Problem Sheet 2.3: Programming in the Lambda Calculus

Exercise 1: The term

$$F \triangleq \lambda x.\lambda y.\ x\ (y\ y\ y)\ x$$

implements a standard function on the Booleans.

a) Calculate the normal form of $F\ \mathsf{true}\ \mathsf{true}$.

b) Calculate the normal form of F applied to the other three combinations of two Boolean values.

c) What is the Boolean function computed by F?

Exercise 2: Recall the definitions for **successor**, **addition**, and **multiplication**.

$$\begin{aligned} \mathsf{succ} &\triangleq \lambda n.\lambda f.\lambda x.f(n\ f\ x) \\ \mathsf{add} &\triangleq \lambda m.\lambda n.\lambda f.\lambda x.m\ f\ (n\ f\ x) \\ \mathsf{mult} &\triangleq \lambda m.\lambda n.\lambda f.\lambda x.m\ (n\ f)\ x \end{aligned}$$

a) Verify that $\mathsf{succ}\ 1 \rightarrow_\beta^* 2$, using the Church encoding of the numbers.

b) Without calculating, write down the normal form of the term $\mathsf{add}\ 2\ 3$. Verify that your answer is correct by performing the beta reductions.

c) Perform the same task for $\mathsf{mult}\ 2\ 2$.