Sensitivity Analysis of Inductive Wireless Power Transfer Links Using the Bilinear Theorem

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***Abstract*—**

***Index Terms*—inductive wireless power transfer, coupling factor, resonance, network sensitivity, bilinear theorem.**

I. Introduction

This work seems to be stopped.

II. Effect of components' variations

The considered network function is denoted by H and it is assumed that it is a transfer function in terms of voltages or currents or immittances. The component value which is subject to variations is denoted by F. The sensitivity is defined as:

$$S_F^H = \frac{\frac{\partial H}{H}}{\frac{\partial F}{F}} = \frac{\partial \ln H}{\partial \ln F}\,. \tag{1}$$

Alternatively, from (1), the variation of the network function with respect to a variation of a component can be obtained as

$$\frac{\partial H}{\partial F} = S_F^H \, \frac{H}{F}\,. \tag{2}$$

In general, for each change in the component value F, a new evaluation of the network function is required, i.e. one has to perform a new analysis of the network. However, by applying the bilinear theorem, it is possible to evaluate the network function H without making any further analysis.

A. The bilinear theorem

The bilinear theorem is reported in the following, while its demonstration is provided in the appendix. Any network function H of a linear and permanent network can be expressed, in terms of the impedance of one of its bipolar component, by the following bilinear expression

$$H = \frac{Z_{th}\, H_0 + Z\, H_\infty}{Z_{th} + Z}\,, \tag{3}$$

where

- Z is the impedance of the F component;
- Z_{th} is the Thevenin impedance at the ends of the component of interest;
- H_0 is the network function evaluated when the component is short–circuited (i.e. when $Z = 0$);
- H_∞ is the network function evaluated when the component is left open circuited (i.e. when $Z = \infty$).

The application of the bilinear theorem provides a simple method for evaluating the sensitivity. By deriving (3) and using (2), we obtain for the sensitivity:

$$S_F^H = \frac{Z\, Z_{th}}{(Z_{th} + Z)}\, \frac{(H_\infty - H_0)}{(H_0\, Z_{th} + H_\infty\, Z)}\,. \tag{4}$$

Alternatively, explicit evaluation of (2) provides

$$\frac{\partial H}{\partial Z} = \frac{(H_\infty - H_0)\; Z_{th}}{(Z_{th} + Z)^2} \tag{5}$$

In order to apply the bilinear to an inductive wireless power transfer network we consider next the case of a ladder network.

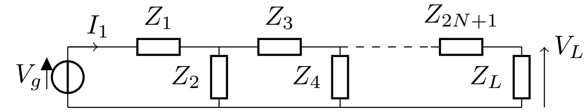

Fig. 1. General ladder network.

III. Sensitivity analysis for a ladder network

The case of a ladder network occurs frequently in inductive power transfer. It is therefore advantageous to consider an example of application for this case. Let us consider, with reference to Fig. 1, as network function $H = V_L/V_g$. It is possible to analyze two different cases, corresponding to Z_i being a series ($i = 2k + 1$) or a shunt ($i = 2k$) component.

A. Series component sensitivity

It is noted that in this case when an element impedance is an open circuit we have $V_L = 0$ and therefore $H_\infty = 0$. Accordingly, from (4), we get

$$S_{Z_{2k+1}}^H = -\frac{Z_{2k+1}}{Z_{2k+1} + Z_{th}}\,. \tag{6}$$

B. Shunt component sensitivity

In this case when we consider a short circuit ($Z = 0$), we have that $H_0 = 0$ and therefore the sensitivity becomes

$$S_{Z_{2k}}^H = \frac{Z_{th}}{Z_{2k} + Z_{th}}\,. \tag{7}$$

We are now in a position to specialize the theory to a simple inductive wireless power transfer link.

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C. Series–series mutually coupled inductors

A series–series mutually coupled inductors is related to the ladder network by

$$\begin{aligned} Z_1 &= j\omega(L_1 - M) + \frac{1}{j\omega C_1} + R_1 \\ Z_2 &= j\omega M \\ Z_3 &= j\omega(L_2 - M) + \frac{1}{j\omega C_2} + R_2 \\ Z_4 &= Z_L \,. \end{aligned} \tag{8}$$

1) Analytical development: In this simple case the sensitivities can also be obtained directly by circuit analysis and by taking the derivatives. Naturally, the results coincide with the proposed approach. By solving the relevant equations the voltage gain is obtained as:

$$\begin{aligned} H &= \frac{V_L}{V_g} = \frac{Z_2\, Z_4}{Z_2\, Z_4 + Z_1\, Z_4 + Z_2\, Z_3 + Z_1\, Z_3 + Z_1\, Z_2} \\ &= \frac{Z_2\, Z_4}{\Delta} \end{aligned} \tag{9}$$

where use has been made of the quantity Δ which is the denominator of the voltage gain appearing in (9)

$$\begin{aligned} \Delta &= Z_2\, Z_4 + Z_1\, Z_4 + Z_2\, Z_3 + Z_1\, Z_3 + Z_1\, Z_2 \\ &= (Z_1 + Z_2)\left(Z_3 + Z_4 + \frac{Z_1 Z_2}{Z_1 + Z_2}\right) . \end{aligned} \tag{10}$$

Evaluation of the derivatives gives

$$\begin{aligned} \frac{\partial H}{\partial Z_1} &= -\frac{Z_2\, Z_4\, (Z_4 + Z_3 + Z_2)}{\Delta^2} \\ \frac{\partial H}{\partial Z_3} &= -\frac{Z_2\, Z_4\, (Z_2 + Z_1)}{\Delta^2} \end{aligned} \tag{11}$$

By using (2) the sensitivities can be recovered.

2) Application of the bilinear theorem: In this case we can use directly equations (6) and (7) without the need to perform the derivatives. In particular, for the series elements we have:

$$\begin{aligned} S^H_{Z_1} &= -\frac{Z_1\, (Z_4 + Z_3 + Z_2)}{\Delta} \\ S^H_{Z_3} &= \frac{(Z_2 + Z_1)\, Z_3}{\Delta} \end{aligned} \tag{12}$$

which shows a different dependence of the sensitivities.

It is noted that with the bilinear theorem it is not possible to evaluate the sensitivity w.r.t. to the coupling M, since it appears in Z_1, Z_2, Z_3.

By applying the equation for the shunt case it is instead possible to evaluate the sensitivity with respect a variation of the load as

$$S^H_{Z_4} = \frac{Z_2\, Z_3 + Z_1\, Z_3 + Z_1\, Z_2}{\Delta} \tag{13}$$

IV. Conclusion

V. Appendix: Relation between the response of a linear circuit and the impedance of one of its branches

Let us consider a linear circuit driven by an arbitrary set of voltage and current generators. The voltages and currents imposed by these generators will be referred as the *circuit inputs*.

Let us also assume that we are interested in a particular voltage or current in the circuit, which will be denoted by x and will be referred as the *circuit response*.

Finally, let us consider an arbitrary chosen bipole of impedance Z contained in the circuit.

We want to determine the relation between x and Z.

To this end, we first determine the parameters V_{th} and Z_{th} of the Thévenin equivalent circuit of the bipole obtained by eliminating the impedance Z (i.e. by replacing Z by an open circuit). Making use of this equivalent circuit, the voltage V at the terminals of the impedance Z can be expressed as

$$V = V_{th} \frac{Z}{Z_{th} + Z} \tag{14}$$

According to the substitution theorem, x does not change if the bipole Z is replaced by a voltage generator V. Since the circuit obtained in this way is linear, by applying the superposition principle, we can express its response as the sum of two contributions

$$x = x_0 + cV = x_0 + cV_{th} \frac{Z}{Z_{th} + Z} \tag{15}$$

where x_0 is the response for $V = 0$, i.e. the response due to the circuit inputs when Z is replaced by a short circuit, and the second addend is the response due to the generator V when the circuit inputs are set to zero. In this term c is a proportionality constant depending on the circuit characteristics.

In order to determine this constant, we can consider the particular case when Z tends to infinite (i.e. when Z is replaced by an open circuit) and consequently V tends to V_{th}. If we denote by x_∞ the circuit response in this condition, from (15) we get

$$x_\infty = x_0 + cV_{th} \tag{16}$$

and, consequenty

$$c = \frac{x_\infty - x_0}{V_{th}} \tag{17}$$

Finally, by replacing (17) into (15) we obtain

$$x = \frac{Z_{th} x_0 + Z x_\infty}{Z_{th} + Z} \tag{18}$$

Let us now consider the special case of a circuit with a single input, i.e. with a single generator, whose voltage or current will be denoted by u. If we denote by $H(Z)$ the network function

$$H(Z) = \frac{x}{u} \tag{19}$$

from (18) we get

$$H(Z) = \frac{Z_{th} H(0) + Z H(\infty)}{Z_{th} + Z} \tag{20}$$

which corresponds to equation VIII.1.3 of Martinelli-Salerno. In particular, for $Z = Z_{th}$ (20) yields

$$H(Z_{th}) = \frac{H(0) + H(\infty)}{2} \tag{21}$$

As an alternative, this relation can be obtained by noting that for $Z = Z_{th}$ equation (15) provides

$$x_{th} = x_0 + c\frac{V_{th}}{2} \tag{22}$$

which, taking (16) into account, can be rewritten as

$$x_{th} = \frac{x_0 + x_\infty}{2} \tag{23}$$

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