Degree Distribution in Quantum Complex Networks

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Waterloo, May 20, 2013

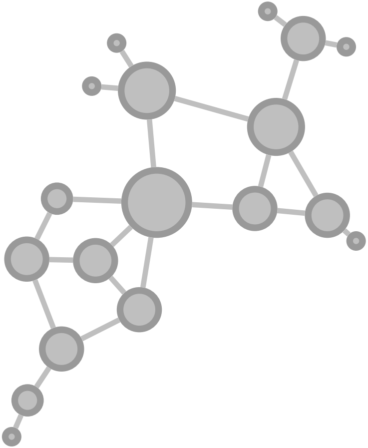
Workshop on
quantum computation
and complex networks

Institute for Scientific Interchange
Quantum physics division

Outline

- Short intro to degree distribution (stochastic random walks)
- Quantum generator and probability distribution (quantum *steady state*)
- Quantum correction to the classical behavior
- Quantumness bounds

The Graph



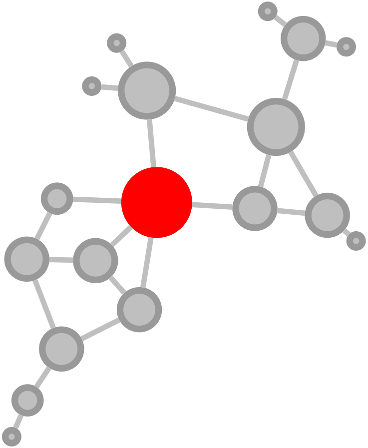
A : Adjacency matrix

D : Matrix with node degrees on the diagonal (sum of A's columns)

L : Laplacian matrix $(A - D)$

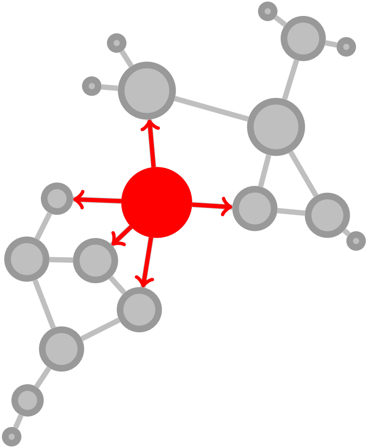
$$A = \begin{pmatrix} & & & & & & \vdots & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & \vdots & & & & & \\ \dots & 1 & 1 & 1 & \dots & & 0 & \dots & 1 & 1 & \dots & \\ & & & & & & \vdots & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & \vdots & & & & & \end{pmatrix}$$

Random Walks



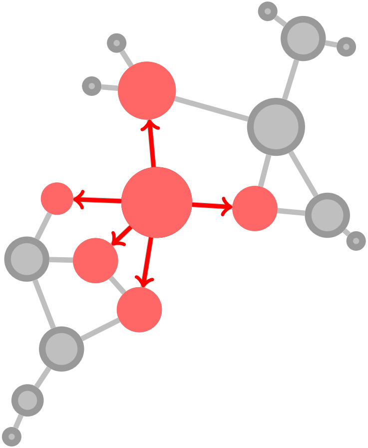
- Start from a node

Random Walks



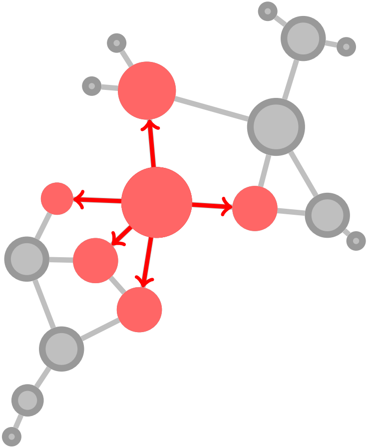
- Start from a node
- Unbiased choise of a neighbor

Random Walks



- Start from a node
- Unbiased choise of a neighbor
- Move to it

Random Walks



- Start from a node
- Unbiased choise of a neighbor
- Move to it

Transition Matrix:

$$\begin{pmatrix} & & & & & \vdots & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & \vdots & & & & \\ \ldots & 1/3 & 1/3 & 1/2 & \ldots & 0 & \ldots & 1/3 & 1/4 & \ldots \\ & & & & & \vdots & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & \vdots & & & & \end{pmatrix} = AD^{-1}$$

Degree and Stochastic Processes

Continuous time random walk.
Stochastic generator:

$$H_C = LD^{-1} = AD^{-1} - 1$$

Probability distribution at time t:

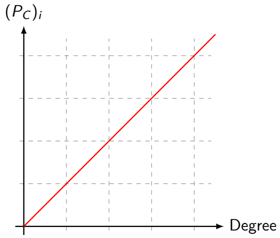
$$P_C(t) = \mathrm{e}^{H_C t} P_C(0)$$

The eigenvector with zero eigenvalue is:

$$D|\hat{1}\rangle = \bar{\phi}_0 = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix}$$

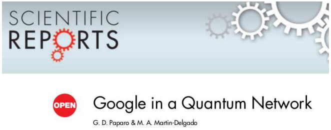
Steady State

$$(P_C)_i(t \to \infty) = \frac{\bar{\phi}_0}{\|\bar{\phi}_0\|_1}$$



Walks: Stochastic vs. Quantum

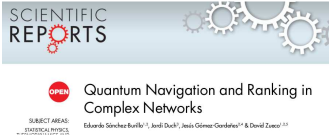
Walks: Stochastic vs. Quantum


SCIENTIFIC REPORTS

OPEN

Google in a Quantum Network

G. D. Paparo & M. A. Martin-Delgado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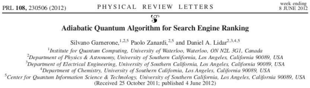
Quantum Navigation and Ranking in Complex Networks

SUBJECT AREAS:
STATISTICAL PHYSICS,

Eduardo Sánchez-Burillo[1,2], Jordi Duch[3], Jesús Gómez-Gardeñes[2,4] & David Zueco[1,2,5]


PRL 108, 230506 (2012) PHYSICAL REVIEW LETTERS week ending 8 JUNE 2012

Adiabatic Quantum Algorithm for Search Engine Ranking

Silvano Garnerone,[1,2,5] Paolo Zanardi,[2,5] and Daniel A. Lidar[2,3,4,5]

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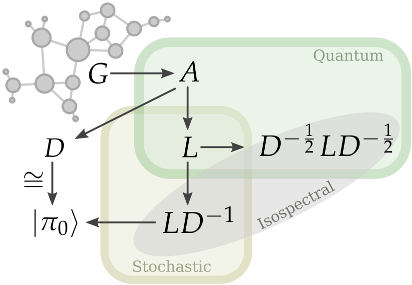
(Received 25 October 2011; published 4 June 2012)

Quantum Dynamics

Generator: Wath is the most natural correspondence between a stochastic and a quantum generator?

Steady State: How to define the long-time behavior of the system?

Quantum Generator



Quantum generator:

$$H_Q = D^{-\frac{1}{2}} L D^{-\frac{1}{2}}$$

with the same spectrum as LD^{-1}.
The probability distribution at time t:

$$(P_Q)_i(t) = |\langle i | \mathrm{e}^{-\mathrm{i}H_Q t} | \Psi_0 \rangle|^2$$

The eigenvector corresponding to the zero eigenvalue is:

$$\bar{\phi}_0 = \begin{pmatrix} \sqrt{d_1} \\ \sqrt{d_2} \\ \vdots \\ \sqrt{d_n} \end{pmatrix}$$

Long Time Behavior

Probability

Use long time average of the probability:

$$(P_Q)_i = \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{d}t \; \left|\langle i|\Psi_t\rangle\right|^2$$

Long Time Behavior

Probability

Use long time average of the probability:

$$(P_Q)_i = \lim_{T \to \infty} \frac{1}{T} \int_0^T \mathrm{d}t \; |\langle i | \Psi_t \rangle|^2$$

Initial State

Evenly distributed initial state:

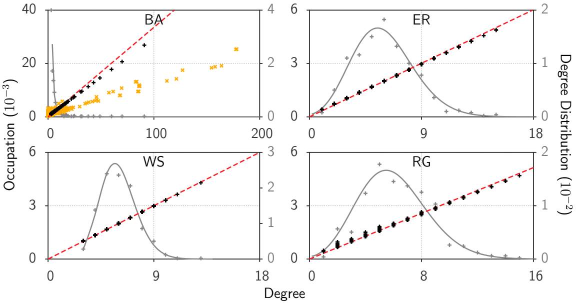
$$|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle$$

Probability Distribution

Limiting probability

Quantum probability distribution and stochastic distribution for long-time averages

P_Q vs. P_C



BA: Barabási-Albert ER: Erdös-Renyi WS: Watts-Strogatz RG: Random Geometric

Quantum Correction

$$(P_Q)_i = \sum_k \langle i | \Pi_k | \Psi(0) \rangle$$

$k \neq 0$ $\quad$ $k = 0$

$$(P_Q)_i = \varepsilon (\tilde{P}_Q)_i + (1 - \varepsilon)(P_C)_i$$

where:

$$\varepsilon = 1 - \left| \langle \phi_0 | \Psi(0) \rangle \right|^2$$

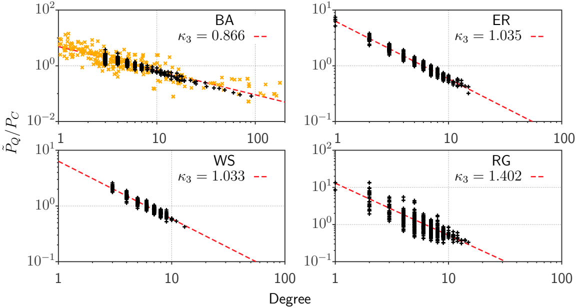
fixing the initial state:

$$\varepsilon = 1 - \frac{\langle \sqrt{d} \rangle^2}{\langle d \rangle} = 1 - \frac{e^{H_{1/2}}}{N}$$

Quantum Correction

network	ε
BA	0.130
ER	0.043
RG	0.040
WS	0.016

Inhomogeneous degree distribution increases ε

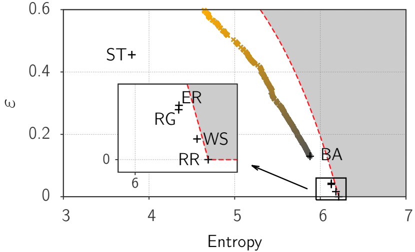


Entropy Bound

The Entropy of the normalized degree illustrates the homogeneity of the degree distribution,

$$\varepsilon \leq 1 - \frac{e^{H_1}}{N}$$

where H_1 is the Shannon Entropy.



Recap

- Relation of quantum walk with topology (degree distribution)
- Separation of quantum walk in classical part and quantum correction through a topological parameter
- Bound to the quantum correction

Thanks to:

- Jacob Biamonte
- Tomi Johnson
- Piotr Migdal
- Sabre Kais

email: `mauro.faccin@isi.it`