Problem Sheet 5.2: Fixed Points and Recursion

Exercise 1:

a) Reduce the fixed-point combinator Y for three β-steps. You can abbreviate $\lambda x.f(xx)$ by W.

b) How is it possible that a term YF has a normal form?

c) Try to predict what the fixed point of the function $\lambda x.\lambda y.y$ is.

d) Compute the normal form of $Y(\lambda x.\lambda y.y)$, if there is one.

Exercise 2: Recall from the lecture on induction that, as in Peano arithmetic, we can define addition recursively if we can add one, subtract one and test for zero:

$$\begin{aligned} \mathsf{add}\; m\; 0 &= m \\ \mathsf{add}\; m\; (n+1) &= \mathsf{succ}\; (\mathsf{add}\; m\; n) \end{aligned}$$

a) Express this definition as an equation that we would like add to satisfy in the λ-calculus. You may assume the existence of the term pred which subtracts one from a Church numeral.

b) Define add as a fixed point of a function.