IIR Filters — Seminar 9 & 10

Professor-Style Theory Summary & Cheat Sheet

1. Discrete-Time LTI Systems

Theory

A causal discrete-time LTI system is described by a linear constant-coefficient difference equation (LCCDE):

$$y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m].$$

Initial rest is assumed unless stated otherwise.

2. Z-Transform and System Function

Theory

The z-transform of a sequence $x[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}.$$

Method

To compute the system function $H(z)$:

1. Take the z-transform of the difference equation.
2. Use z-shift property: $\mathcal{Z}\{x[n-k]\} = z^{-k}X(z)$.
3. Solve for $H(z) = \dfrac{Y(z)}{X(z)}$.

Example / Result

$$y[n] = ay[n-1] + x[n] \quad \Rightarrow \quad H(z) = \frac{1}{1 - az^{-1}}.$$

3. Poles, Zeros, and Stability

Theory

For

$$H(z) = \frac{B(z)}{A(z)},$$

zeros are roots of $B(z)$ and poles are roots of $A(z)$.

Theory

A causal LTI system is BIBO stable if and only if all poles satisfy

$$|p_i| < 1.$$

Zeros do not affect stability.

4. Impulse Response and Convolution

Theory

The impulse response $h[n]$ is the output of the system to $x[n] = \delta[n]$.

Method

Any input can be decomposed as:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

By linearity and time invariance:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

5. First-Order IIR Systems

Theory

A first-order IIR system has the form:

$$y[n] = ay[n-1] + bx[n].$$

Example / Result

Impulse response:

$$h[n] = ba^n u[n], \qquad |a| < 1.$$

Step response:

$$y[n] = \frac{1-a^{n+1}}{1-a}.$$

6. Second-Order Resonators

Theory

Complex conjugate poles at

$$z = re^{\pm j\omega_0}$$

produce damped oscillations.

Method

The corresponding denominator polynomial is:

$$A(z) = 1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}.$$

Example / Result

Impulse response:

$$h[n] = Cr^n \cos(\omega_0 n + \phi)u[n].$$

7. Delayed Feedback Systems

Theory

A delayed feedback system:

$$y[n] = -ay[n-K] + x[n]$$

has system function

$$H(z) = \frac{1}{1 + az^{-K}}.$$

Example / Result

Poles satisfy:

$$z^K = -a$$

and are equally spaced in angle on a circle of radius $|a|^{1/K}$.

8. Exam Strategy

Method

1. Write the difference equation.
2. Compute $H(z)$.
3. Find poles and zeros.
4. Check stability.
5. Interpret time-domain behavior from pole locations.