

Problem Sheet 4.2

$$\begin{aligned}
 \text{a) } & (\lambda f x. f f x) \left(\lambda a. \left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) a) \right) \right) \right) \\
 & T = \left(\lambda a. \left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) a) \right) \right) \right) \\
 & \lambda x. T T x \xrightarrow{\beta} \\
 & \lambda x. \left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) T) \right) \right) x \xrightarrow{\beta} \\
 & \lambda x. \left((\lambda y. y) ((\lambda z. z) T) \right) x \xrightarrow{\beta} \\
 & \lambda x. T x \xrightarrow{\beta} \\
 & \lambda x. \left(\left(\lambda a. \left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) a) \right) \right) \right) x \right) \xrightarrow{\beta} \\
 & \lambda x. \left(\left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) x) \right) \right) \right) \xrightarrow{\beta} \\
 & \lambda x. \left((\lambda y. y) ((\lambda z. z) x) \right) \xrightarrow{\beta} \\
 & \lambda x. ((\lambda z. z) x) \xrightarrow{\beta} \\
 & \lambda x. x
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (\lambda f x. f f x) \left(\lambda a. \left((\lambda x. x) \left((\lambda y. y) ((\lambda z. z) a) \right) \right) \right) \xrightarrow{\beta} \\
 & (\lambda f x. f f x) \left(\lambda a. \left((\lambda x. x) ((\lambda y. y) a) \right) \right) \xrightarrow{\beta} \\
 & (\lambda f x. f f x) \left(\lambda a. ((\lambda x. x) a) \right) \xrightarrow{\beta} \\
 & (\lambda f x. f f x) (\lambda a. a) \xrightarrow{\beta} \\
 & \lambda x. (\lambda a. a) (\lambda a. a) x \xrightarrow{\beta} \\
 & \lambda x. ((\lambda a. a) x) \xrightarrow{\beta} \\
 & \lambda x. x
 \end{aligned}$$