

Problem Sheet 8.2

Exercise 1

- a) A lambda term is in normal form if there are no applications left to reduce the term. This means that a lambda term is in normal form if it has variables, like x , which are already normal, abstractions, like $\lambda x.N$ where N is normal or MN expressions where both are normal and M is not an abstraction. Therefore, if a term is in normal form it must have the following shape:

$$\lambda x_1 \dots \lambda x_n. x N_1 \dots N_k$$

Where each N_i must be in normal form.

- b) $o \rightarrow o$
 $\lambda x^o. x$

$$o \rightarrow o \rightarrow o$$

$$\lambda x^o. \lambda y^o. x$$

$$\lambda x^o. \lambda y^o. y$$

$$(o \rightarrow o) \rightarrow o \rightarrow o$$

Church Numerals

$$\lambda x^{o \rightarrow o}. x$$

$$(o \rightarrow o) \rightarrow o$$

None