

1 Axioms

- $L_0 : \varphi \vee \neg\varphi$
- $L_1 : \varphi \rightarrow (\psi \rightarrow \varphi)$
- $L_2 : (\psi \rightarrow (\varphi_1 \rightarrow \varphi_2)) \rightarrow ((\psi \rightarrow \varphi_1) \rightarrow (\psi \rightarrow \varphi_2))$
- $L_3 : (\varphi \wedge \psi) \rightarrow \varphi$
- $L_4 : (\varphi \wedge \psi) \rightarrow \psi$
- $L_5 : \varphi \rightarrow (\psi \rightarrow (\psi \wedge \varphi))$
- $L_6 : \varphi \rightarrow (\varphi \vee \psi)$
- $L_7 : \psi \rightarrow (\varphi \vee \psi)$
- $L_8 : (\varphi_1 \rightarrow \varphi_3) \rightarrow ((\varphi_2 \rightarrow \varphi_3) \rightarrow ((\varphi_1 \vee \varphi_2) \rightarrow \varphi_3))$
- $L_9 : \neg\varphi \rightarrow (\varphi \rightarrow \psi)$
- $L_{10} : \forall\nu\varphi(\nu) \rightarrow \varphi(\tau)$
- $L_{11} : \varphi(\tau) \rightarrow \exists\nu\varphi(\nu)$
- $L_{12} : \text{If } \nu \notin \text{free}(\psi), \text{ then } \forall\nu(\psi \rightarrow \varphi(\nu)) \rightarrow (\psi \rightarrow \forall\nu\varphi(\nu))$
- $L_{13} : \text{If } \nu \notin \text{free}(\psi), \text{ then } \forall\nu(\varphi(\nu) \rightarrow \psi) \rightarrow (\exists\nu\varphi(\nu) \rightarrow \psi)$
- $L_{14} : \tau = \tau$
- $L_{15} : (\tau_1 = \tau'_1 \wedge \dots \wedge \tau_n = \tau'_n) \rightarrow R(\tau_1, \dots, \tau_n) \rightarrow R(\tau'_1, \dots, \tau'_n)$
- $L_{16} : (\tau_1 = \tau'_1 \wedge \dots \wedge \tau_n = \tau'_n) \rightarrow F(\tau_1, \dots, \tau_n) = F(\tau'_1, \dots, \tau'_n)$

2 Inference rules

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} \text{ Modus Ponens (MP)}$$

$$\frac{\varphi}{\forall\nu\varphi} \text{ Generalisation } (\forall) *$$

* if ν does not occur free in any non-logical axiom.