

Problem Sheet 8.2: Normalisation

Exercise 1: The weak normalization theorem tells us that every term has a normal form. We know from the Church-Rosser theorem that a term's normal form is unique. Therefore in the typed λ -calculus, the possible terms of a given type are just the normal forms. If we can work out all the normal forms of a type then we know what values that type possesses. In this exercise you will do just that.

a) Convince yourself that if a term is in normal form, it has the shape:

$$\lambda x_1. \dots \lambda x_n. x N_1 \dots N_k$$

where each N_i is in normal form. (Either n or k may be zero, that is, a term might just look like $x N_1 \dots N_k$ or like $\lambda x_1. \lambda x_2. x$, for instance.)

b) Using this characterization, write down all the closed normal forms (i.e. normal forms with no free variables) of the following types:

- $o \rightarrow o$
- $o \rightarrow o \rightarrow o$
- $(o \rightarrow o) \rightarrow o \rightarrow o$
- $(o \rightarrow o) \rightarrow o$