Problem Sheet 3.3

Exercise 1

a) $(\lambda xyz.\, xz(yz))KK \xrightarrow[\beta]{}$

$\lambda z.\, Kz(Kz) \xrightarrow[\beta]{}$

$\lambda z.\, ((\lambda xy.\, x)z)(Kz) \xrightarrow[\beta]{}$

$\lambda z.\, (\lambda y.\, z)(Kz) \xrightarrow[\beta]{}$

$\lambda z.\, z$

b) $(\lambda xyz.\, xz(yz))IK \xrightarrow[\beta]{}$

$\left(\lambda yz.\, IZ(yz)\right)K \xrightarrow[\beta]{}$

$\lambda z.\, (\lambda x.\, x)z(Kz) \xrightarrow[\beta]{}$

$\lambda z.\, z(Kz) \xrightarrow[\beta]{}$

$\lambda z.\, z(\lambda y.\, z)$

c) $\left(\lambda xyz.\, xz(yz)\right)SS \xrightarrow[\beta]{}$

$(\lambda yz.\, Sz(yz))S \xrightarrow[\beta]{}$

$\lambda z.\, Sz(Sz) =$

$\lambda z.\, (\lambda xyz.\, xz(yz))z(Sz) \equiv_\alpha$

$\lambda z.\, \left(\lambda xyw.\, xw(yw)\right)z(Sz) \xrightarrow[\beta]{}$

$\lambda z.\, \left(\lambda yw.\, zw(yw)\right)(Sz) \xrightarrow[\beta]{}$

$\lambda z.\, \lambda w.\, zw(Szw) =$

$\lambda z.\, \lambda w.\, zw\left((\lambda xyz.\, xz(yz))zw\right) \equiv_\alpha$

$\lambda z.\, \lambda w.\, zw\left((\lambda xyt.\, xt(yt))zw\right) \xrightarrow[\beta]{}$

$\lambda z.\, \lambda w.\, zw((\lambda yt.\, zt(yt))w) \xrightarrow[\beta]{}$

$\lambda z.\, \lambda w.\, zw(\lambda t.\, zt(wt))$

Exercise 2

a) $(\lambda x.\, \lambda y.\, xyy)W \xrightarrow[\beta]{}$

$\lambda y.\, Wyy \xrightarrow[\beta]{}$

$\lambda y.\, ((\lambda x.\, \lambda y.\, xyy)yy) \xrightarrow[\beta]{}$

$\lambda y.\, ((\lambda y.\, yyy)y) \xrightarrow[\beta]{}$

$\lambda y.\, yyy$

b) $WWW =$

$(\lambda x.\, \lambda y.\, xyy)WW \xrightarrow[\beta]{}$

$(\lambda y.\, Wyy)W =$

$(\lambda y.(\lambda x.\lambda y.xyy)yy)W \equiv_\alpha$

$(\lambda y.(\lambda z.\lambda a.zaa)yy)W \underset{\beta}{\rightarrow}$

$(\lambda y.(\lambda a.yaa)y)W \underset{\beta}{\rightarrow}$

$(\lambda y.yyy)W \underset{\beta}{\rightarrow}$

WWW

This expression does not have a normal form.