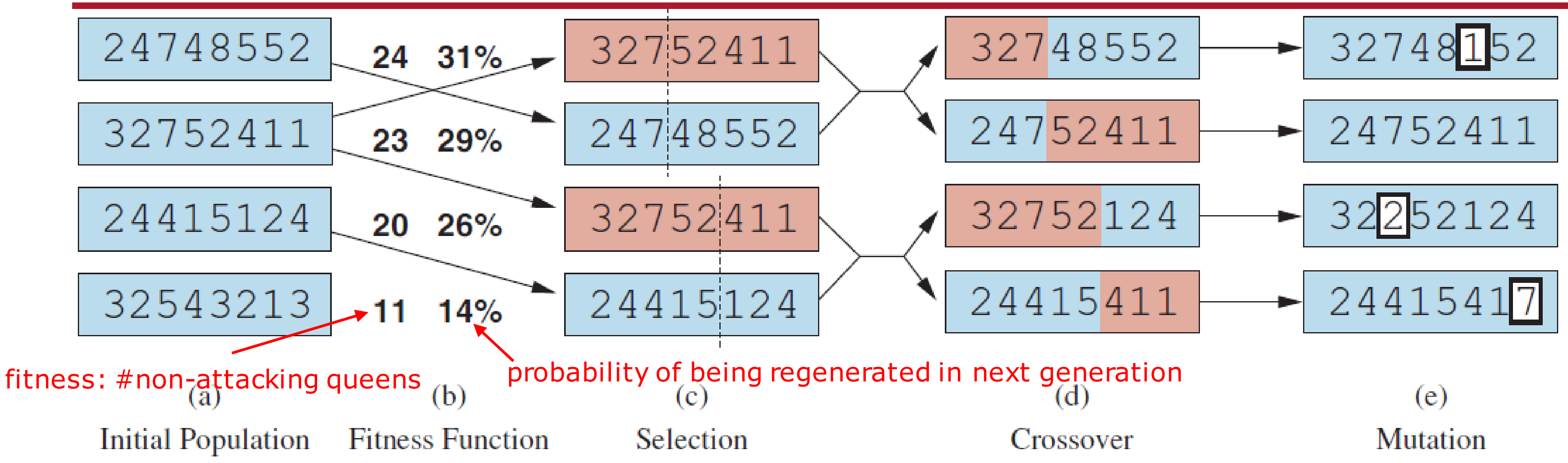
- Each state is rated by the evaluation function called fitness function. Fitness function should return higher values for better states:

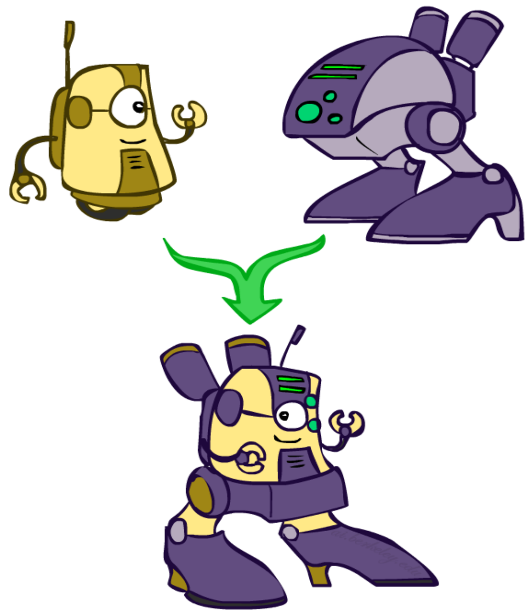
Fitness(X) should be greater than Fitness(Y) !!

[Fitness(x) = 1/Cost(x)]



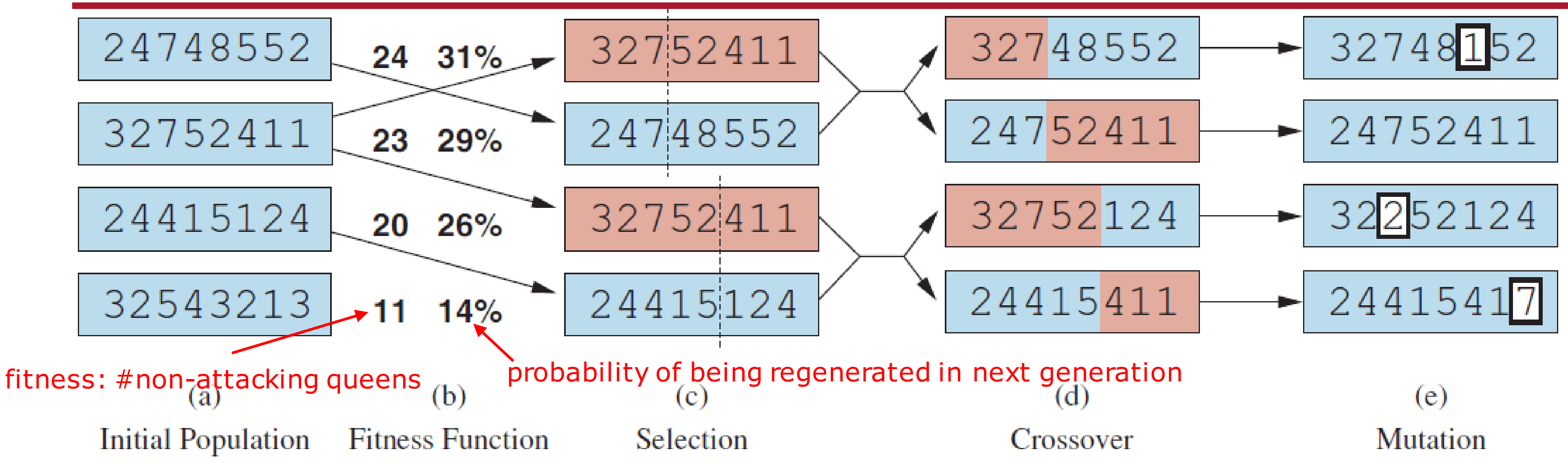
Genetic Algorithms on 8-Queens Problem

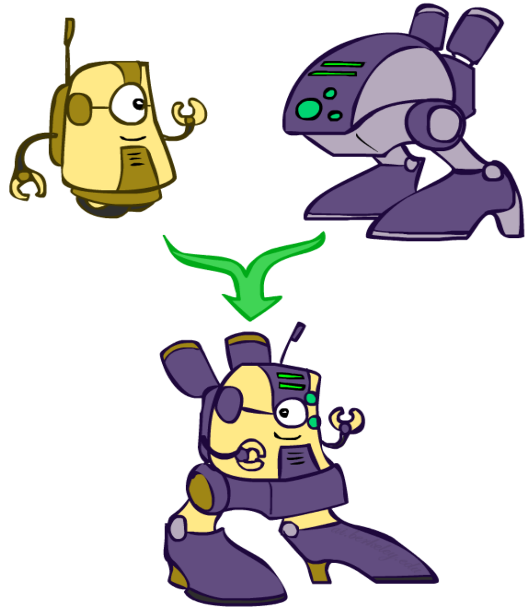




- **Initial Population**: Start with *K* randomly individuals
- **Fitness Function**: Find the number of **non-attacking pairs of queens** for each individual. Higher fitness values are better. Min = 0; Max = 28. **Why?**
- A possible fitness function is the number of non-attacking pairs of queens that we are interested to maximize which has the maximum value $C_2^8 = \frac{8*7}{2} = \mathbf{28}$.
- We have 8 ways to choose the first queen in the pair, and 7 ways to choose the second queen, different from the first. But then we divide by 2, because each pair of queens {*X*,*Y*} was counted twice: taking *X* as the first queen and *Y* as the second, and taking *Y* as the first queen and *X* as the second.

Genetic Algorithms on 8-Queens Problem

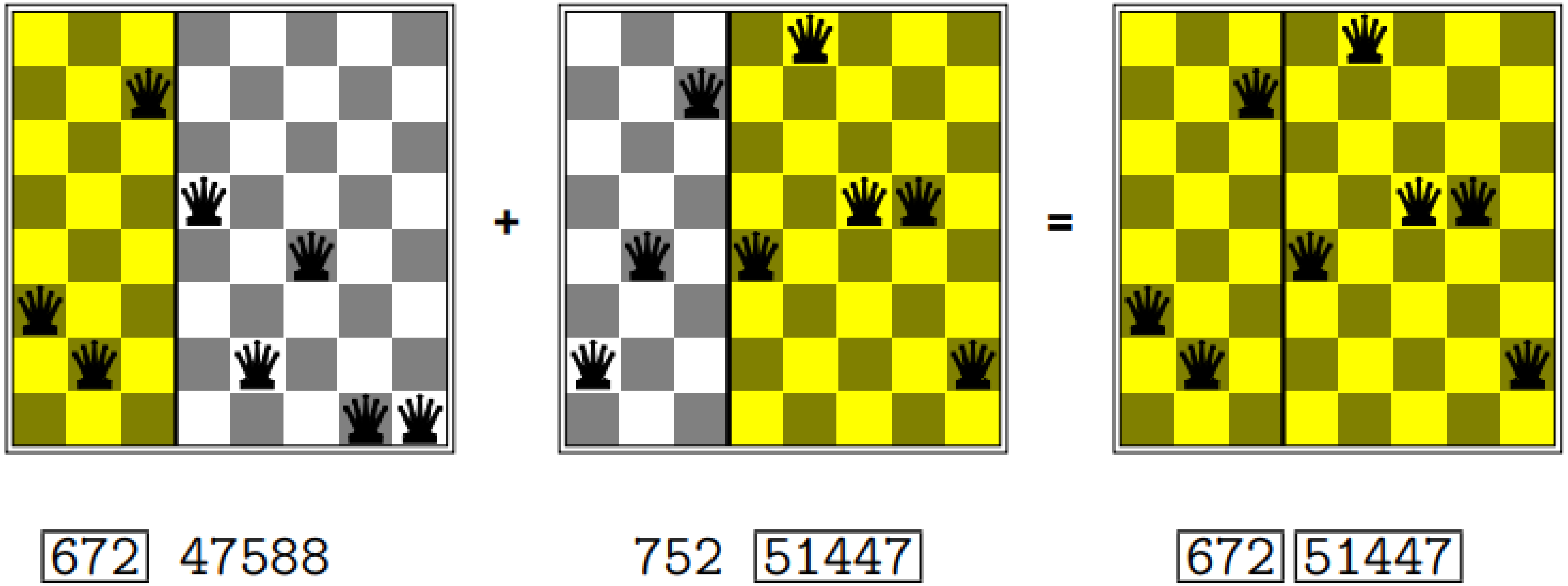




- **Initial Population**: Start with *K* randomly individuals
- **Fitness Function**: Find the number of **non-attacking pairs of queens** for each individual. Higher fitness values are better. Min = 0; Max = 28
- **Selection**: Compute the probability of each individual, e.g., 24 / (24 + 23 + 20 + 11) = 31%
  - Select from *p* individuals with higher probabilities ***OR***
  - Randomly select *n* individuals and then select the *p* most fit ones as parents with higher probabilities
- **Crossover**: Split each of the parent individuals and recombine the parts to form two children.
- **Mutation**: Determine the mutation rate that every bit in its composition is flipped with probability equal to the mutation rate.

Genetic Algorithms on 8-Queens Problem

i'th character = row where i'th queen is located



GAs are suitable to generate a variety of good solutions, but not in finding the optimal solution

8-Queens Problem

map(fun, iter) function returns a map object(which is an iterator) of the results after applying the given function to each item of a given iterable (list, tuple etc.)

```python
def init_population(pop_number, gene_pool, state_length):
    """Initializes population for genetic algorithm
    pop_number  :  Number of individuals in population
    gene_pool   :  List of possible values for individuals
    state_length:  The length of each individual"""
    g = len(gene_pool)
    population = []
    for i in range(pop_number):
        new_individual = [gene_pool[random.randrange(0, g)] for j in range(state_length)]
        population.append(new_individual)

    return population
```

init_population(100, range(8), 8)

[0, 0, 5, 1, 7, 2, 2, 5]

***args. It is used to pass a variable number of arguments to a function**

```python
def genetic_algorithm(population, fitness_fn, gene_pool=[0, 1], f_thres=None, ngen=1000, pmut=0.1):
    """[Figure 4.8]"""
    for i in range(ngen):
        population = [mutate(recombine(*select(2, population, fitness_fn)), gene_pool, pmut)
                      for i in range(len(population))]

        fittest_individual = fitness_threshold(fitness_fn, f_thres, population)
        if fittest_individual:
            return fittest_individual

    return max(population, key=fitness_fn)
```

genetic_algorithm(population, fitness, gene_pool=range(8), f_thres=25)

The number of **non-attacking pairs of queens**

```python
def fitness_threshold(fitness_fn, f_thres, population):
    if not f_thres:
        return None

    fittest_individual = max(population, key=fitness_fn)
    if fitness_fn(fittest_individual) >= f_thres:
        return fittest_individual

    return None
```

```python
def select(r, population, fitness_fn):
    fitnesses = map(fitness_fn, population)
    sampler = weighted_sampler(population, fitnesses)
    return [sampler() for i in range(r)]
```

utils.py

```python
def recombine(x, y):
    n = len(x)
    c = random.randrange(0, n)
    return x[:c] + y[c:]
```

A New Child

pmut = percent for non-mutation

```python
def mutate(x, gene_pool, pmut):
    if random.uniform(0, 1) >= pmut:
        return x

    n = len(x)
    g = len(gene_pool)
    c = random.randrange(0, n)
    r = random.randrange(0, g)

    new_gene = gene_pool[r]
    return x[:c] + [new_gene] + x[c + 1:]
```

The random.randrange(start, stop, step) method returns a randomly selected element from the specified range. <u>stop is not included</u>.

8-Queens Problem

```
from search import *

def fitness(q):
    non_attacking = 0
    for row1 in range(len(q)):
        for row2 in range(row1+1, len(q)):
            col1 = int(q[row1])
            col2 = int(q[row2])
            row_diff = row1 - row2
            col_diff = col1 - col2

            if col1 != col2 and row_diff != col_diff and row_diff != -col_diff:
                non_attacking += 1

    return non_attacking

def main():
    """Initializes population for genetic algorithm
        pop_number   :  Number of individuals in population
        gene_pool    :  List of possible values for individuals
        state_length:   The length of each individual
    """

    population = init_population(100, range(8), 8)
    print(population[:5]) #Display the first 5 individuals. Each individual is a solution state for the 8-Queens Problem

    solution = genetic_algorithm(population, fitness, gene_pool=range(8), f_thres=25)
    print(solution)

    print(fitness(solution))

if __name__ == "__main__":
    main()
```

Genetic Algorithms

[PyGAD - Python Genetic Algorithm! — PyGAD 2.18.1 documentation]

Summary

- Many configuration and optimization problems can be formulated as local search

- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches