Problem Sheet 2.3

Exercise 1

a) $\lambda x.\lambda y.x\ (y\ y\ y)\ x\ true\ true$
$\lambda y.true\ (y\ y\ y) true\ true$
$true(true\ true\ true) true$
$\lambda x.\lambda y.x\ (\lambda x.\lambda y.x\ true\ true)\ \lambda x.\lambda y.x$
$\lambda x.\lambda y.x\ (\lambda y.true\ true)\ \lambda x.\lambda y.x$
$\lambda x.\lambda y.x\ true\ \lambda x.\lambda y.x$
$\lambda y.true\ true$
$true$

b) $\lambda x.\lambda y.x\ (y\ y\ y)\ x\ true\ false$
$\lambda y.true\ (y\ y\ y) true\ false$
$true(\lambda x.\lambda y.y\ false\ false) true$
$true\ false\ true$
$\lambda x.\lambda y.x\ false\ true$
$\lambda y.false\ true$
$false$

$\lambda x.\lambda y.x\ (y\ y\ y)\ x\ false\ true$
$false(\lambda x.\lambda y.x\ true\ true) false$
$false\ true\ false$
$\lambda y.y\ false$
$false$

$\lambda x.\lambda y.x\ (y\ y\ y)\ x\ false\ false$
$false(\lambda x.\lambda y.y\ false\ false) false$
$false\ false\ false$
$false$

Exercise 2

a) $succ\ 1 \underset{\beta}{\rightarrow}^{*} 2$
$\lambda n.\lambda f.\lambda x.f(nfx)\ 1 \underset{\beta}{\rightarrow}$
$\lambda f.\lambda x.f(1fx) \triangleq$
$\lambda f.\lambda x.f\big((\lambda f.\lambda x.fx)fx\big) \underset{\beta}{\rightarrow}$
$\lambda f.\lambda x.f\big((\lambda x.fx)x\big) \underset{\beta}{\rightarrow}$
$\lambda f.\lambda x.f(fx) \triangleq 2$

b) $add\ 2\ 3$

$\lambda m.\lambda n.\lambda f.\lambda x.mf(nfx)\ 2\ 3 \underset{\beta}{\rightarrow}$

$\lambda f.\lambda x.2f(3fx) \triangleq$

$\lambda f.\lambda x.2f\left(\left(\lambda f.\lambda x.f\left(f(fx)\right)\right)fx\right)\underset{\beta}{\rightarrow}$

$\lambda f.\lambda x.2f\left(f\left(f(fx)\right)\right)\underset{\beta}{\rightarrow}$

$\lambda f.\lambda x\left(\lambda x.f(fx)\right)\left(f\left(f(fx)\right)\right)\underset{\beta}{\rightarrow}$

$\lambda f.\lambda xf(f\left(f\left(f(fx)\right)\right)) \triangleq 5$

c) $mult\ 2\ 2$

$\lambda m.\lambda n.\lambda f.\lambda x.m(nf)x\ 2\ 2 \underset{\beta}{\rightarrow}$

$\lambda f.\lambda x.2(2f)x \triangleq$

$\lambda f.\lambda x.2\left(\lambda x.f(fx)\right)x \triangleq \lambda f.\lambda x.\left(\lambda f.\lambda x.f(fx)\right)\left(\lambda x.f(fx)\right)x \underset{\beta}{\rightarrow}$

$\lambda f.\lambda x.\left(\lambda x.f\left(f\left(f(fx)\right)\right)\right)x \underset{\beta}{\rightarrow}$

$\lambda f.\lambda x.f\left(f\left(f(fx)\right)\right) \triangleq 4$