ChatGPT

Thought for 16s

Short answer: yes—but only if you're careful.

The math

Acceleration is the second time-derivative of displacement, so in the Fourier domain

$$\hat{a}(\omega) = -\omega^2 \hat{x}(\omega) \quad \Rightarrow \quad \hat{x}(\omega) = -\frac{\hat{a}(\omega)}{\omega^2}.$$

So for each FFT bin at frequency f_k (with $\omega_k = 2\pi f_k$):

- **Complex spectrum:**
  $\hat{X}_k = -\dfrac{\hat{A}_k}{(2\pi f_k)^2}$ for $f_k > 0$.
  (This introduces a 180° phase flip; magnitudes scale by $1/(2\pi f)^2$.)
- **Magnitude (amplitude) spectrum:**
  $|\hat{X}(f)| = \dfrac{|\hat{A}(f)|}{(2\pi f)^2}$.
- **Power spectral density (PSD):**
  $S_{xx}(f) = \dfrac{S_{aa}(f)}{(2\pi f)^4}$.
  (For amplitude spectral density, ASD $= \sqrt{\mathrm{PSD}}$:
  $\mathrm{ASD}_x(f) = \mathrm{ASD}_a(f)/(2\pi f)^2$.)

Practical gotchas (important!)

1. **DC and low-frequency blow-up:** Division by f^2 explodes near $f = 0$.
   - Detrend/remove the mean, and usually **high-pass** the acceleration (or the resulting displacement) before/after integration.
   - Set the $f = 0$ bin to zero.
2. **Gravity & tilt:** A vertical accelerometer sees g. If the device tilts, gravity projects into the axis and looks like low-frequency "motion."
   - Remove g (mean) and, if possible, IMU orientation to compensate; otherwise rely on a conservative high-pass.

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ChatGPT can make mistakes. Check important info.