

FIR Filters — Seminar 6

Professor-Style Theory Summary & Cheat Sheet

1. FIR Systems and Impulse Responses

Theory

A finite-impulse-response (FIR) discrete-time LTI system is defined by

$$y[n] = \sum_{k=0}^M b_k x[n - k].$$

The impulse response has *finite* duration:

$$h[n] = 0 \quad \text{for } n \notin \{0, \dots, M\}.$$

Initial rest is assumed.

FIR = no feedback;
response ends after M samples.

Example / Result

For

$$h[n] = \delta[n - 1] - 2\delta[n - 4],$$

the difference equation is

$$y[n] = x[n - 1] - 2x[n - 4].$$

2. Impulse Response Construction

Theory

The impulse response is the output when

$$x[n] = \delta[n].$$

A shifted delta produces delayed samples:

$$\delta[n - k] \Rightarrow h[k].$$

Each delta term reveals one tap.

Method

To obtain $h[n]$ from a difference equation:

1. Set $x[n] = \delta[n]$.
2. Substitute into $y[n] = \sum b_k x[n - k]$.
3. Read off nonzero samples at $n = k$.

Example / Result

For

$$y[n] = x[n] + 2x[n - 2] - 4.5x[n - 3] + 5x[n - 5],$$

the impulse response is

$$h[n] = \delta[n] + 2\delta[n - 2] - 4.5\delta[n - 3] + 5\delta[n - 5].$$

3. Convolution Representation**Theory**

Any input can be decomposed as

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - k].$$

By linearity & time invariance:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n - k].$$

Output = weighted,
shifted
copies of
 $h[n]$.

Method

Convolution with FIR:

1. Identify nonzero $h[k]$.
2. For each k , form $x[n - k]$.
3. Multiply by $h[k]$ and sum.

Example / Result

Two-tap FIR:

$$h[n] = \delta[n] - 2\delta[n - 3].$$

Output:

$$y[n] = x[n] - 2x[n - 3].$$

4. Block Diagrams of FIR Filters**Theory**

An M -tap FIR filter uses:

M delays, $M + 1$ multipliers, M adders.

A k -sample delay implements $x[n - k]$.

Each tap corresponds to one coefficient b_k .

Method

To draw the structure:

1. Create delay chain: $x[n] \rightarrow z^{-1} \rightarrow z^{-1} \dots$
2. Tap outputs at $x[n-k]$.
3. Multiply each tap by b_k .
4. Sum all branches into $y[n]$.

Example / Result

For coefficients $(1, 0, 2, -4.5, 0, 5)$: Delays produce $x[n], x[n-1], \dots, x[n-5]$; taps at $k = 0, 2, 3, 5$.

5. Frequency Response of FIR Filters

Theory

For an FIR filter,

$$H(e^{j\omega}) = \sum_{k=0}^M b_k e^{-j\omega k}.$$

Complex exponentials are eigenfunctions:

$$x[n] = C e^{j\omega n} \Rightarrow y[n] = H(e^{j\omega}) C e^{j\omega n}.$$

Filter acts as gain & phase shift at frequency ω .

Method

To compute $y[n]$ for $x[n] = C e^{j\omega n}$:

1. Evaluate $H(e^{j\omega}) = \sum b_k e^{-j\omega k}$.
2. Multiply amplitude: $A = |H| |C|$.
3. Add phase: $\varphi = \arg(C) + \arg(H)$.
4. Output: $y[n] = A e^{j(\omega n + \varphi)}$.

Example / Result

Given

$$y[n] = 2x[n] + 4x[n-1] + 2x[n-2], \quad x[n] = 11e^{j(0.3\pi n + 0.5\pi)},$$

$$H(e^{j0.3\pi}) = 2 + 4e^{-j0.3\pi} + 2e^{-j0.6\pi},$$

$$y[n] = H(e^{j0.3\pi}) 11e^{j(0.3\pi n + 0.5\pi)}.$$

6. Sinusoids Through FIR Filters

Theory

A cosine can be written as:

$$A \cos(\omega n + \phi) = \Re\{Ae^{j(\omega n + \phi)}\}.$$

FIR filters preserve frequency; amplitude and phase are modified by $H(e^{j\omega})$.

Method

To obtain output for a cosine input:

1. Convert to complex exponential.
2. Multiply by $H(e^{j\omega})$.
3. Convert back to cosine using real part.

Example / Result

Input:

$$x[n] = A \cos(\omega n + \phi).$$

Output:

$$y[n] = |H(e^{j\omega})| A \cos(\omega n + \phi + \arg H(e^{j\omega})).$$

7. Time Shifts and Scaling

Theory

Time shift:

$$x[n - n_0] \Rightarrow y[n - n_0].$$

Scaling:

$$\alpha x[n] \Rightarrow \alpha y[n].$$

LTI
property
used
repeatedly
in FIR
exercises.

Example / Result

If $y_1[n]$ is output for $x_1[n]$, then

$$x_2[n] = \alpha x_1[n - n_0] \Rightarrow y_2[n] = \alpha y_1[n - n_0].$$