FLAME DYNAMICS CHARATERISATION BY CHAOTIC ANALYSIS OF IMAGE SEQUENCES

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1. INTRODUCTION

The goal of the present paper is to characterise the working condition of a combustion system through the analysis of the flame dynamics. This aim is useful to develop systems for monitoring and controlling the combustion phenomena in order to improve the effectiveness of the process and the environmental impact. The combustion is the result of various non-linear processes and includes fluid-dynamical, chemical and physical aspects which render analysis and prediction difficult.

Our approach is intended to identify the working conditions of the combustion plant by the analysis of the signals which are provided by specific sensors (in this case we used a fast digital camera) and whose fluctuations are related to the kind of fuel being used. In order to classify the different conditions, we applied a new analysis technique based on non-linear dynamics which resulted more accurate for this application than others used in similar studies.

2. STATE OF ART OF CHAOTIC ANALYSIS APPLIED TO THE COMBUSTION

In this section we present some of the latest and most important studies of the combustion approached with non-linear dynamic analysis. In general the application of Chaos Theory to real systems introduces a number of problems due to the unavailability of the model of the differential equations and to the presence of noise. This kind of analysis is very innovative in the field of combustion as the first applications we found took place as late as in the current decade.

In 1993 Gorman and Robbins tested a burner who worked with a mixture of methane and air. They [1] acquired images of flames with a fast digital camera in order to classify the four fundamental states of the flames observed during the operations. They built the *attractors* (the representation of the dynamic evolution of the system in the phase space, see next paragraph) by means of signals obtained by extracting targets from the images and noticed some morphological characteristics that emerged from their observations and allowed them to recognise the kind of flame. In spite of this, they could not find any dynamic invariants which could enable them to classify these states; they tried to calculate Lyapunov exponents, global, local and fractal dimensions, without satisfactory results.

The same researchers, studying the burner led to work with *lean mixture*, succeeded eventually in identifying the transition state before flame blowing-out, measuring the amplitudes and the slopes of some characteristic peaks of Power Spectra [2].

In 1995 Daw and Thomas [3] studied the pulsed combustion through the chaotic analysis in order to characterise some particular undesired states which produce peaks of temperature and high NO_x emissions. They were able to recognise these conditions by the entropy and fractal dimension. Based on these results, they [4] could later work out an automatic control system for pulsed combustion burners which changes the working conditions when the plant approaches the critic states.

In general the main problem to solve in these studies is to formalize the deterministic behaviour showed by the chaotic approach with some effective and significant parameters.

All the chaotic studies were conducted by trying to classify the different states by the calculation of the so-called *dynamic invariants*, measurements of the non-linear dynamics independent of initial conditions.

But the *classic* dynamic invariants (Lyapunov, Kolmogorov entropy and fractal dimensions [5,6,7,8,9,10]) present the problem that they need a high number of samples and are very sensitive to noise.

The technique of classification we used in this experiment was introduced in 1996 by Annunziato and Abarbanel [11] with reference to the identification of oil flow regimes in industrial piping. They were able to work out a system which identifies the oil flow regimes through the calculation of some morphological characteristics of the attractors obtained by a signal provided by means of differential pressure transducers. The main advantage of this systems is that it obtains results in real time trough fast calculation, low statistics and low computational cost.

The good results obtained were a starting-point for deepening these studies and for applying this technique to the recognition of the classes of behaviour of other kind of processes [12].

3. SETTING OF THE PROBLEM

The opportunity to extend the use of this methodology to the study of combustion phenomena was derived by a scientific collaboration between CAM-Technology Company (Milan) and ENEA regarding the development of new mixing technology (Mec-System, patent of CAM-TEC) [13]. The Company proved (trough extended experiments) that these fuels have very good performances in terms of limited emissions and good stability when emulsified with the Mec-System mixing technology.

The present investigation is included in a series of studies on the combustion of these fuels (chemical aspects, temperature and velocity mapping, dynamics) Particularly, the goal of the research reported in this paper is the development of a better knowledge of the differences in the flame dynamics and flame diffusion in the combustion chamber for conventional and emulsified (in different ways) fuels. In order to characterize the flame dynamics we used image sensors and chaotic analysis.

The flame dynamics was captured through a fast digital camera installed in front of an optical window on the wall of the combustion chamber and it was connected to an acquisition board, that works at maximum frequency of 830 fps. Since we did not know the characteristic frequencies of the process we were forced to work at maximum frequency, reducing the length of the continue acquisitions.

The sequences of images, of dimensions of 128x128 pixels and of 255 levels of gray, were initially elaborated with traditional image analysis methods in order to extract some basic information about the localization of the most interesting zones of the flame (we calculated the standard deviation and the mean value on the whole sequence of images) and the signal of the gray-level intensity of some selected zones.

Then we chose, as target on which to apply the chaotic analysis, the pixel which assumes the maximum value of the deviation standard because it certainly stands on the flame front. For this zone is the most representative of the flame because of the intermediate reactions which occur on it.

The signals were then filtered with a moving average filter (recommended for nonlinear analysis [14]) in order to reduce external noise and finally normalised with mean value μ= 0 and standard deviation σ=1.

A first interesting result we obtained on the data collected on the industrial burner is that the attractor of the intensity signals related to small zones of the flames is very similar to the one computed on signals related to large zones (1 pixel, 4x4 pixels, 7x7 pixels, 10x10 pixels - see fig. 1). This demonstrates that the local dynamics of the flame contains most of the information we can obtain from the analysis of the full flame dynamics and that the local choice of the target (1 pixel) is important for shifting in the interested area.

The test matrix is composed by a series of about 50 tests made on 5 different fuels (3 for civil use and 2 for industrial use) everyone tested in 3 different working conditions: pure, water emulsified, water and stabilisation additive emulsified. One of our goals was to establish a relation between the chaotic behaviour (dynamics invariants), the flame morphological aspects and the emissions data acquired with a special equipment.

Besides, we needed to reach this aim with fast and simple calculations, to obtain results in real time and thus to design a control system able to identify the performance of the plant and to actuate some control devices for improving and optimising the process.

4. CLASSIFICATION

4.1 Chaotic analysis

For the chaotic processes it is impossible to predict their evolution on long time because they strictly depend on the initial conditions and are irregular in time;

however, they have a regular structure in phase space that shows a deterministic behaviour, where phase space is determined by a variable and its successive derivatives. This structure, named *attractor*, represents the non-linear dynamic evolution of the system and is built by the set of trajectories and orbits described by the state vector **x**(t).

So, every chaotic process has a characteristic dimension determined by the number of differential equations needed to represent it. It is possible to demonstrate that this number is always at least equal to three for chaotic processes [14].

The main problem for real applications of chaotic analysis is due to the unavailability of the differential equation system, so that we can neither derive **x**(t) nor build the attractors.

In order to solve the problem, in 1981 Takens and Manè [15] introduced the *Embedding Theorem*; some of main consequences are :

- it is possible to build the attractors simply by replacing the derivatives with the delayed repetitions of only one variable of the system (s(t), s(t+T),…, s(t+nT) , *Pseudo Embedded Space*);
- in order to evaluate the main characteristic of a process it is sufficient to built the attractor of the only one of the variables above indicated, because each of them is strictly connected to the others and “contributes” to the other attractors.

In the same time this theorem represented a great revolution for real applications because it became possible to study a real system with chaotic analysis simply by using data acquired with measuring instruments.

So the attractor is obtained by using the signal values delayed of a time lag, characteristic of the system. The reconstruction procedure consists of the determination of the optimal time lag (T) and the dimensionality of the system (n) which provides the best and most defined structure of the attractor.

In the nonlinear dynamics literature, several methods have been developed in order to determine these parameters. The time lag is derived by the Average Mutual Information (AMI) a sort of nonlinear autocorrelation function of the signal and the dimension is derived by the Global (/Local) False Nearest Neighbors (GFNN/LFNN) [14]. We used a tool (CSP) developed by Institute of Nonlinear Science (UCSD, Ca) in order to compute these parameters.

The results showed homogenous characteristics over all the tests because everyone had a time lag between 12 and 20 msec and both global and local dimensions of 4-6. These similitudes represent an advantage because it is possible to assume a constant time lag for all the conditions by simplifying the analysis.

4.2 The attractor shape description

Although the attractor appears as a very complicated solid with an undefined shape, when we compare attractors related to different conditions it is often possible to note some differences due to their deterministic behaviour. The common problem, in these cases, is to find a formalism for these differences in order to obtain an accurate characterisation. The techniques reached in literature usually consist of the calculation of the dynamic invariants which have the disadvantage of being very sensitive to the noise unavoidably introduced by the measuring instruments, and of requiring large statistics not always available.

The proposed method has been developed on the morphological characteristics of the attractor, using and enlarging a technique developed by Annunziato, Abarbanel [11] for the morphological characterisation of two-dimensional attractors.

One of the main problems is connected with the great influence of the time lag (T) on the attractor. In figure 2 we represent an attractor with different T, just to show how its choice changes strongly the shape of the attractor and how the attractor loses its coherent structure when its value increases above a characteristic value (optimal time lag).

The use of different time lags for each test can induce large errors connected to a not exact choice of the time lag, in the same time the use of a constant time lag is not correct because each test has its specific lag.

The idea was to compute a sort of moments of inertia, extending in order and dimensions. We built a series of shape descriptors, named *dynamic moments* [11]; everyone computed by varying the time lag in order to obtain the evolution of every shape characteristic in function of T.

Generally, the order of dynamics moments used as discriminants should be equal to the number of the dimensions of the chaotic process. However, with respect to the classification if the chaotic process has high dimensions but the classes are enough well separated, it is possible to extract discriminant characteristics by computing the dynamics moments of lower dimensions.

For instance in the case under examination, as it will report in the paragraph 5, we used the dynamic moments of third order although the dimensions of the process were equal to 4 or 6.

For example in fig. 3 we show the visual comparison of three attractors, projected on two dimensions, related to the three different mixtures of the same fuels (pure, emulsified and additive). Observing the picture it is evident that the pure case presents the attractor broader than the others around the principal bisector. This morphological characteristic it is common to all fuels and it suggested to us the choice of the discriminant parameters.

In the following we first propose the dynamic moments in two and three dimensions. Then we extend the calculation to a generic number n of dimensions. In every case the technique consists in individuating some symmetry references from whom the distances of every point of the attractor are computed. Obviously for two and three dimensions we have easily individuated geometric references , while for n dimensions we lose every real representation and we can just reduce the calculation to an analytic procedure.

Sometimes, depending on the processes we are studying, it could be enough to compute the dynamic moments in two or three dimensions, if some differences appear on the lowest dimensions. In other cases it might be necessary to look for the discriminant characteristics in the highest dimensions and to extend the calculations to extract the finest characteristics of the attractors.

In general we do not know neither which are the dimensions and how many they are, nor of which morphological characteristics the differences consist, and therefore we are compelled to make vary attempts before individuating the parameters we need.

4.3 Two dimensions

When we work in two dimensions we are projecting the attractor on the plane, so we consider only two components of the signal: x_i=s(it) and y_i=s(it+T) where t is the acquisition time and T is the time lag which we vary from 0 to a high value that makes the components totally independent (we recommend at least 150).

We assumed, as symmetry references, two axes, the bisector of first-third (called *principal axis*) and second-fourth quadrant, and the origin. From them, we compute the distance of every point (i) of the attractor:

$$d_{1,i} = \frac{\sqrt{2}}{2}\left(x_i - y_i\right) \qquad [4.1]$$

$$d_{2,i} = \frac{\sqrt{2}}{2}\left(x_i + y_i\right) \qquad [4.2]$$

$$d_{3,i} = \sqrt{x_i^2 + y_i^2} \qquad [4.3]$$

With every distance it is possible to define some moments of order j:

$$M_{m,j}(T) = \frac{\sum_{i=1}^{N} d_{m,i}{}^{j}}{N} \qquad [4.4]$$

where N is the number of samples and m=1,2,3 the distance considered.

Starting from T=0, the attractor is compressed on the principal axis (fig. 4.1); when T increases, these moments describe the morphological evolution during the unfolding process of the attractor. The moments evolve from the linear value (for T=0) to nonlinear one. Finally we can outline that the even moments are ever positive and describe the scatter of the attractor, while the odd moments are symmetry descriptors.

It is possible also to compute mixed moments like the following:

$$M_{s,4} = \frac{\sum_{i=1}^{N} d_{1,i}{}^{3} d_{2,i}}{N} \qquad [4.5]$$

This is a descriptor of the symmetry with respect to the principal axis, but the zones of the attractor which are more distant from the secondary axis are more weighted.

4.4 Three dimensions

Although 2D moments can be accurate enough to characterize chaotic processes, sometimes it could be necessary to extend moment calculation to higher dimensions in order to have parameters more sensitive to the fine characteristics of the attractors.

In three dimensions we introduce a third component z_i=s(it+2T) and three symmetry references represented by three perpendicular planes along the directions of the attractor whose descriptions (naming *directrix* the bisector on the space whose attractor unfold around) and equations are:

- plane A (perpendicular to plane xy and crossing directrix): x-y=0;
- plane B (perpendicular to directrix and crossing the origin): x+y+z=0;
- plane C (perpendicular to planes A and B): x+y-2z=0.

We computed the three distances from these planes and the distance from origin:

$$d_{A,i} = \frac{\sqrt{2}}{2}\left(x_i - y_i\right) \qquad [4.6]$$

$$d_{B,i} = \frac{\sqrt{3}}{3}\left(x_i + y_i + z_i\right) \qquad [4.7]$$

$$d_{C,i} = \frac{\sqrt{6}}{6}\left(x_i + y_i - 2z_i\right) \qquad [4.8]$$

We can introduce, with the same notations of 2D, some dynamic moments of generic order j:

$$d_{O,i} = \sqrt{x_i^2 + y_i^2 + z_i^2} \qquad [4.9]$$

$$M_{h,j}(T) = \frac{\sum_{i=1}^{N} d_{h,i}{}^{j}}{N} \qquad [4.10]$$

where h=A,B,C and O refers to the distance considered.

In the same way of 2D, these moments can be mixed in various forms in order to extract specific morphological characteristics.

4.5 Generic N dimensions

Since chaotic processes can be very complicated, they might need very high dimensions to be described; so it’s important to have at one’s disposal a strong instrument able to discover the morphological differences on the highest dimensions. Besides, even if 2D and 3D moments resulted accurate enough in some precedent studies, their use is from a theoretic point of view a prioritary renounce on the part of the information, as the chaotic processes have at least dimensions equal to three.

In this case we cannot obviously obtain any help from geometric references and the calculations consist just of an analytic procedure which is an extension of the 2D cases.

The idea of the generalisation derived from the observation of the distances from origin and from the directrix computed on 2D and 3D (see eq. 4.3, 4.9 and eq. 4.1, 4.6). Comparing them it is possible to note that we can generalise them to any number of dimensions, in this way:

$$d_{O,i} = \sqrt{\sum_{k=1}^{d} \left(s\left(i + (k-1)T\right)\right)^2} \qquad [4.11]$$

$$d_{AX,i} = \frac{\sqrt{d}}{d} \sqrt{\sum_{k=1}^{d-1} \left[\left(s\left(i+(k-1)T\right) - s\left(i+kT\right)\right)^2 + \ldots + \left(s\left(i+(k-1)T\right) - s\left(i+(d-1)T\right)\right)^2\right]} \qquad [4.12]$$

where d is the dimension considered (global dimension of the process).
As before, we can now define some dynamic moments of generic order j:

$$M_{h,j}(T) = \frac{\sum_{i=1}^{N} {d_{h,i}}^{j}}{N} \qquad [4.13]$$

where h=O or AX.

We can consider these moments a valid and strong instrument for chaotic analysis because they should reveal the most specific and remote characteristic of the dynamic evolution.

4.6 The unfolding descriptors

Next problem to solve is to extract from the computed moments some discriminant parameters in order to classify the processes examined. The idea is to find parameters which should be independent from the time lag because we have seen it could change significantly between different tests.

Independently of the dimensions considered, the dynamic moment in general appears like a curve composed by a first zone approximately linear and a second zone in which it fluctuates in a not correlated pattern. The two zones sometimes are separated by the first maximum or minimum, otherwise the curve simply degenerates without peaks appearing. In all case it is possible to separate the two behaviours at the “transition time lag” of about 20-50.

If we analyse the attractor shape during this increase of the time lag, the first zone corresponds to the creation of the attractor structure and the second zone corresponds to a degenerative process in which the attractor loses the coherence in the trajectories and it assumes a shape typical of very high (not correlate) time lags.

From this analysis we conclude that: a) the first zone is very stable and it represents the unfolding process from the linear contraction (T=0) to the nonlinear structure, b) the second zone, corresponding to the first maximum or minimum (if it exists, otherwise we consider the point from where the curve degenerates), represents a *transition lag* in which the process of genesis of the structure of the attractor is inverted.

At the moment, the relation between the *transition lag* and the "optimal time lag" [14] it is not very clear although a strong relation is expected.

From the comparison of the curves of some moments computed for different tests it is possible to distinguish the different dynamic behaviours. So we need to extract some parameters (dynamic invariants) in order to formalise the differences and to be able to make a classification of the processes.

It is difficult to give a general and exact algorithm, valid in all cases, because it depends on the kind of curves related to the specific experiment, but surely it should concern the characterisation of the described first zone.

If all tests show a very clear transition lag and a sufficiently linear first zone it is possible to use the slope of the first zone as the linear regression from 0 up to the transition lag (used in the application [11]). In our case we could take advantage of the homogeneous characteristics of the tests studied which assumed very similar optimal time lags. So we could assume as dynamic invariants the value of the moments at T=15, which it represented a valid value for all tests (for little change of T the attractor shape change in a negligible way).

5. RESULTS

Particularly, the fig. 3 shows the most evident characteristic which distinguishes the three conditions (true for each fuel), the amplitude around the principal axis.

With reference to the classification both options, we needed to introduce the dynamic moments of the third order ($M_{A,3}$, $M_{B,3}$).

Particularly, in the case of the classification option a) the use of these moments allows to obtain the separation of the three classes of conditions, as it is shown in fig. 4 where the symbol identifies the fuel and the texture the combustion condition. However the vertical dotted lines on the graph point out that $M_{1,2}(T)$ is able to accurately classify the three classes with few errors; in fact we obtained a total effectiveness of 92% of recognition over all tests. The class of pure fuel is well separated from the others, while the emulsified and additive classes are partially superimposed. This confirms that the principal difference of performance depends from the water being mixed with the fuel or not.

The figure 5 shows results of the classification option b), related to the typology of the fuel. In this case three of them (A, C and D) are well separated, while B and E are located in the same zone, as distinct from the others. This characteristic may be explained with the fact that B and E fuels differ only in sulphur content. It is probably necessary to analyse higher dimension moments in order to formalise this finer difference. However, by associating B and E in the same class, the effectiveness of the classification resulted of 100% for emulsified and additive and 95% for pure fuel.

Beside, we want to underline that in the case of option a), if we exam in different graphs only the results of each fuel in the different conditions the classes are completely separated and the effectiveness of recognition is 100% over all test.

Finally, we compared the proposed technique with other techniques based on chaotic analysis (fractal dimension, Lyapunov coefficients and dimension, Kolmogorov entropy) and with the more traditional image analysis. This comparison clearly demonstrated that, at least in the specific application under examination, the proposed methodology gives best results in terms of effectiveness and accuracy of classification (see following table).

Fuel B / **Methods**	Pure	Emulsified	Additive Emuls	Proposed by
Dynamic moments	19.9	10.2	12.6	Present paper
Lyapunov max espon.	0.37	0.48	0.47	Gorman et. Al.
Lyapunov dimensions	3.98	3.99	3.93	Gorman et Al.
Entropia Kolmorogov	10.0	10.0	10.0	Daw et Al.
Fractal dimensions	Not univocal result	Not univocal result	Not univocal result	Daw et Al.

We consider these results as encouraging; however we are trying to improve them by having at our disposal longer sequences of data and by using neural networks to formalise the information derived by the dynamic moments.

6. CONCLUSIONS

The main conclusion is that the proposed approach, based on the description of the morphology evolution of the attractor in its unfolding process, has a very interesting potential for identifying the behaviour of a system.

More investigations are necessary to verify the possibility to extend this conclusion to other conditions and applications and to understand the physical meaning of the transition lag.

An important aspect to be underlined is that this method required a low computational cost. In fact we used only a small area of the image (just 1 pixel), relatively short image sequence (less than 4000 frames in less than 5 seconds) and the calculations to obtain the morphological parameters resulted very short. These characteristics and the obtained results can give the opportunity to use this methodology for the design of systems for the diagnostics of the combustion plants.

In order to give an idea of the potentiality of this technique the graph of fig. 4 has been redesigned plotting each different test by a circle having diameter proportional to the NO_x reduction (fig. 6). A system based only on the control of the specific position of the current working point on the graph should inform the operator of the actual state (normal or critical) of the plant and suggest specific actions in order to obtain desired performances.

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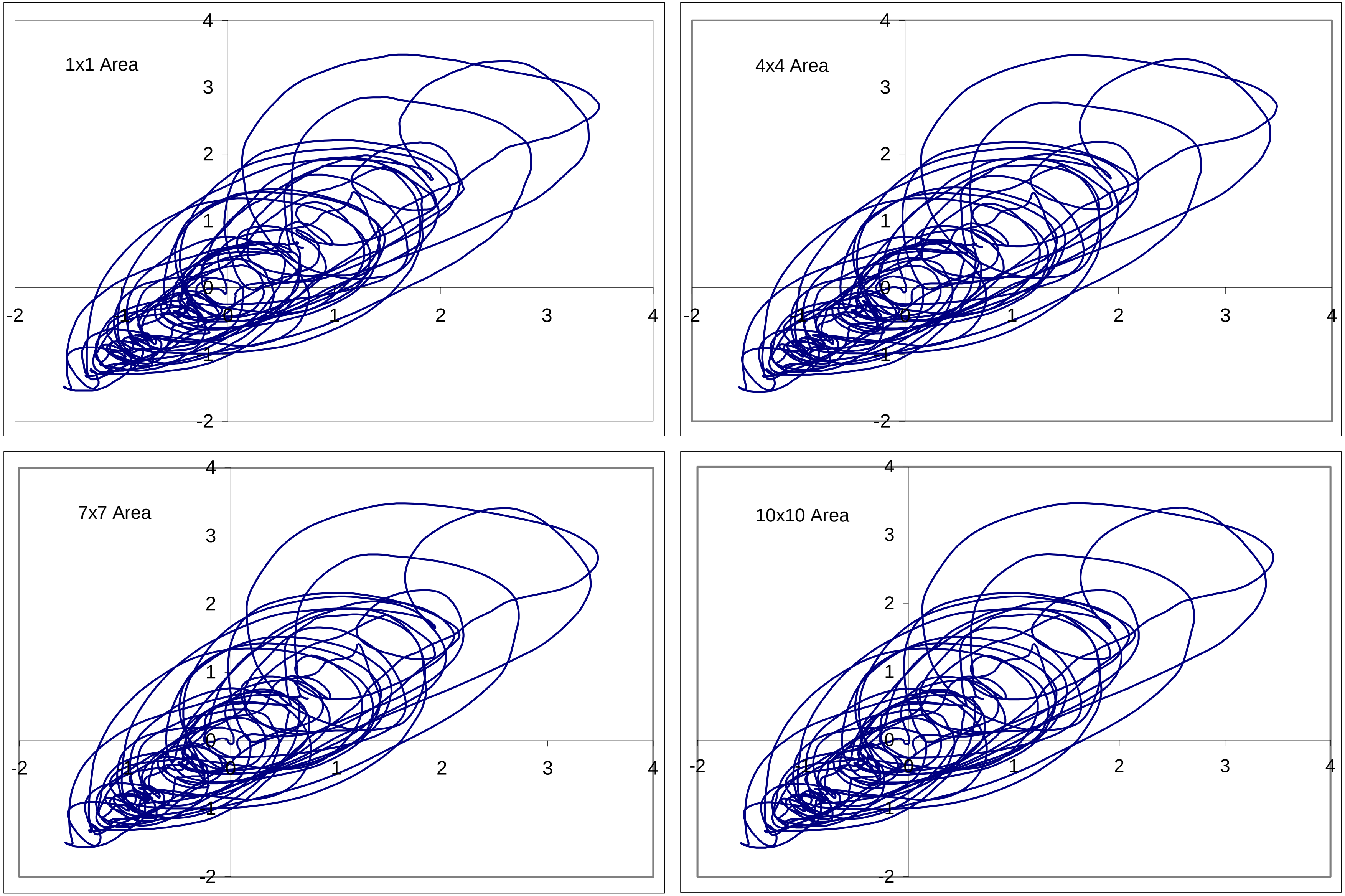

Figure 1. Examples of attractors reconstructed from signals of the intensity related to zones of the flames with different area: 1x1 pixels, 4x4, 7x7, 10x10. The shape of these attractors are very similar.

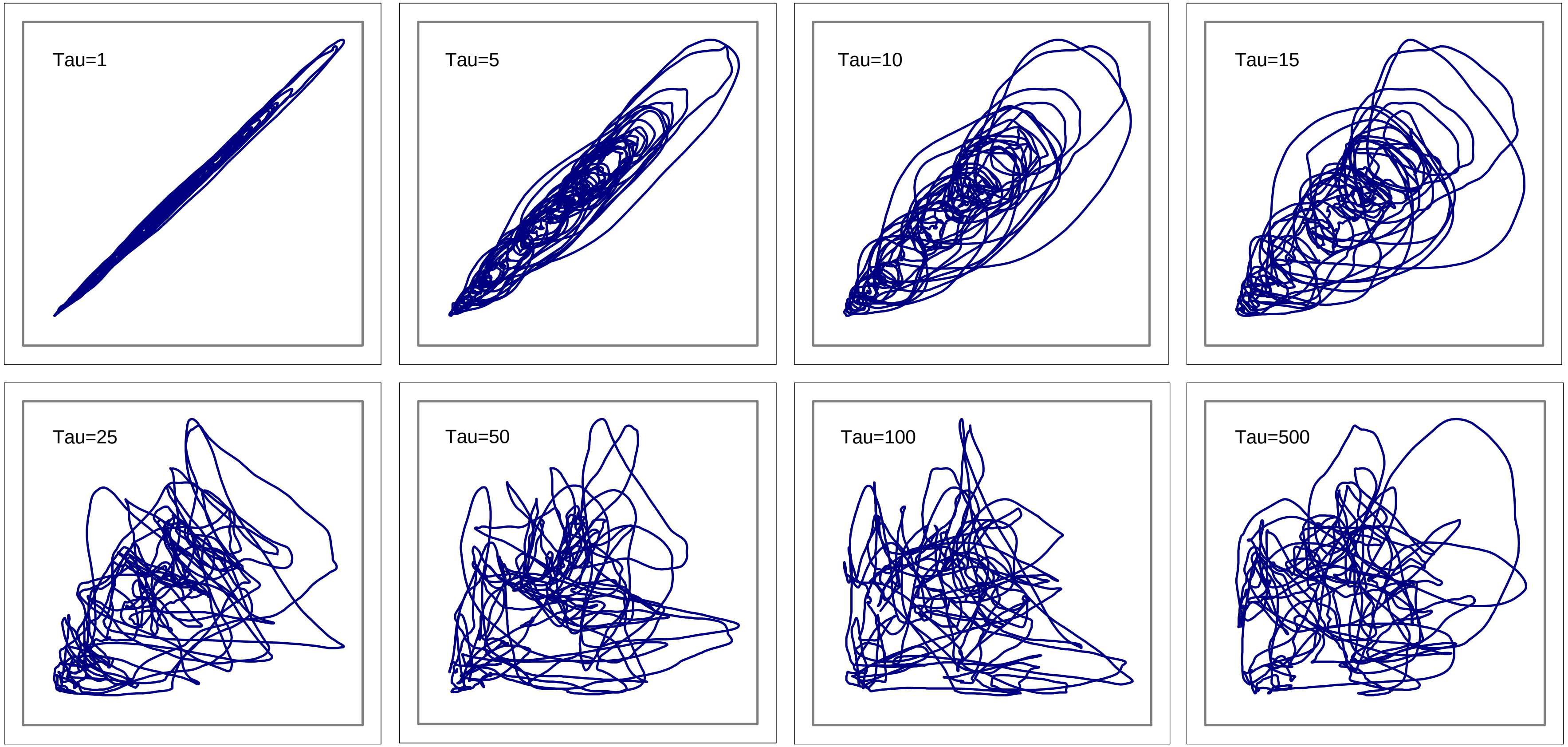

Figure 2. Unfolding process of an attractor of a additive-emulsified fuel.

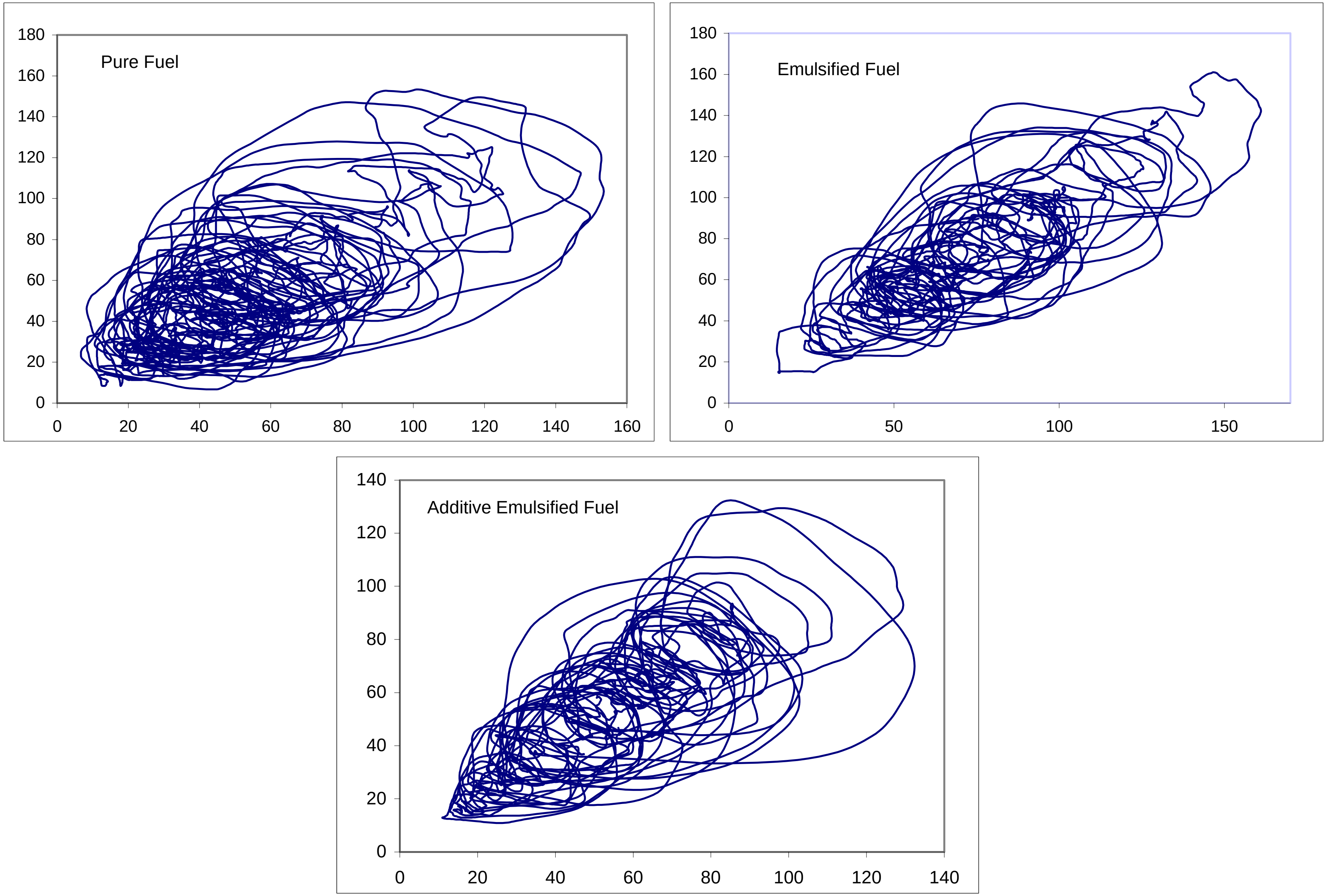

Figure 3. Examples of attractors related to the different conditions:pure, emulsified and additive emulsified.

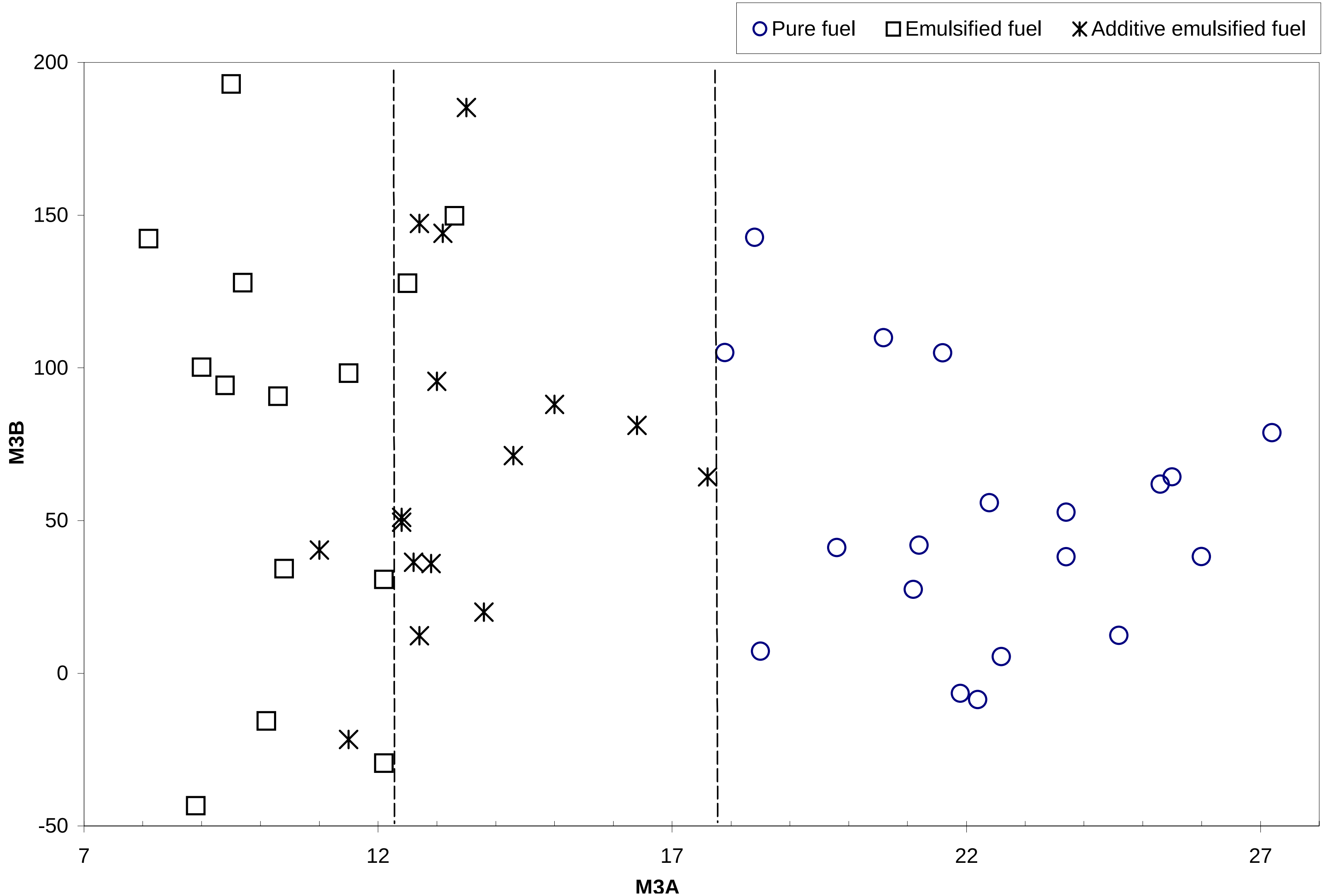

Figure 4. Classification according to the combustion conditions of the fuels: pure, emulsified and additive emulsified.

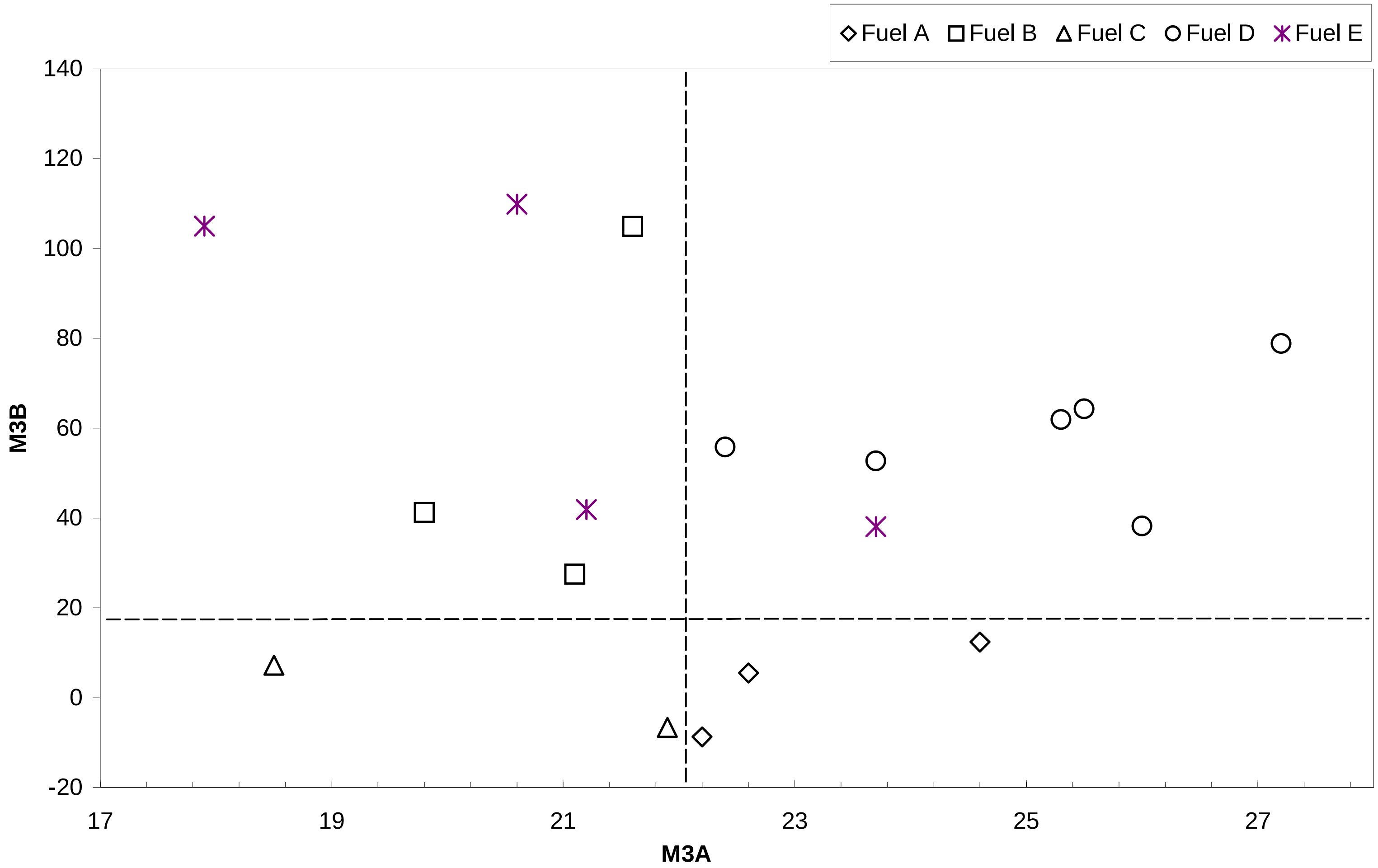

Figure 5. Classification according to the typology of the fuels: A,C,D are well separated, B and E are partially superimposed.

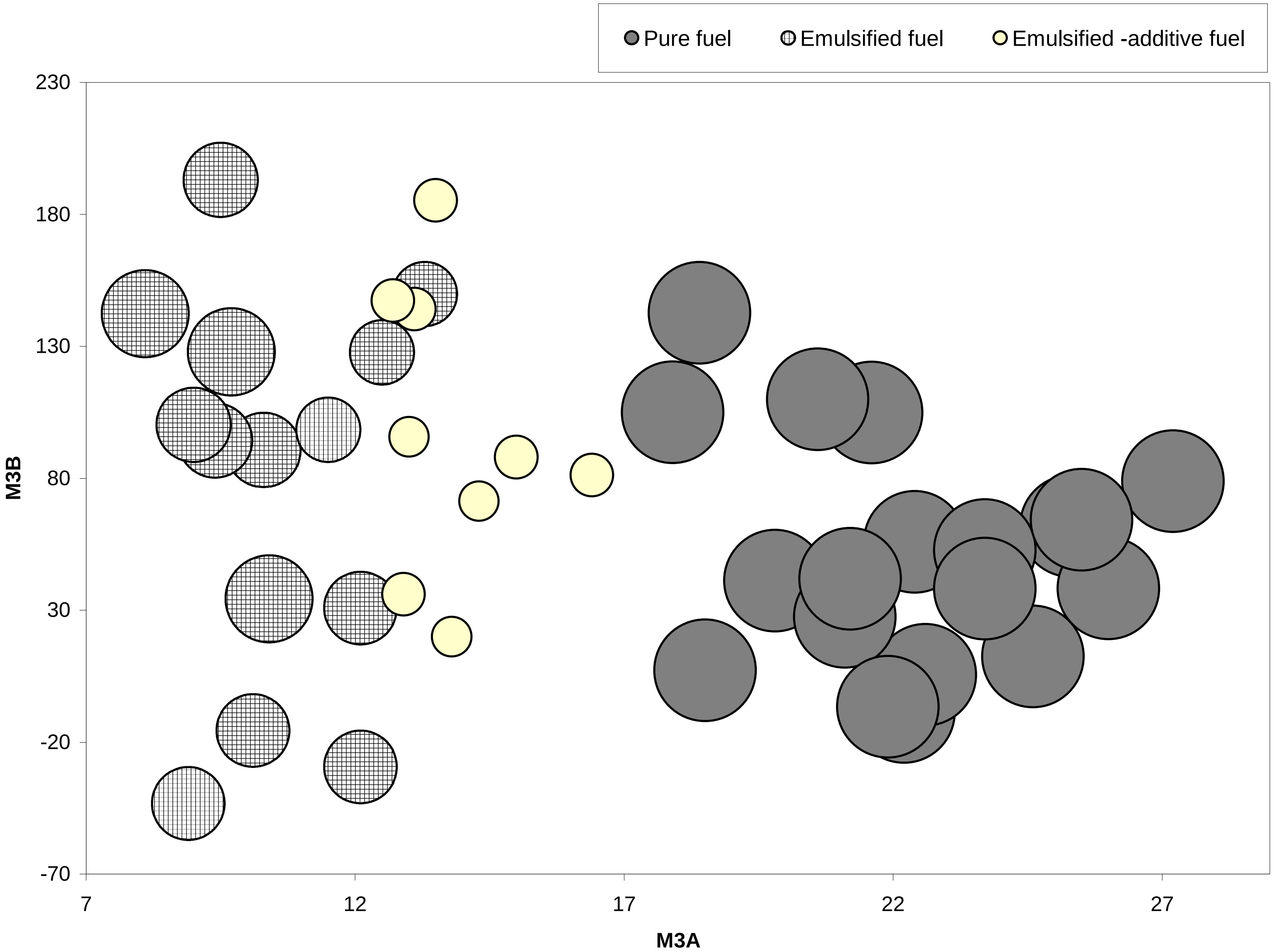

Figure 6. Classification of the combustion conditions according to the NO_x emission reduction; the circles represent the dynamic moments of the tests and their diameters are proportional to the NO_x emissions.