1 Vector analysis

1.1 General

1.1.1 Gradient

$$grad\, w = \nabla w = \left(\frac{dw}{dl} \mathbf{u}_l \right)_{max} \tag{1}$$

maximum directional derivative.

1.1.2 Divergence

$$div\, \mathbf{F} = \nabla \cdot \mathbf{F} = \lim_{\Delta\tau \to 0} \frac{1}{\Delta\tau} \oiint \mathbf{F} \cdot d\mathbf{s} \tag{2}$$

Outward flow per unit volume

1.1.3 Curl

$$curl\, \mathbf{F} = \nabla \times \mathbf{F} = \lim_{\Delta s \to 0} \frac{1}{\Delta s} \left[\mathbf{u}_n \oint \mathbf{F} \cdot d\mathbf{l} \right]_{max} \tag{3}$$

maximum circulation per unit area

1.1.4 Laplacian

$$div\, (grad\, w) = \nabla^2 w = \nabla \cdot \nabla w \tag{4}$$

1.2 Rectangular coordinates

$$\nabla w = \mathbf{u}_x \frac{\partial w}{\partial x} + \mathbf{u}_y \frac{\partial w}{\partial y} + \mathbf{u}_z \frac{\partial w}{\partial z} \tag{5}$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{6}$$

$$\nabla \times \mathbf{F} = \mathbf{u}_x \left(\frac{\partial F_z}{\partial\, y} - \frac{\partial F_y}{\partial\, z} \right) + \mathbf{u}_y \left(\frac{\partial F_x}{\partial\, z} - \frac{\partial F_z}{\partial\, x} \right) + \mathbf{u}_z \left(\frac{\partial F_y}{\partial\, x} - \frac{\partial F_x}{\partial\, y} \right) \tag{7}$$

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \tag{8}$$

1.3 Cylindrical coordinates

$$\nabla w = \mathbf{u}_\rho \frac{\partial w}{\partial \rho} + \mathbf{u}_\phi \frac{1}{\rho}\frac{\partial w}{\partial \phi} + \mathbf{u}_z \frac{\partial w}{\partial z} \tag{9}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho F_\rho\right) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \tag{10}$$

$$\begin{aligned}
\nabla \times \mathbf{F} \quad = \quad & \mathbf{u}_\rho \left(\frac{1}{\rho}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right) && (11)\\
& + \mathbf{u}_\phi \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho}\right) && (12)\\
& + \mathbf{u}_z \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho F_\phi\right) - \frac{1}{\rho}\frac{\partial F_\rho}{\partial \phi}\right] && (13)
\end{aligned}$$

$$\nabla^2 w = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial w}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 w}{\partial \phi^2} + \frac{\partial^2 w}{\partial z^2} \tag{14}$$

1.4 Spherical coordinates

$$\nabla w = \mathbf{u}_r \frac{\partial w}{\partial r} + + \mathbf{u}_\theta \frac{1}{r}\frac{\partial w}{\partial \theta} + \mathbf{u}_\phi \frac{1}{r \sin\theta}\frac{\partial w}{\partial \phi} \tag{15}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(F_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi} \tag{16}$$

$$\begin{aligned}
\nabla \times \mathbf{F} \quad = \quad & \mathbf{u}_r \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(F_\phi \sin\theta\right) - \frac{\partial F_\theta}{\partial \phi}\right] && (17)\\
& + \mathbf{u}_\theta \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}\left(r F_\theta\right)\right] && (18)\\
& + \mathbf{u}_\phi \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r F_\theta\right) - \frac{\partial F_r}{\partial \theta}\right] && (19)
\end{aligned}$$

$$\nabla^2 w = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial w}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 w}{\partial \phi^2} + \frac{\partial^2 w}{\partial z^2} \tag{20}$$

$$\nabla^2 w = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial w}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial w}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 w}{\partial \phi^2}\,. \tag{21}$$