

Entropic entanglement criteria in phase space

① Entanglement is hard to measure!

Two systems:
 A and B
 x_A, p_A
 x_B, p_B
 $[x_j, p_k] = i\hbar \delta_{jk}$

"Husimi" distributions
 Q_A and Q_B

Measure entanglement?
 Can be measured jointly in experiments:
 e.g., quantum optics, ultracold atoms

=) measure x_A, p_A, x_B, p_B simultaneously

③ Our method

We use "twisted coordinates"

Compare: Q_{\pm} against: Q_A and Q_B

$Q_{\pm} = Q_A + Q_B$

Our witness:

$$S(Q_{\pm}) \geq \ln(e^{S(Q_A)} + e^{S(Q_B)}) \geq 1 + \ln 2$$

"Strong" criteria "Weak" criteria

Violation = entanglement

② Entropy

In information theory, entropy measures "missing" information

for example: unfair coin flip

Entropy

Probability (Heads)

Always tails → - no missing info!

Why entropies?
 Entropy can be used to witness entanglement!

From our Husimi distributions, we can measure an entropy:
 $S = -\int \frac{dx_j dp_j}{2\pi} Q_j \ln Q_j$

④ Example states

NOON states
 Q_{\pm} , Q_A , Q_B

Schrödinger cat states
 Q_{\pm} , Q_A , Q_B

Strong criteria: violated

Weak criteria: violated

⑤ Future direction: incorporate memory; add more modes

Applications: Spinor Bose-Einstein condensates