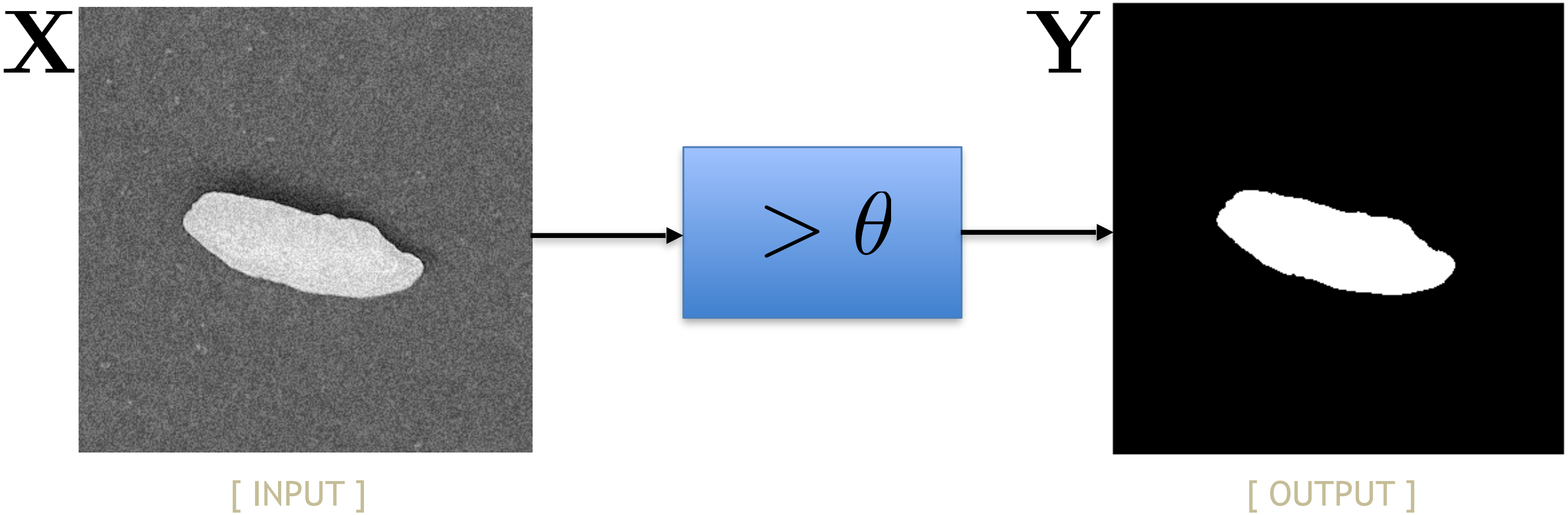
Segmentación por umbral

Método de Otsu

Segmentación por Umbral

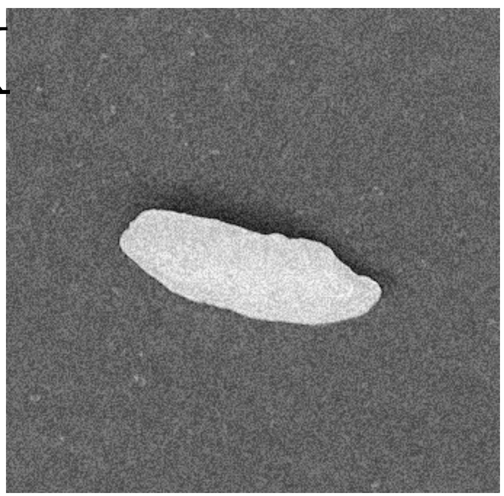
- Los tonos de gris mayores que un umbral pertenecen a la región segmentada, mientras que el resto pertenece al fondo.



Segmentación por Umbral

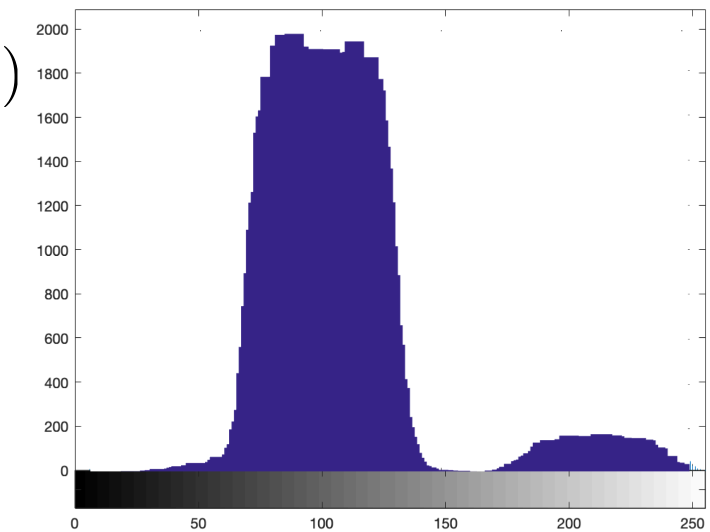
Para escoger el umbral se analiza el histograma

$\mathbf{X}$

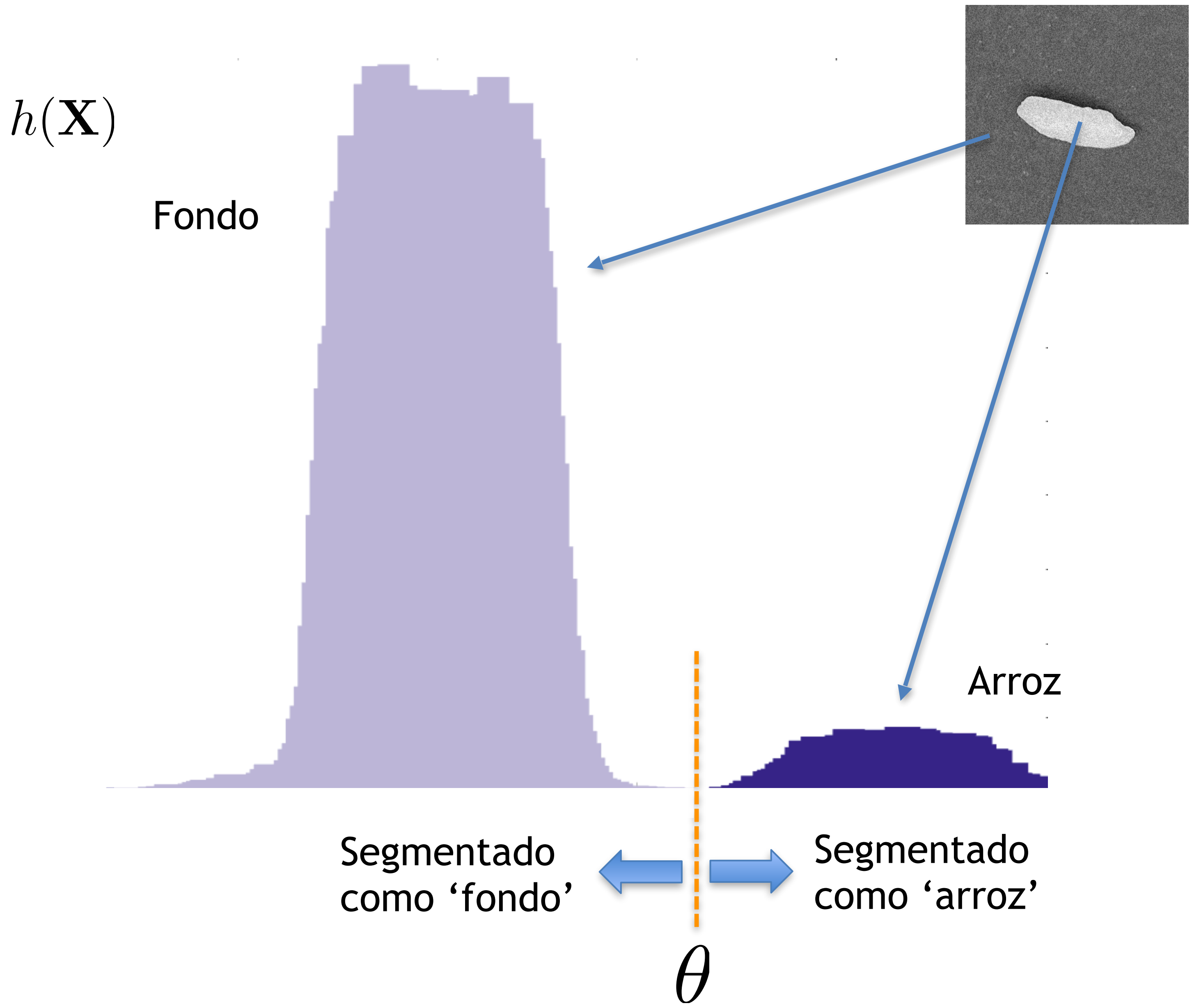


[INPUT]

$h(\mathbf{X})$

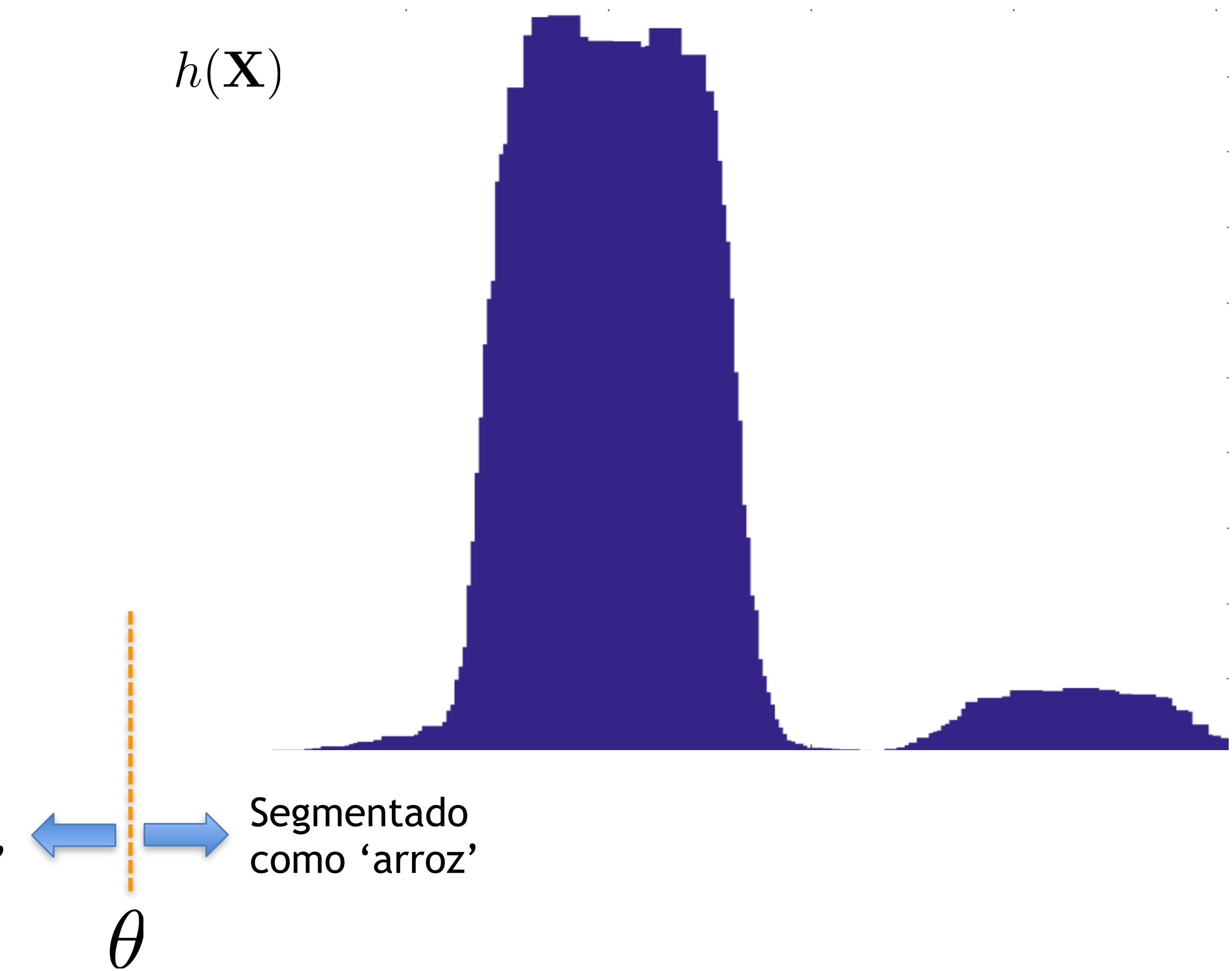


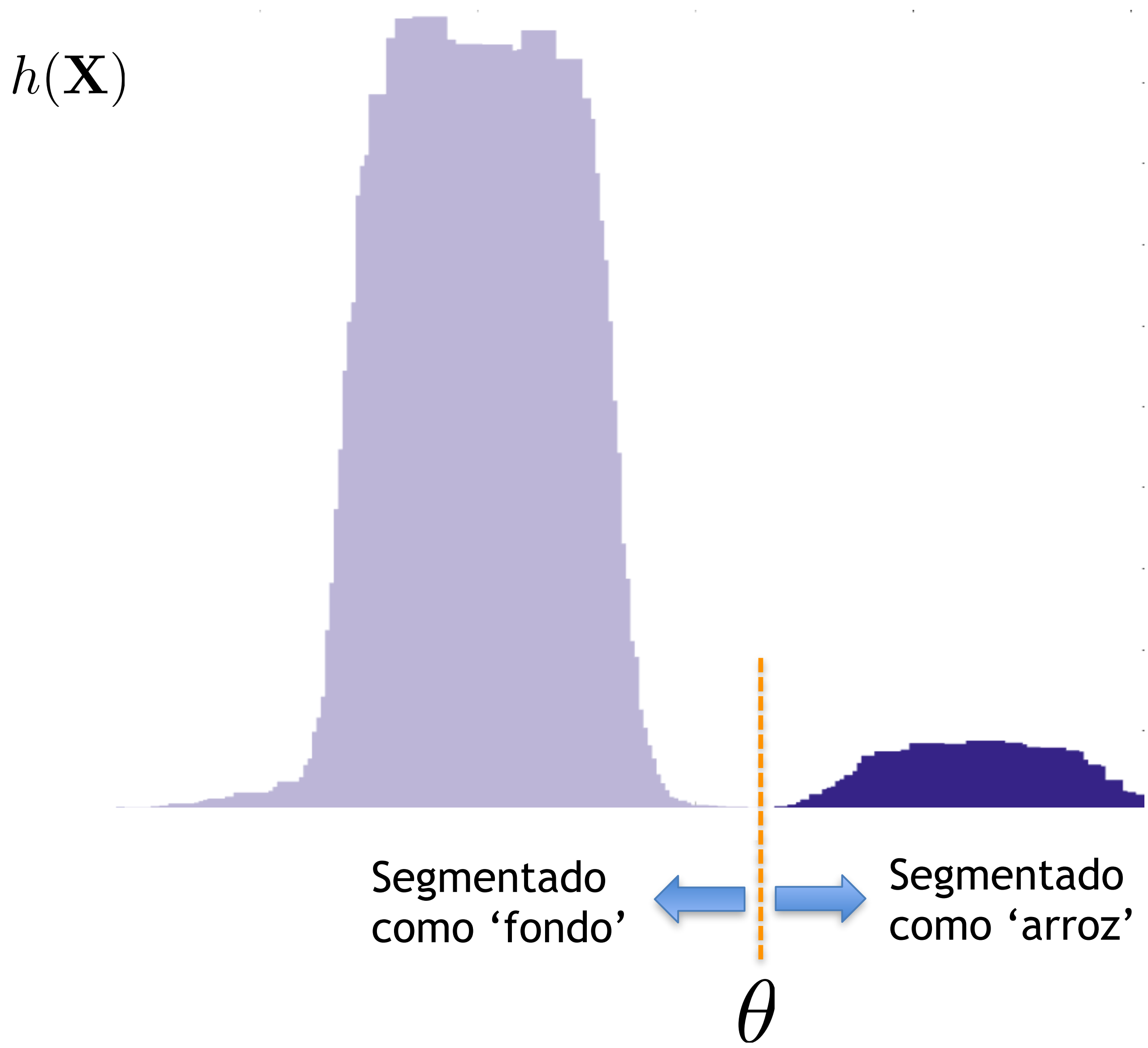
[HISTOGRAMA]


$h(\mathbf{X})$
Fondo
Arroz
Segmentado como 'fondo'
Segmentado como 'arroz'
θ

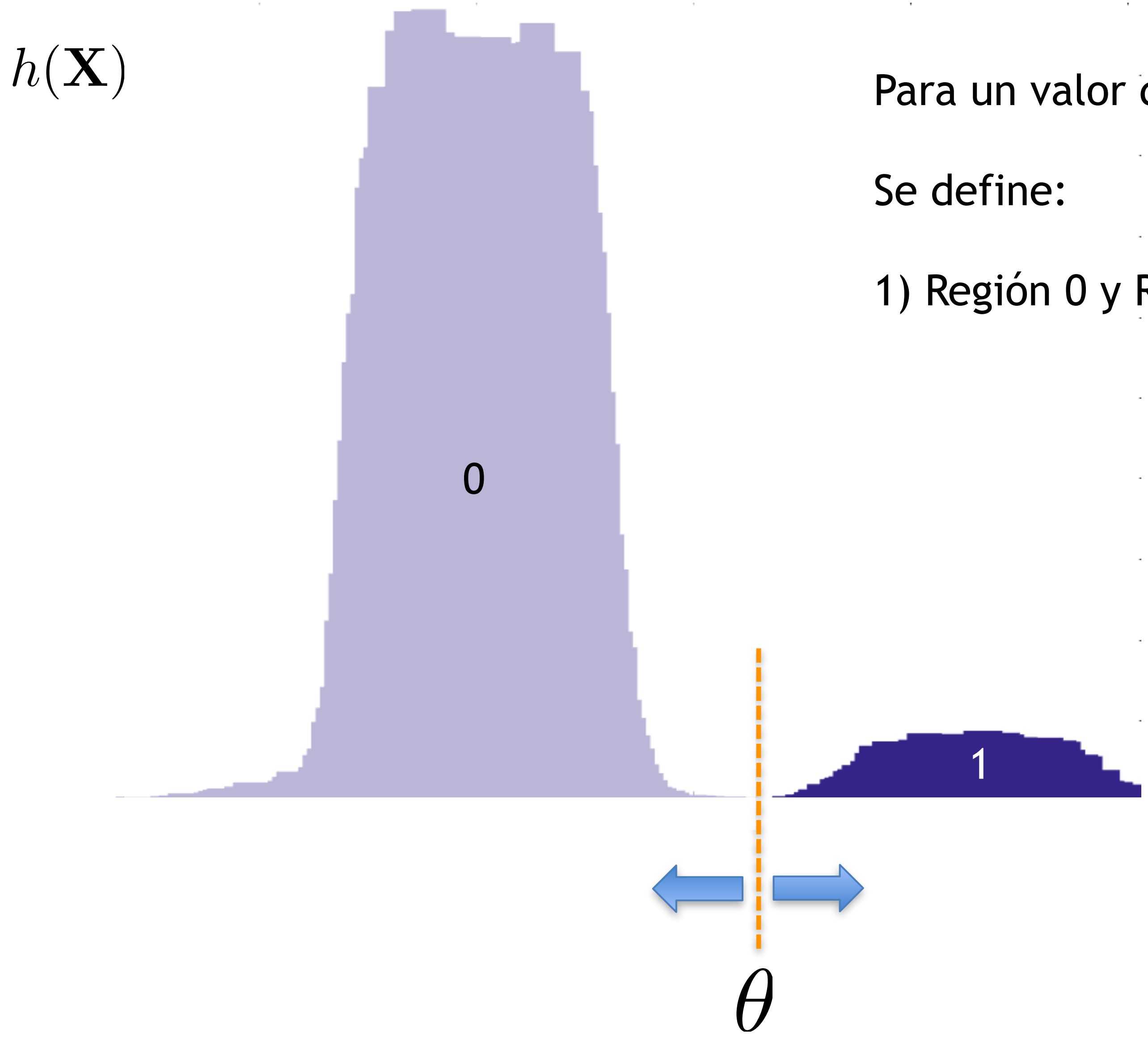
Método para estimar θ de manera automática

(Método de Otsu)


$h(\mathbf{X})$
Segmentado
como 'arroz'
θ



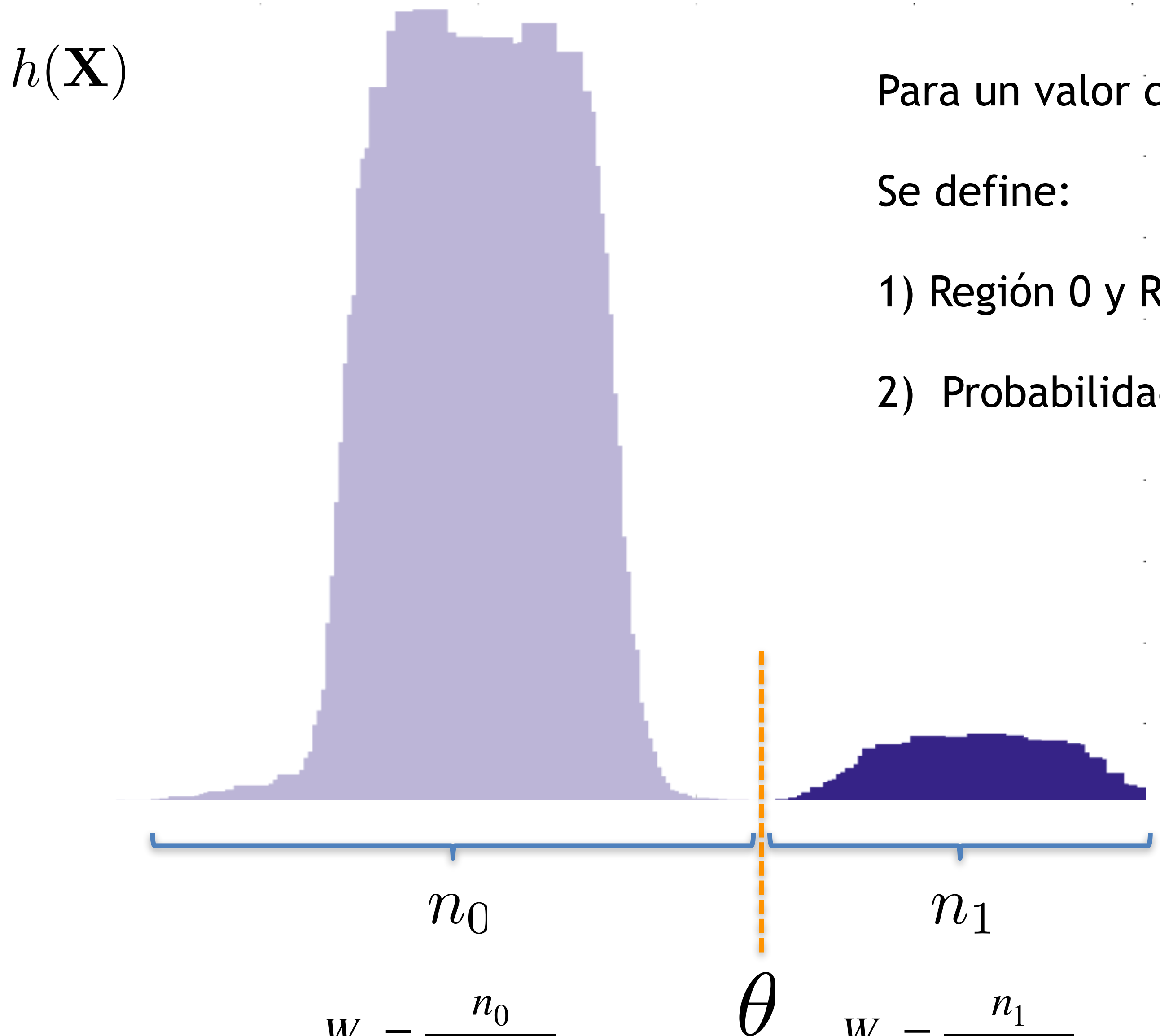
$h(\mathbf{X})$
Segmentado como 'fondo'
Segmentado como 'arroz'
θ



Para un valor de θ

Se define:

1) Región 0 y Región 1



Para un valor de θ

Se define:

1) Región 0 y Región 1

2) Probabilidades W_0, W_1

$$W_0 = \frac{n_0}{n_0 + n_1} \qquad W_1 = \frac{n_1}{n_0 + n_1}$$

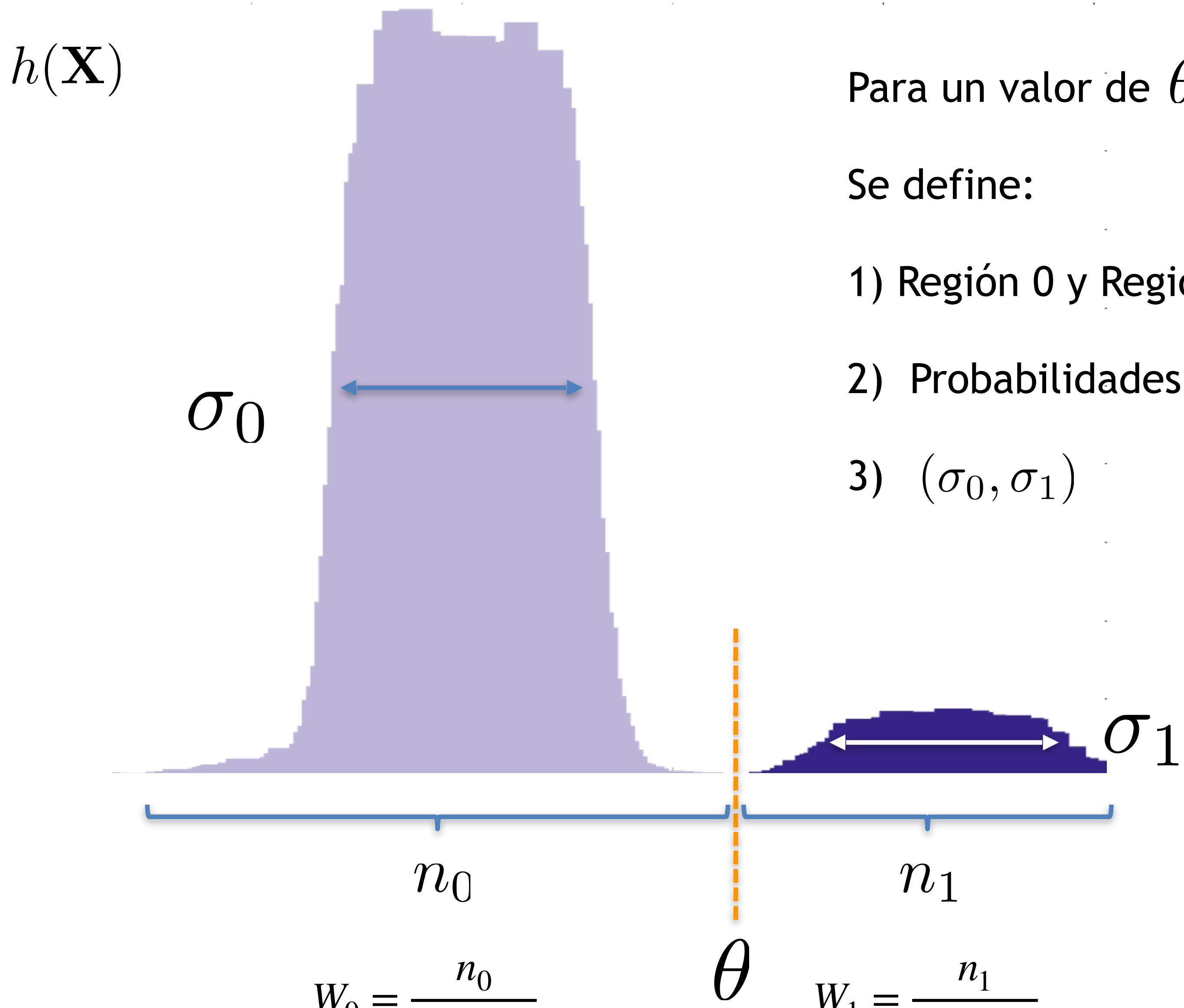
Para un valor de θ

Se define:

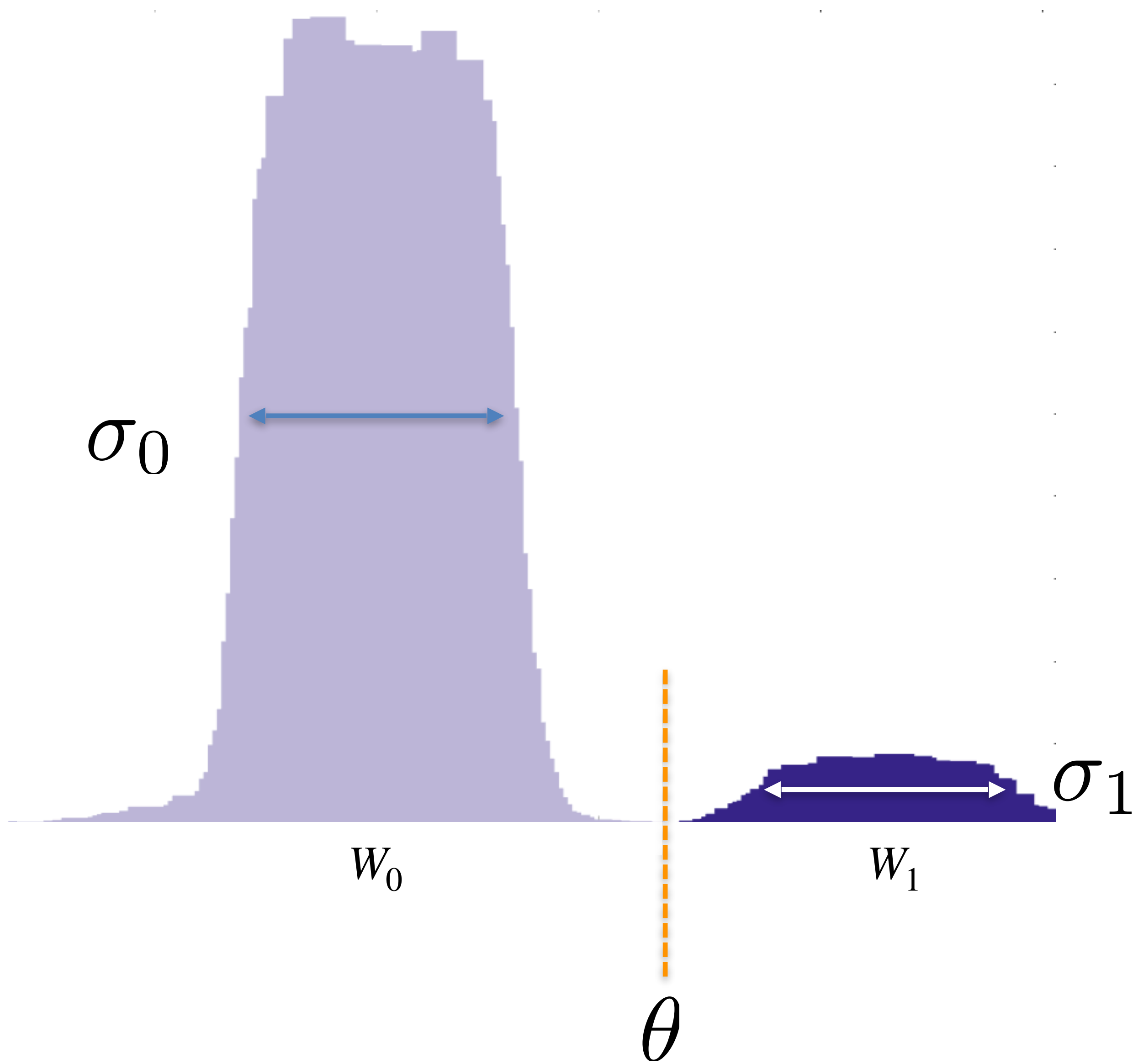
1) Región 0 y Región 1

2) Probabilidades W_0, W_1

3) (σ_0, σ_1)

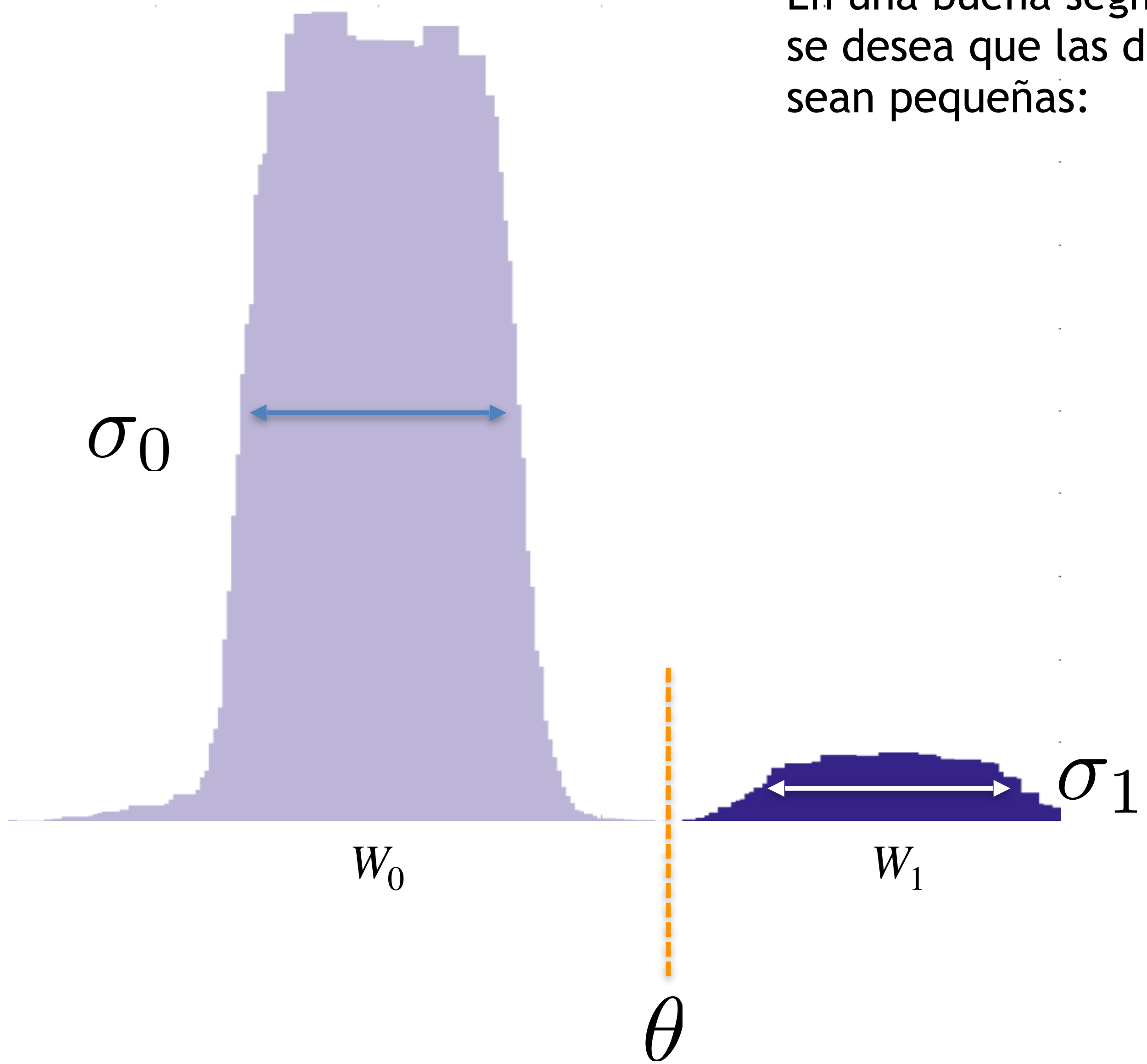


$$W_0 = \frac{n_0}{n_0 + n_1} \qquad W_1 = \frac{n_1}{n_0 + n_1}$$



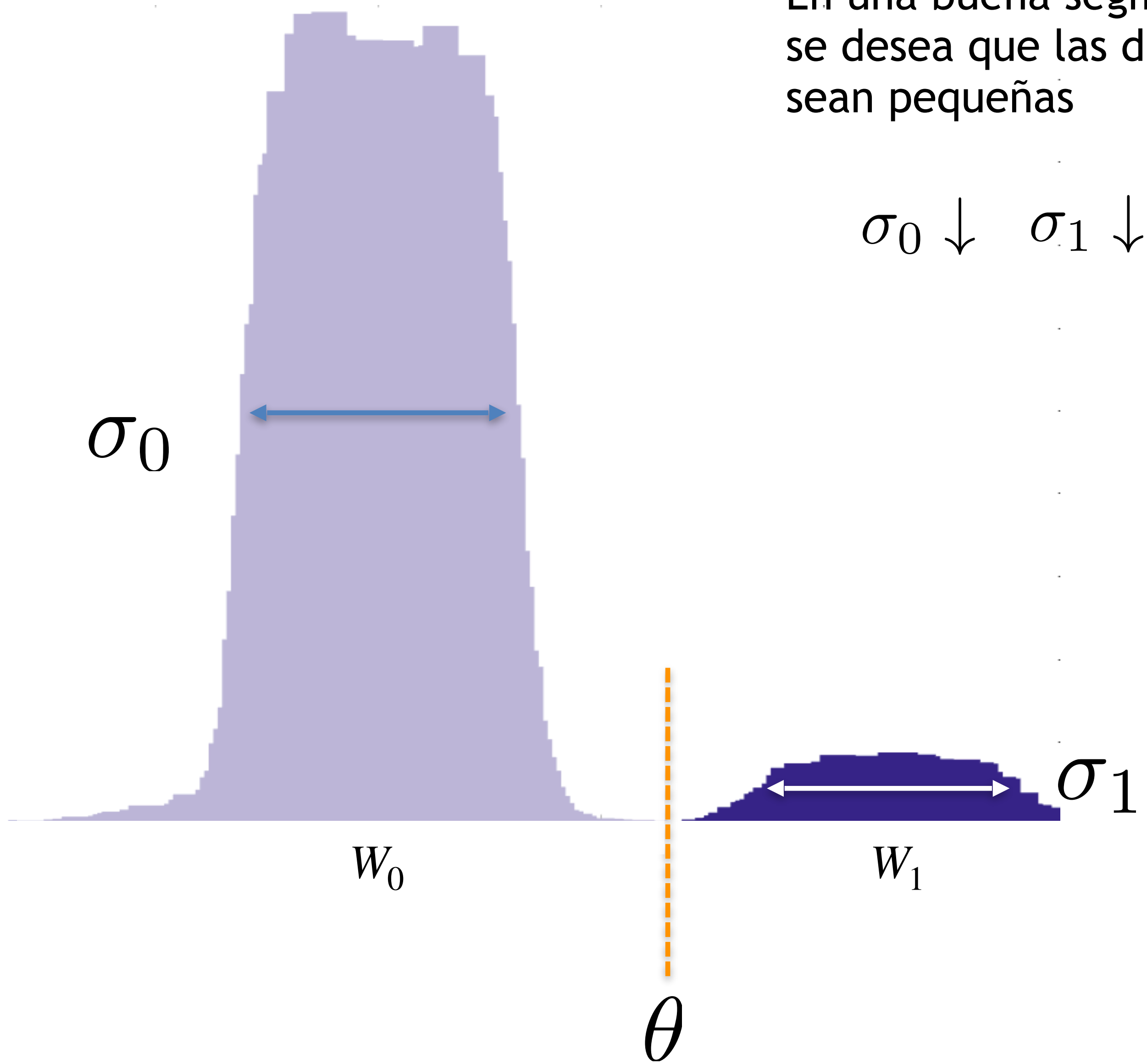
σ_0
σ_1
W_0
W_1
θ

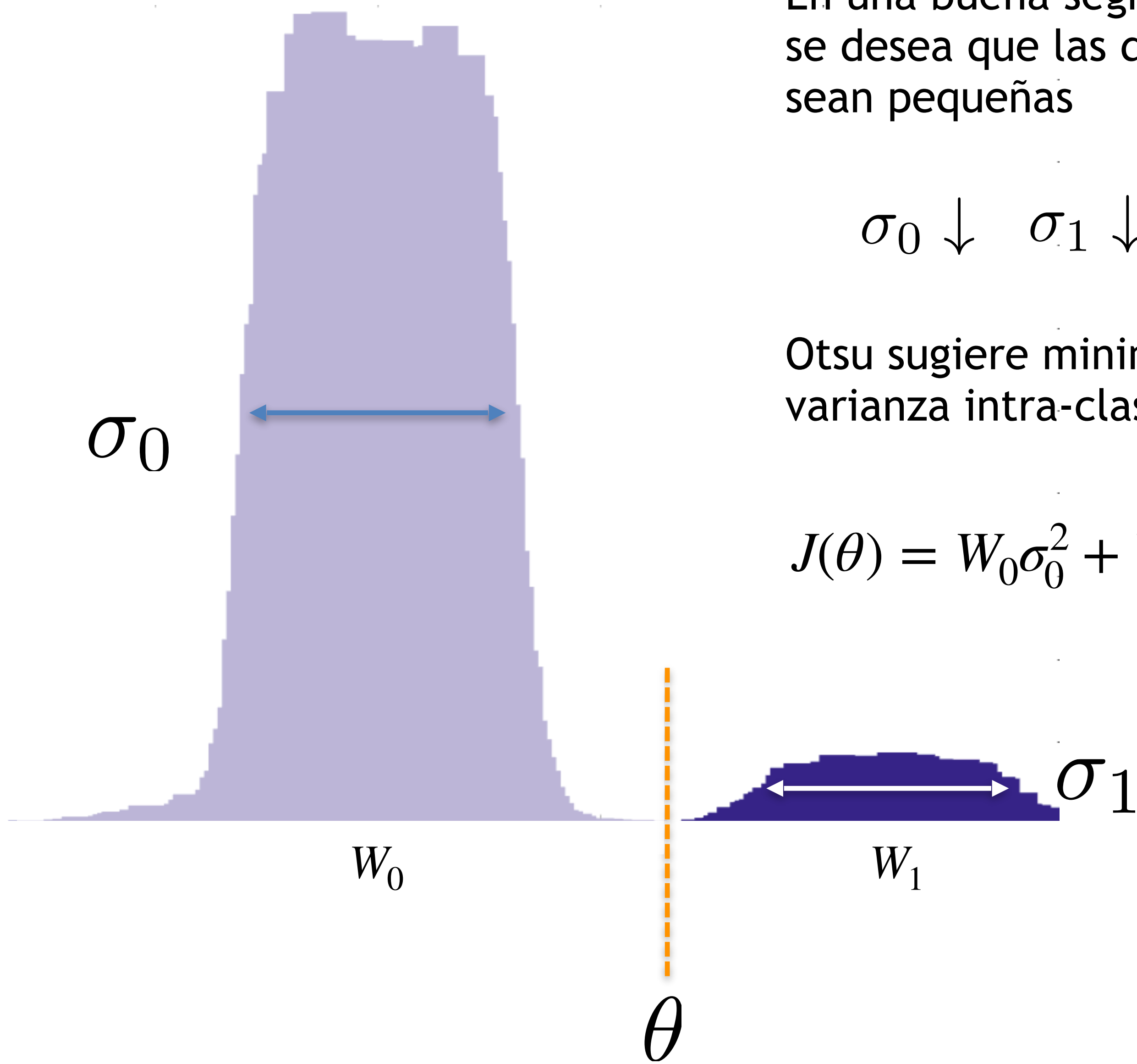
En una buena segmentación, se desea que las dispersiones sean pequeñas:



En una buena segmentación, se desea que las dispersiones sean pequeñas

$$\sigma_0 \downarrow \quad \sigma_1 \downarrow$$



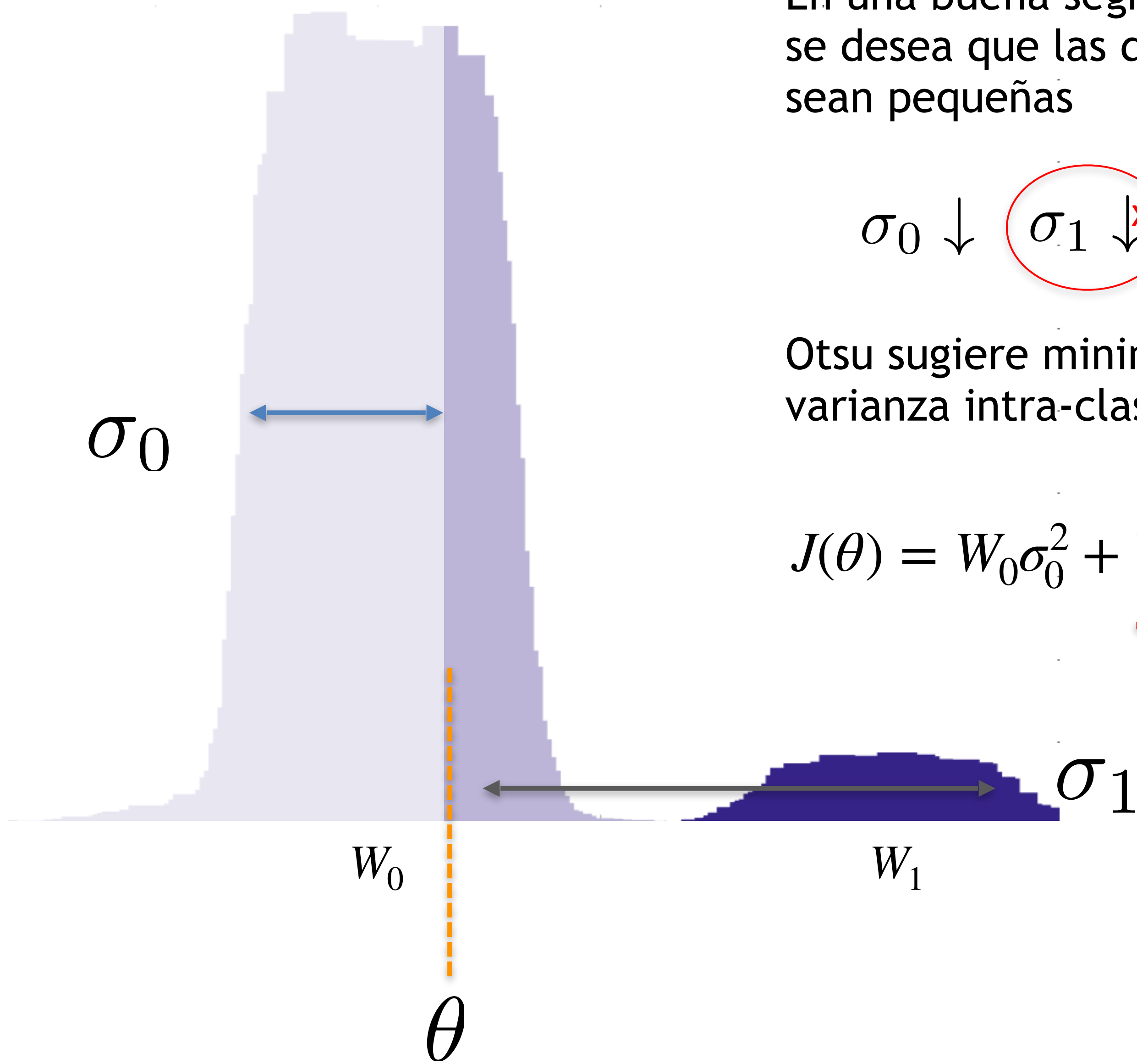


En una buena segmentación, se desea que las dispersiones sean pequeñas

$$\sigma_0 \downarrow \quad \sigma_1 \downarrow$$

Otsu sugiere minimizar la varianza intra-clase:

$$J(\theta) = W_0\sigma_0^2 + W_1\sigma_1^2 \rightarrow \min$$



En una buena segmentación, se desea que las dispersiones sean pequeñas

$$\sigma_0 \downarrow \quad \sigma_1 \downarrow$$

x

Otsu sugiere minimizar la varianza intra-clase:

$$J(\theta) = W_0\sigma_0^2 + \underline{W_1\sigma_1^2} \rightarrow \min$$

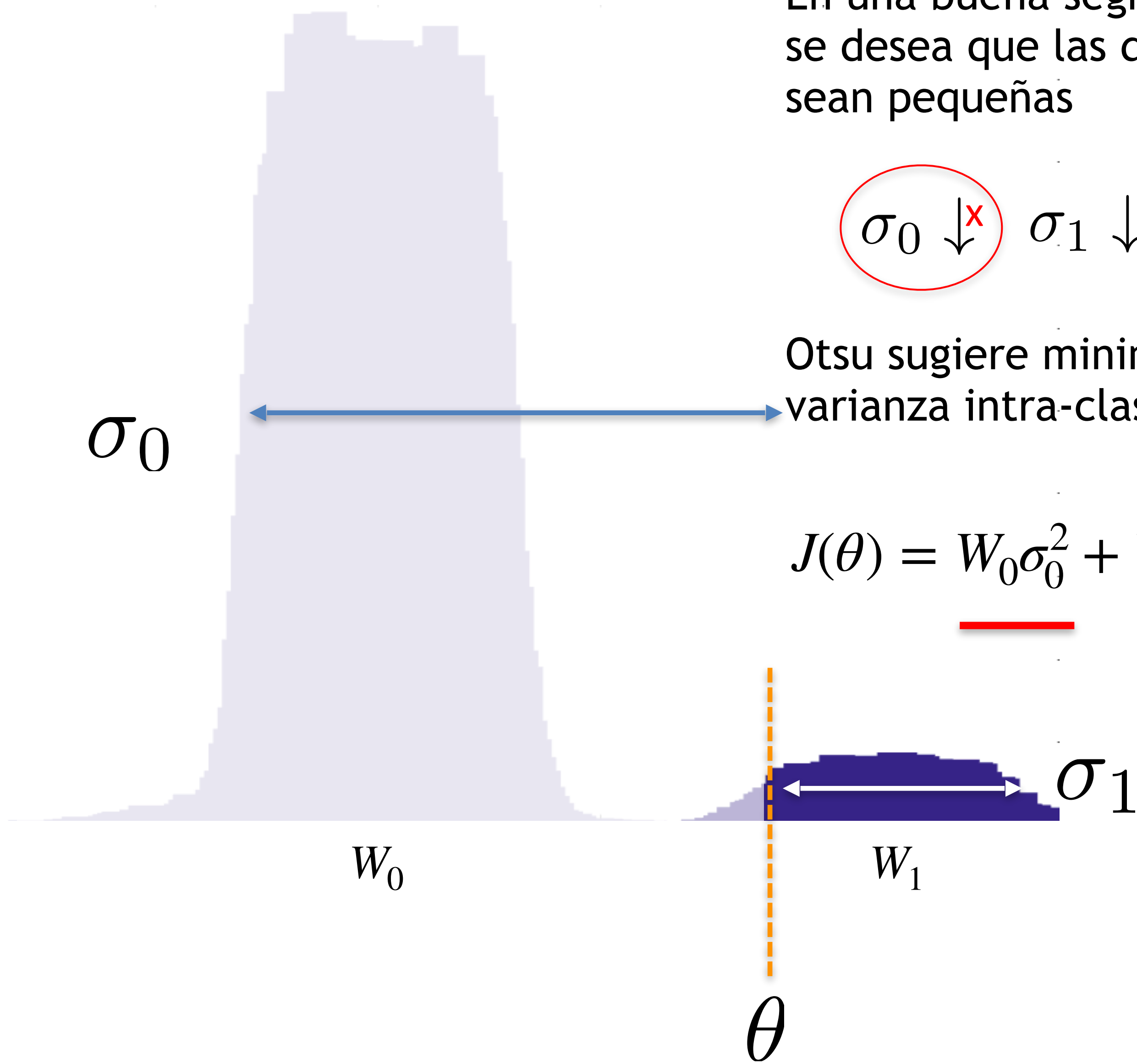
x

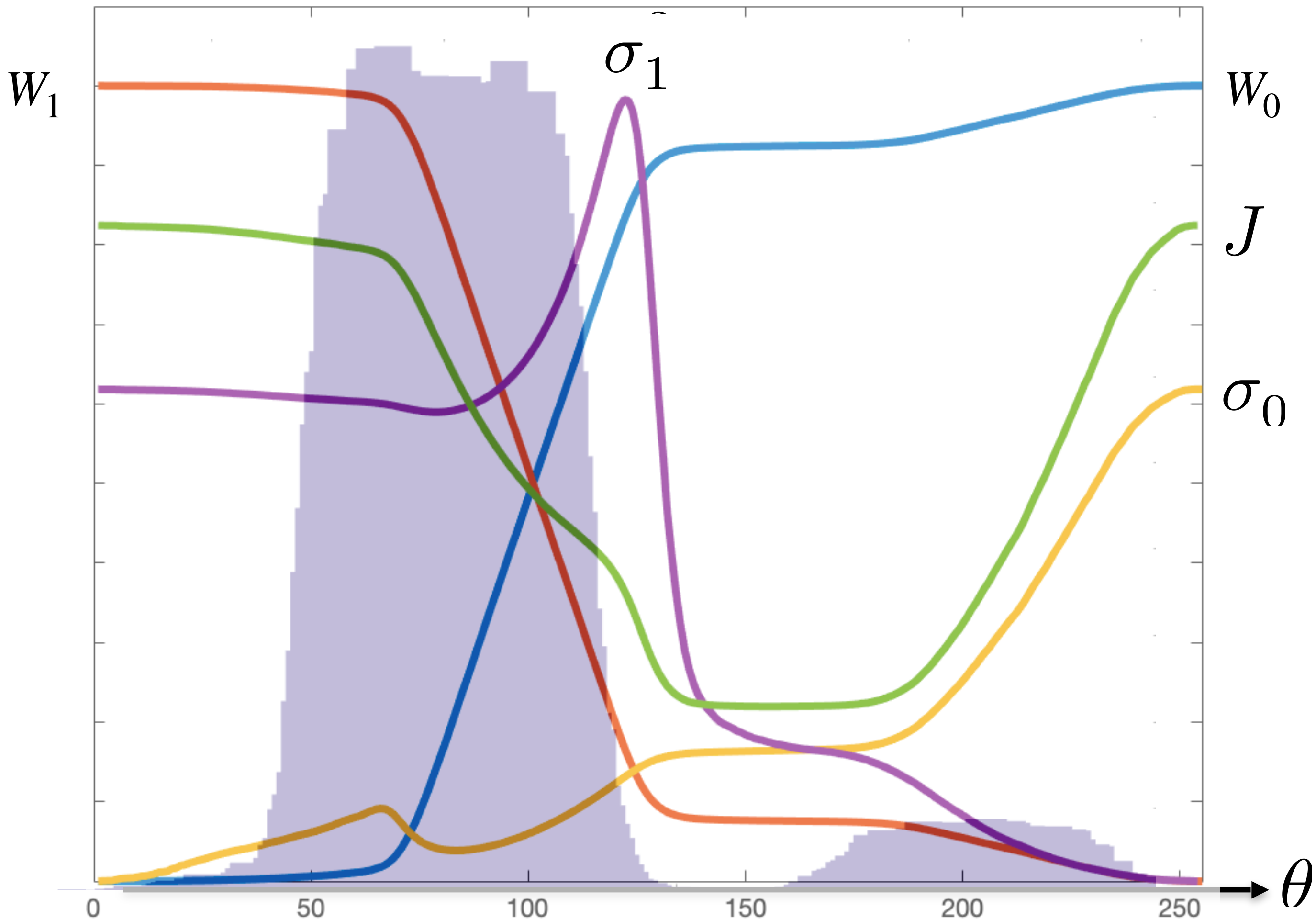
En una buena segmentación, se desea que las dispersiones sean pequeñas

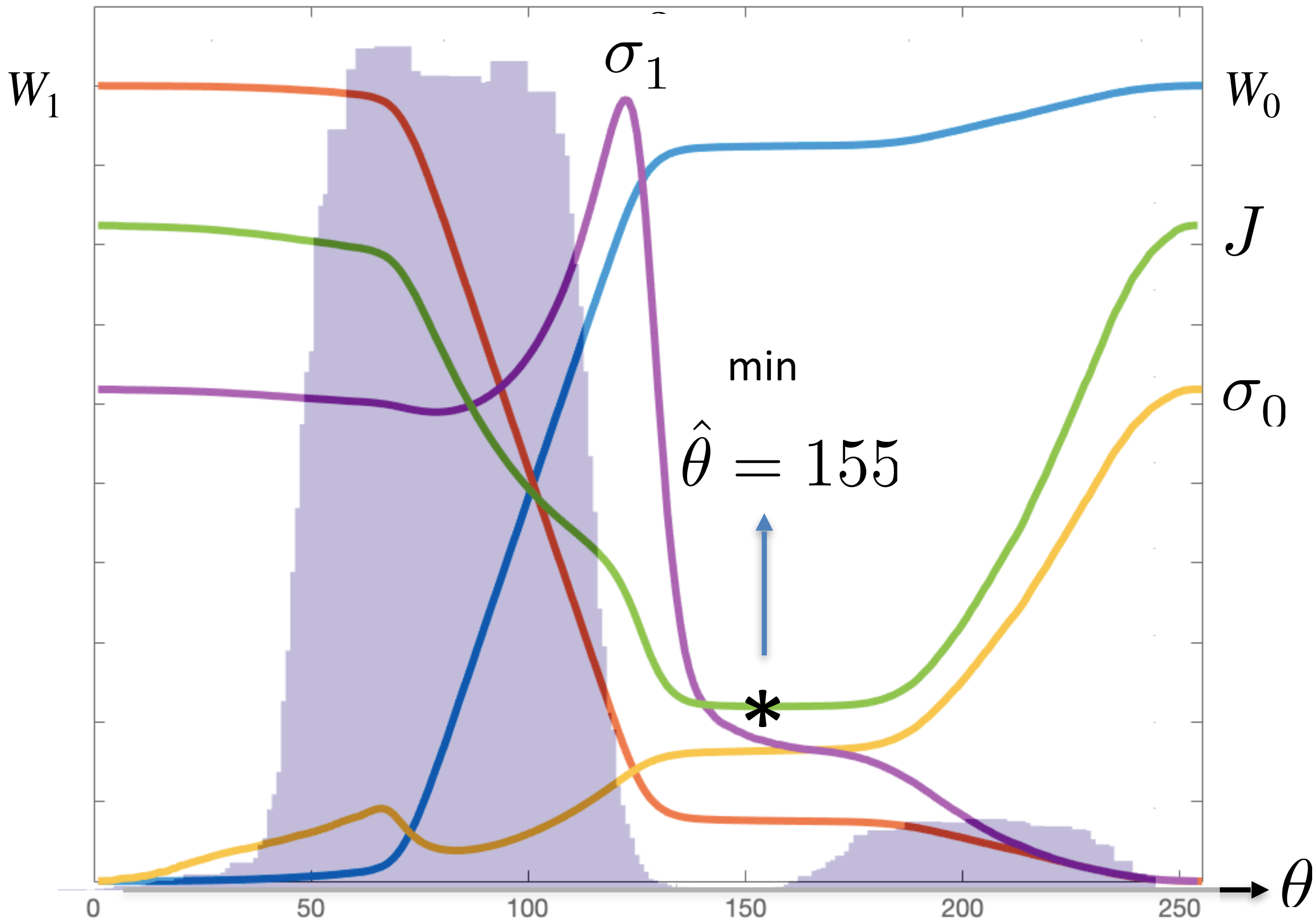
$$\sigma_0 \downarrow \quad \sigma_1 \downarrow$$

Otsu sugiere minimizar la varianza intra-clase:

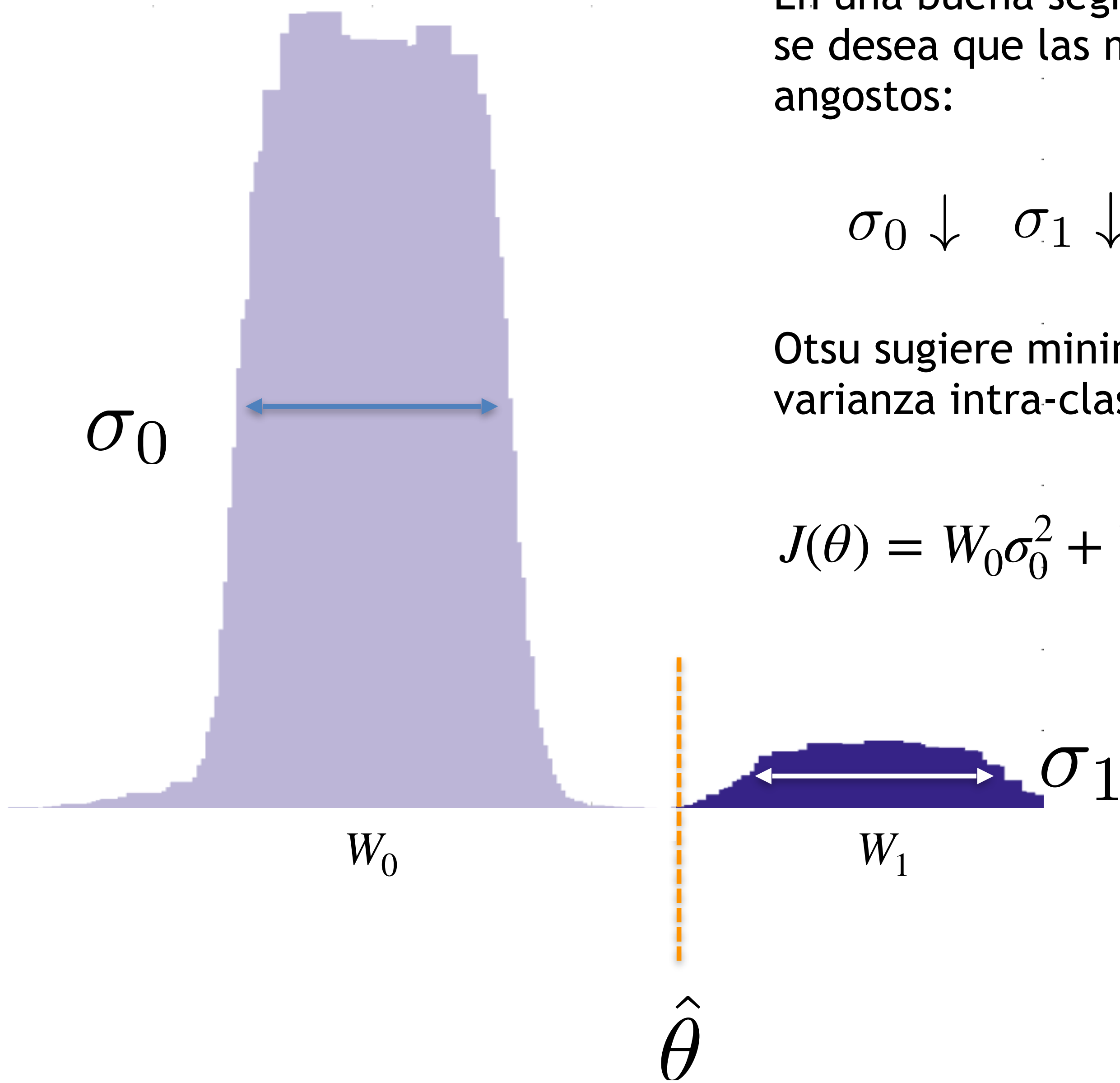
$$J(\theta) = W_0\sigma_0^2 + W_1\sigma_1^2 \rightarrow \min$$




W_1
σ_1
W_0
J
σ_0
θ
0
50
100
150
200
250



W_1
σ_1
W_0
J
min
$\hat{\theta} = 155$
σ_0
*
θ
0
50
100
150
200
250



En una buena segmentación, se desea que las modas sean angostos:

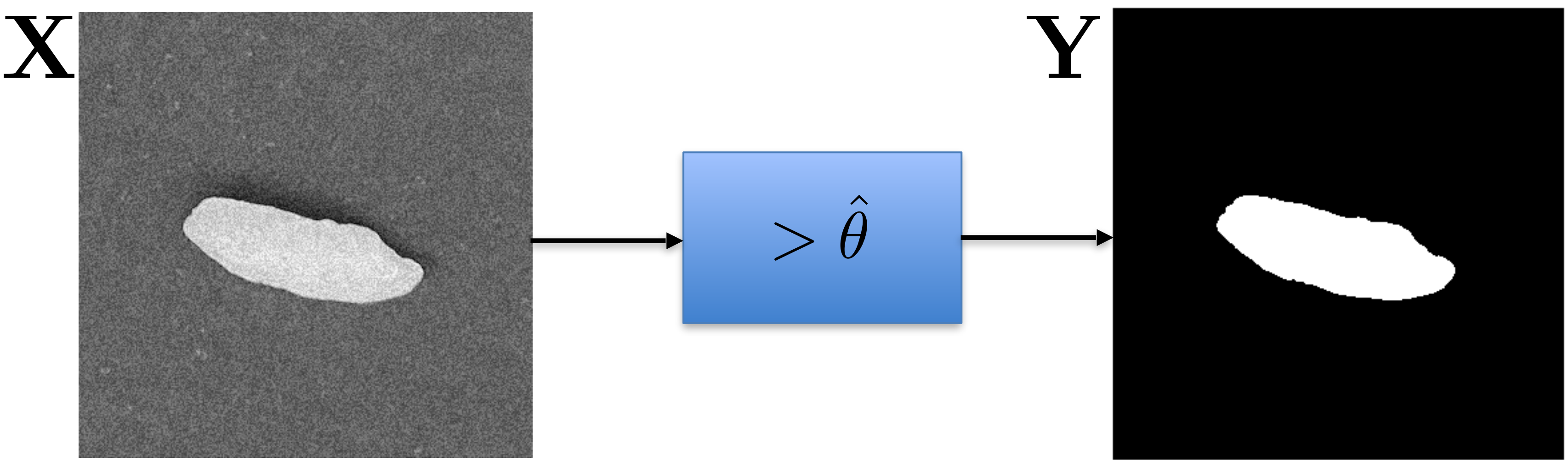
$$\sigma_0 \downarrow \quad \sigma_1 \downarrow$$

Otsu sugiere minimizar la varianza intra-clase:

$$J(\theta) = W_0\sigma_0^2 + W_1\sigma_1^2 \rightarrow \min$$

Segmentación por Umbral

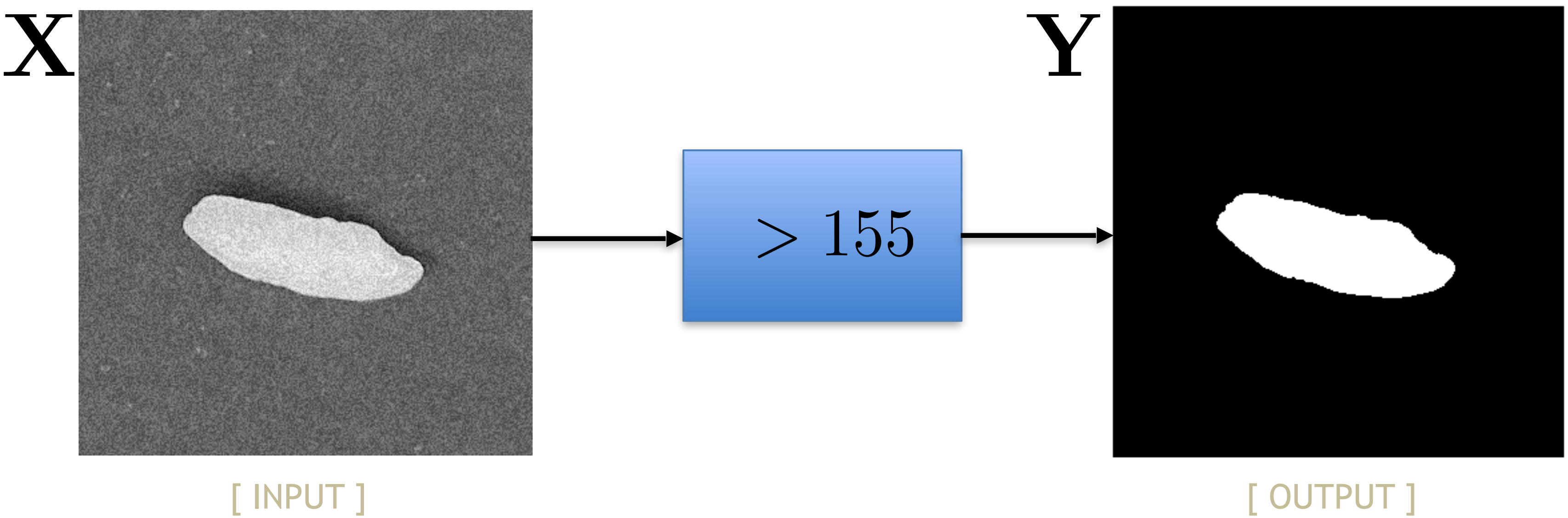
- Los tonos de gris mayores que un umbral pertenecen a la región segmentada, mientras que el resto pertenece al fondo.



[INPUT] [OUTPUT]

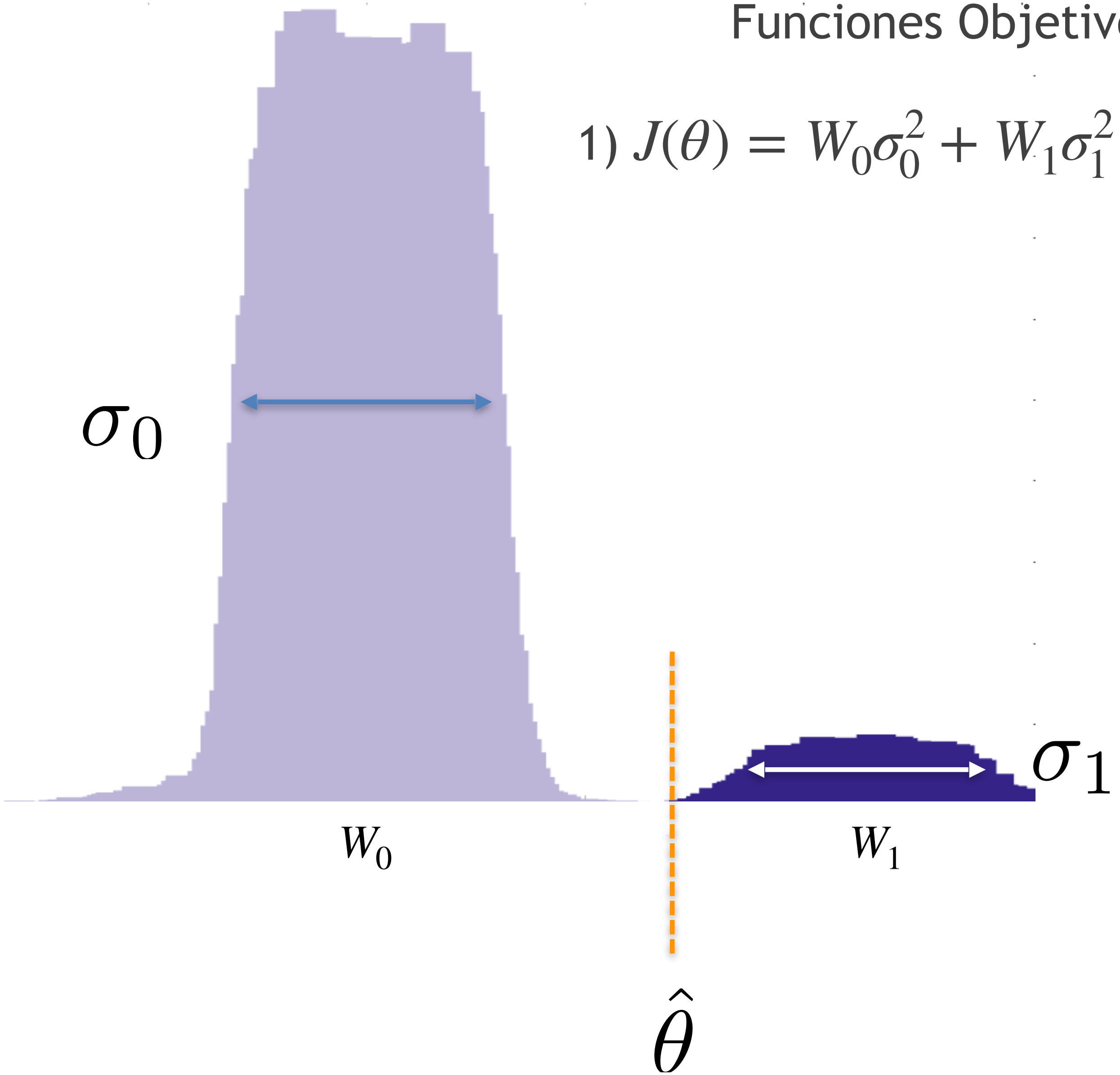
Segmentación por Umbral

- Los tonos de gris mayores que un umbral pertenecen a la región segmentada, mientras que el resto pertenece al fondo.



Funciones Objetivo

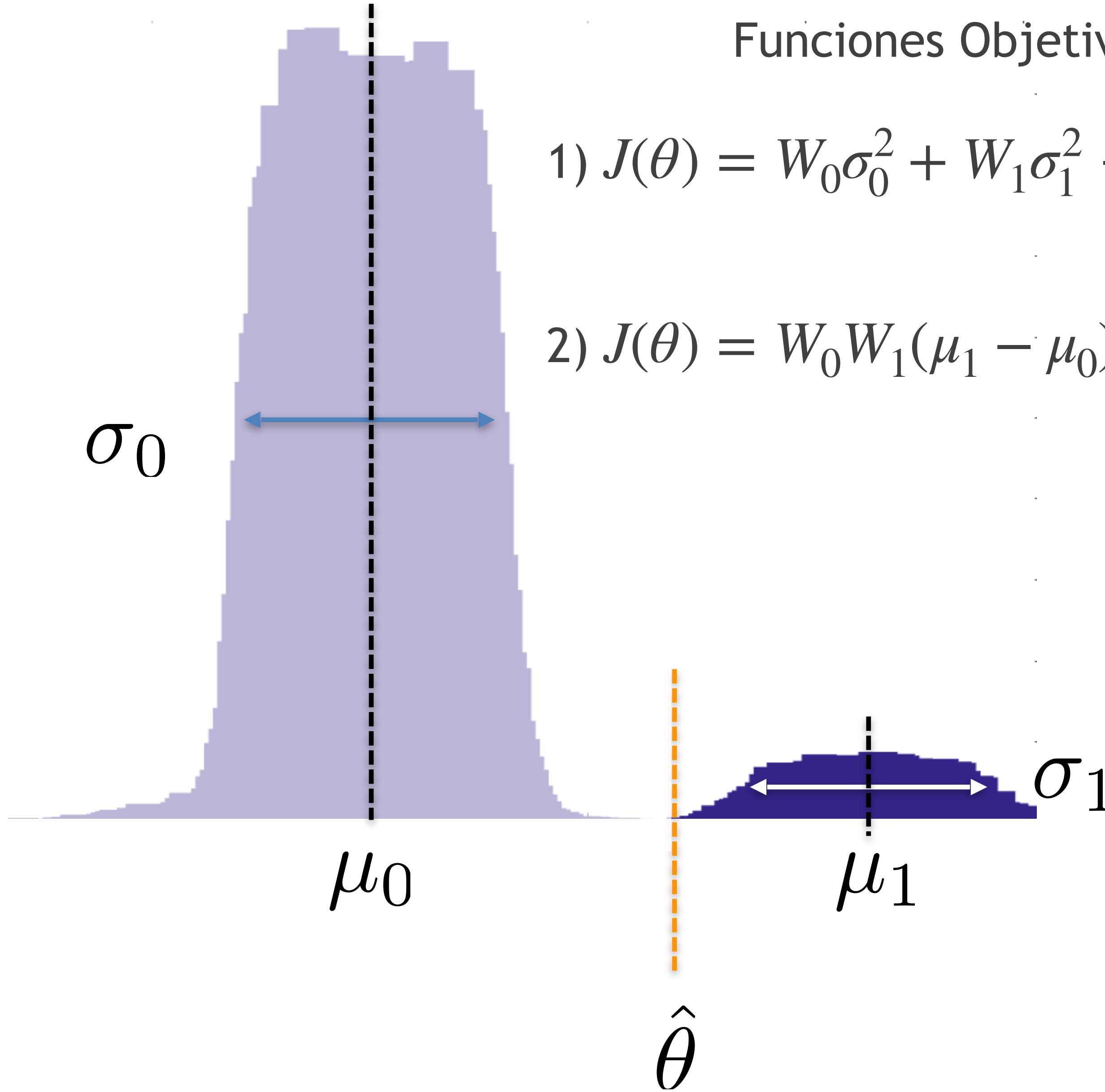
1) $J(\theta) = W_0\sigma_0^2 + W_1\sigma_1^2 \rightarrow \min$



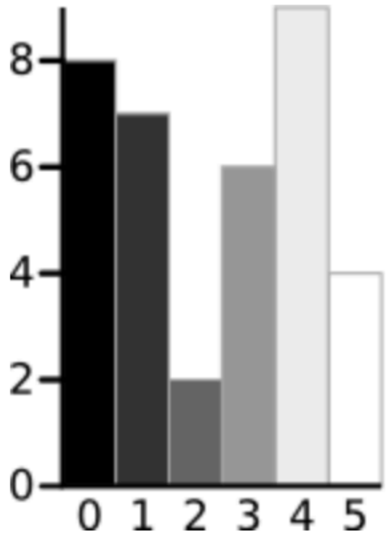
Funciones Objetivo

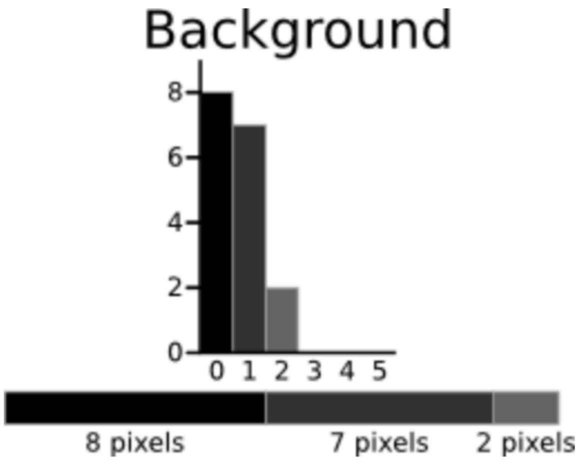
1) $J(\theta) = W_0\sigma_0^2 + W_1\sigma_1^2 \rightarrow \min$

2) $J(\theta) = W_0W_1(\mu_1 - \mu_0)^2 \rightarrow \max$



Ejemplo

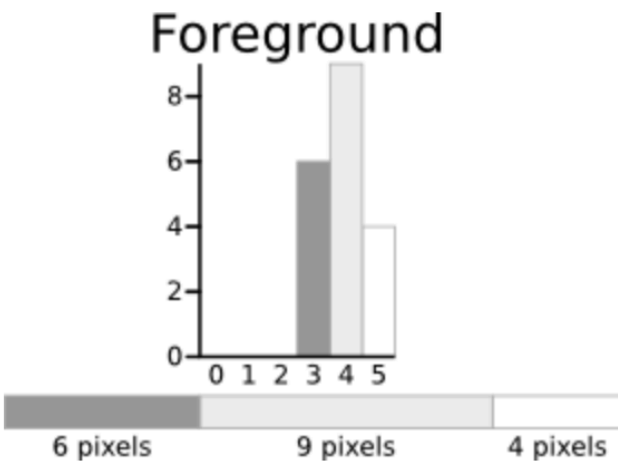




$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

$$\text{Mean} \quad \mu_b = \frac{(0 \times 8) + (1 \times 7) + (2 \times 2)}{17} = 0.6471$$

$$\begin{aligned} \text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2 \times 8) + ((1-0.6471)^2 \times 7) + ((2-0.6471)^2 \times 2)}{17} \\ &= \frac{(0.4187 \times 8) + (0.1246 \times 7) + (1.8304 \times 2)}{17} \\ &= 0.4637 \end{aligned}$$

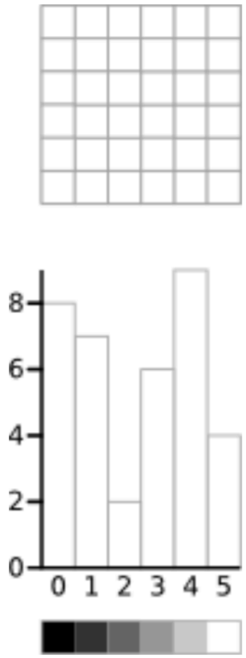
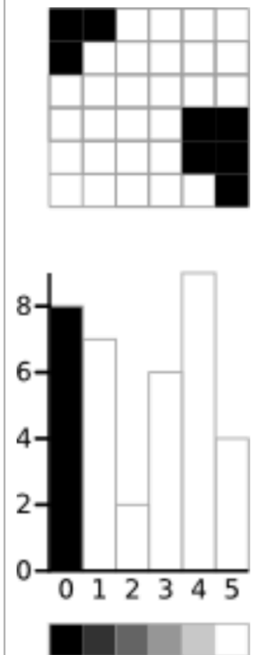
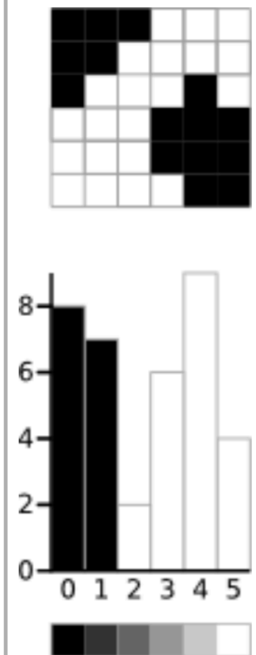
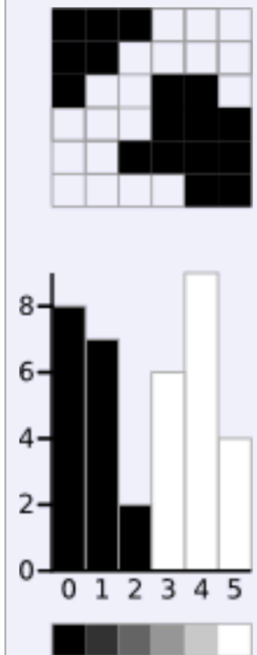
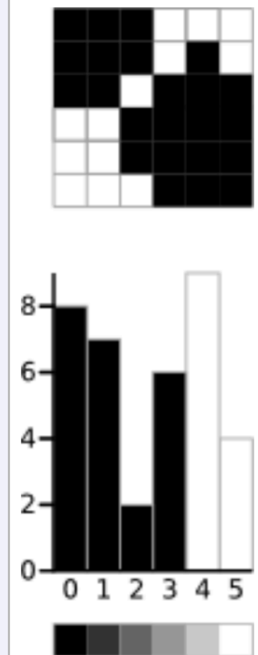
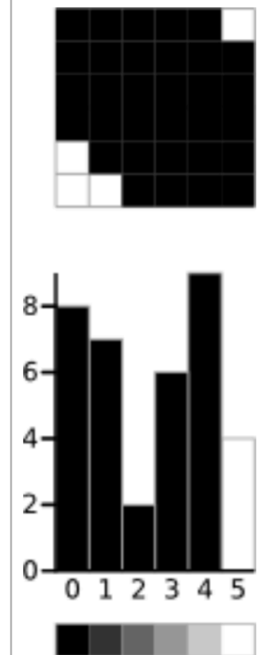


$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

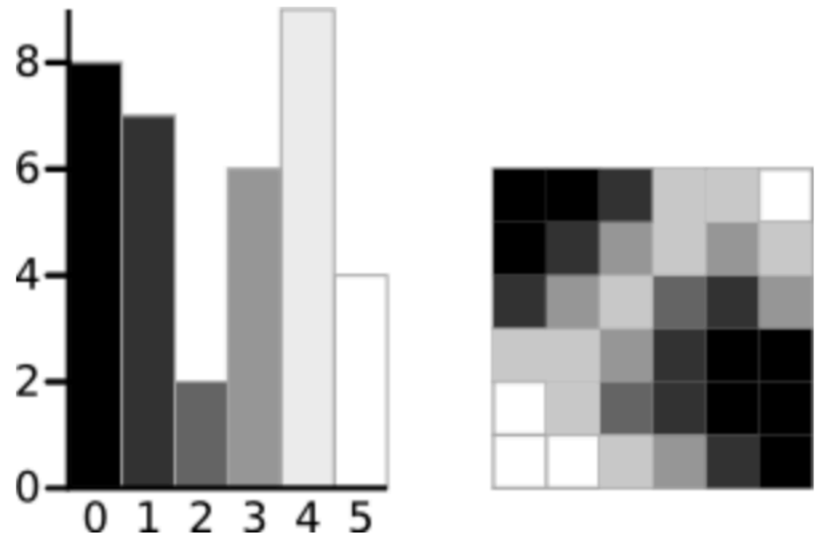
$$\text{Mean} \quad \mu_f = \frac{(3 \times 6) + (4 \times 9) + (5 \times 4)}{19} = 3.8947$$

$$\begin{aligned} \text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2 \times 6) + ((4-3.8947)^2 \times 9) + ((5-3.8947)^2 \times 4)}{19} \\ &= \frac{(4.8033 \times 6) + (0.0997 \times 9) + (4.8864 \times 4)}{19} \\ &= 0.5152 \end{aligned}$$

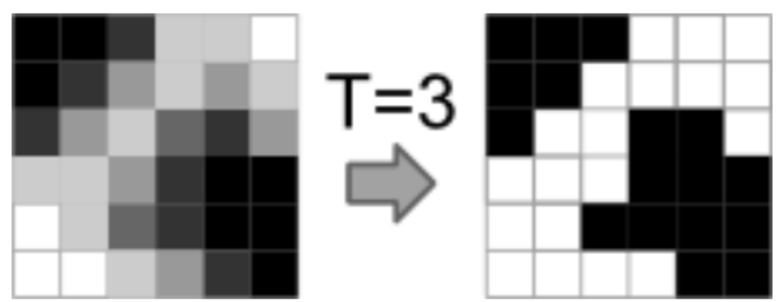
Varianza ***intra*** clase: $\sigma_W^2 = W_b\sigma_b^2 + W_f\sigma_f^2 = 0.4722 \cdot 0.4637 + 0.5278 \cdot 0.5152 = 0.4909$

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
						
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2{}_b = 0$	$\sigma^2{}_b = 0$	$\sigma^2{}_b = 0.2489$	$\sigma^2{}_b = 0.4637$	$\sigma^2{}_b = 1.4102$	$\sigma^2{}_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2{}_f = 3.1196$	$\sigma^2{}_f = 1.9639$	$\sigma^2{}_f = 0.7755$	$\sigma^2{}_f = 0.5152$	$\sigma^2{}_f = 0.2130$	$\sigma^2{}_f = 0$
Varianza intra clase	$\sigma^2{}_W = 3.1196$	$\sigma^2{}_W = 1.5268$	$\sigma^2{}_W = 0.5561$	$\boldsymbol{\sigma^2{}_W = 0.4909}$	$\sigma^2{}_W = 0.9779$	$\sigma^2{}_W = 2.2491$

Varianza ***intra*** clase: $\sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2$



8
6
4
2
0
0 1 2 3 4 5



T=3

Ejemplo

- Minimizar σ_w^2 es equivalente a maximizar σ_B^2

Varianza total: $\sigma^2 = \sigma_W^2 + \sigma_B^2$

Varianza intra clase: $\sigma_W^2 = W_b\sigma_b^2 + W_f\sigma_f^2$

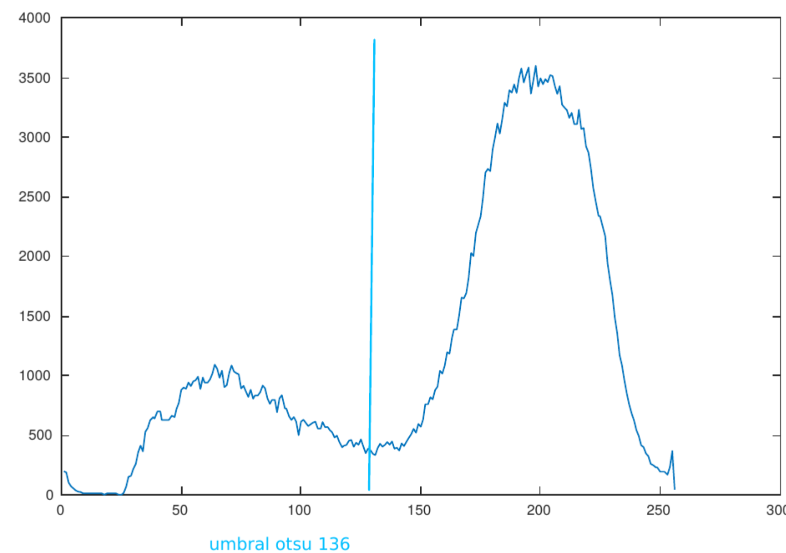
Varianza inter clase: $\sigma_B^2 = \sigma^2 - \sigma_W^2$

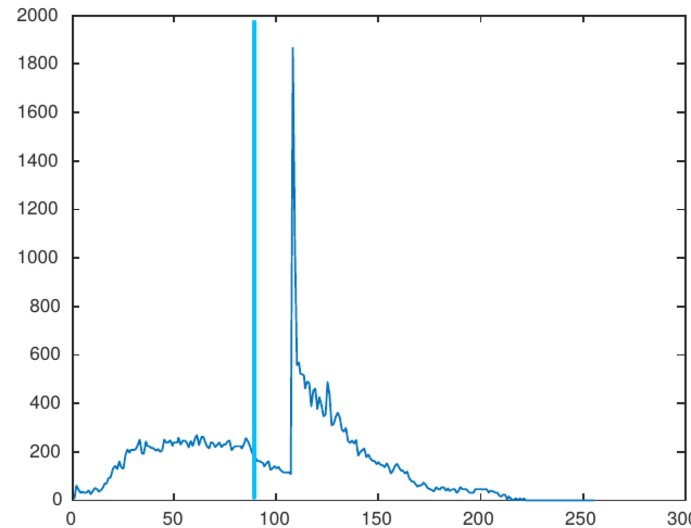
$$= W_b(\mu_b - \mu)^2 + W_f(\mu_f - \mu)^2$$
$$= W_b\, W_f\, (\mu_b - \mu_f)^2$$

donde $\mu = W_b\mu_b + W_f\mu_f$ y $W_b + W_f = 1$

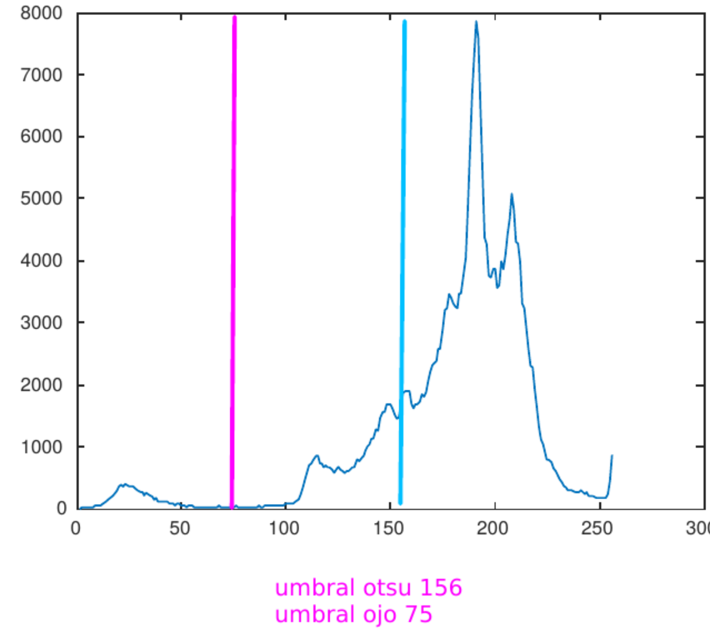
Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Varianza intra clase	σ^2_W = 3.1196	σ^2_W = 1.5268	σ^2_W = 0.5561	**σ^2_W = 0.4909**	σ^2_W = 0.9779	σ^2_W = 2.2491
Varianza inter clase	σ^2_B = 0	σ^2_B = 1.5928	σ^2_B = 2.5635	**σ^2_B = 2.6287**	σ^2_B = 2.1417	σ^2_B = 0.8705

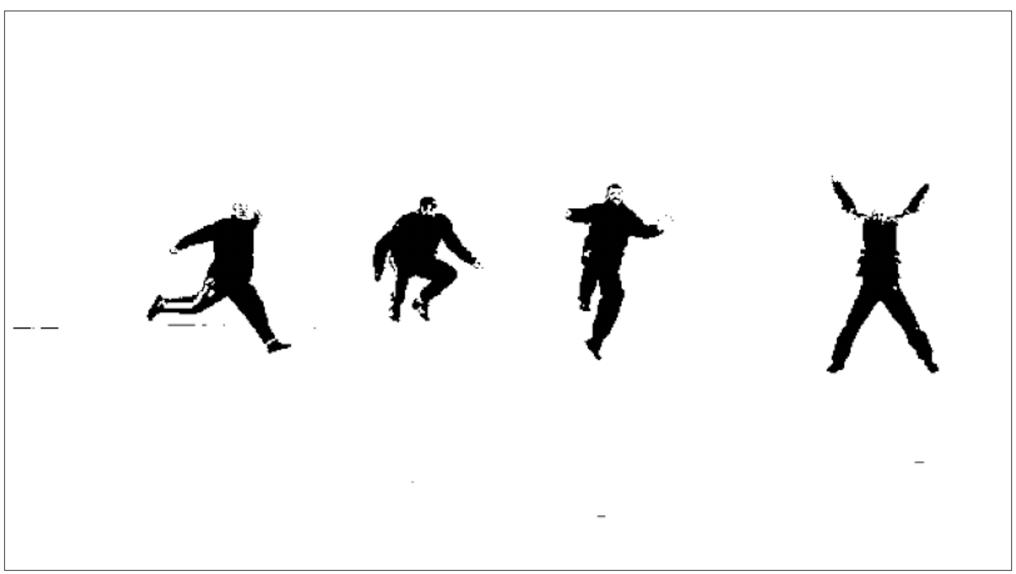
Ejemplos


4000
3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250
300
umbral otsu 136


2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250
300

umbral 90


8000
7000
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
250
300
umbral otsu 156
umbral ojo 75



Derivaciones

- V_t

$$= \frac{\sum_{i=1}^{N} (P_i - \mu_t)^2}{N} = \frac{\sum_{i=1}^{N} (P_i^2 - 2 \times P_i \times \mu_t + \mu_t^2)}{N}$$

$$= \frac{(\sum_{i=0}^{N} P_i^2) - N \times \mu_t^2}{N} = \frac{\sum_{i=0}^{N} P_i^2}{N} - \mu_t^2$$

P_i es cada pixel de la imagen sobre un total de N

$$\mu_t = W_1 \mu_1 + W_2 \mu_2 \,, \;\; W_1 + W_2 = 1$$

$$\mu_1 = \frac{\sum_{P_i \in C_1} P_i}{W_1 \times N} \,, \mu_2 = \frac{\sum_{P_i \in C_2} P_i}{W_2 \times N}$$

- V_w

$$= W_1 \times V_1 + W_2 \times V_2$$

$$= W_1 \times \left(\frac{\sum_{P_i \in C_1} (P_i - \mu_1)^2}{N \times W_1} \right) + W_2 \times \left(\frac{\sum_{P_i \in C_2} (P_i - \mu_2)^2}{N \times W_2} \right)$$

$$= \frac{\sum_{P_i \in C_1} (P_i - \mu_1)^2}{N} + \frac{\sum_{P_i \in C_2} (P_i - \mu_2)^2}{N}$$

$$= \frac{(\sum_{P_i \in C_1} P_i^2) - N \times W_1 \times \mu_1^2}{N} + \frac{(\sum_{P_i \in C_2} P_i^2) - N \times W_2 \times \mu_2^2}{N}$$

$$= \frac{\sum_{P_i \in C_1} P_i^2}{N} - W_1 \times \mu_1^2 + \frac{\sum_{P_i \in C_2} P_i^2}{N} - W_2 \times \mu_2^2$$

Derivaciones

- $V_b = V_t - V_w$

$$= \left(\frac{\sum_{i=1}^{N} P_i^2}{N} - \mu_t^2\right) - \left(\frac{\sum_{P_i \in C_1} P_i^2}{N} - W_1 \times \mu_1^2 + \frac{\sum_{P_i \in C_2} P_i^2}{N} - W_2 \times \mu_2^2\right)$$

$$= W_1 \times {\mu_1}^2 + W_2 \times {\mu_2}^2 - {\mu_t}^2$$

$$= W_1 \times {\mu_1}^2 + W_2 \times {\mu_2}^2 - 2 \times {\mu_t}^2 + {\mu_t}^2$$

$$\because \mu_t = W_1 \times \mu_1 + W_2 \times \mu_2$$

$$\because W_1 + W_2 = 1$$

$$= W_1 \times {\mu_1}^2 + W_2 \times {\mu_2}^2 - 2 \times (W_1 \times \mu_1 + W_2 \times \mu_2) \times \mu_t + (W_1 + W_2) \times {\mu_t}^2$$

$$= W_1 \times ({\mu_1}^2 - 2 \times \mu_1 \times \mu_t + {\mu_t}^2) + W_2 \times ({\mu_2}^2 - 2 \times \mu_2 \times \mu_t + {\mu_t}^2)$$

$$= W_1 \times (\mu_1 - \mu_t)^2 + W_2 \times (\mu_2 - \mu_t)^2$$

$$\because \mu_t = W_1 \times \mu_1 + W_2 \times \mu_2$$

$$= W_1 \times [(1 - W_1) \times \mu_1 - W_2 \times \mu_2]^2 + W_2 \times [W_1 \times \mu_1 - (1 - W_2) \times \mu_2]^2$$

$$\because W_1 + W_2 = 1$$

$$= W_1 \times [(W_2) \times \mu_1 - W_2 \times \mu_2]^2 + W_2 \times [W_1 \times \mu_1 - (W_1) \times \mu_2]^2$$

$$= W_1 \times W_2 \times (\mu_1 - \mu_2)^2 \times (W_2 + W_1)$$

$$\because W_1 + W_2 = 1$$

$$= W_1 \times W_2 \times (\mu_1 - \mu_2)^2$$

- V_b

$$= W_1 \times (\mu_1 - \mu_t)^2 + W_2 \times (\mu_2 - \mu_t)^2$$

$$= W_1 \times W_2 \times (\mu_1 - \mu_2)^2$$

Referencias

1. Nobuyuki Otsu (1979), “A Threshold Selection Method from Gray-Level Histograms” – https://ieeexplore.ieee.org/document/4310076