Maintenance Cost Optimization for Multiple Components Using a Condition Based Method

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***Abstract*—Since maintenance planning directly affect the availability and the lifecycle cost of components and systems, it has become a topic of great interest among researchers and industry practitioners in recent years. Preventive maintenance techniques can be adopted in order to determine a convenient maintenance schedule, reducing the number of unexpected failure events. The implementation of a preventive maintenance approach may also provide other benefits such as increase in equipment availability, reduction in maintenance costs and increase in equipment lifetime. In this scenario, the application of PHM (Prognostics and Health Monitoring) techniques can be thought as a powerful tool to support the implementation of a CBM (Condition Based Maintenance) approach. The problem of CBM optimization can be formulated as finding the optimum maintenance schedule for a set of components so that the average maintenance cost per unit of time is minimized. In this paper, a maintenance cost optimization method for multiple components is presented. The proposed method uses information on the health condition of each component and takes into account the economic benefits of repairing multiple components at the same time instead of scheduling maintenance interventions for different components in different time instants, based on individual optimization recommendations. A numerical example is presented to illustrate the application of the proposed method.**

***Keywords*—Prognostics; Health Monitoring; Optimization; Maintenance Planning; Multiple Components.**

I. INTRODUCTION

Condition Based Maintenance (CBM) is a proactive maintenance strategy that helps maintenance planners to schedule maintenance interventions based on the assessment of the health condition of components. The health condition is obtained by analyzing data such as temperature, pressure, current and vibration [1]. The main objective of Condition Based Maintenance methods is to determine an optimal maintenance policy to minimize the maintenance cost based on condition information [2].

Many papers have been published presenting quantitative methods for maintenance optimization based on failure probabilities distributions [3], [4], [5]. However, in most of the existing work reported in the literature, the maintenance interventions are optimized separately for each monitored component.

When multiple components are considered, maintenance cost optimization methods must consider the economic benefits of repairing multiple components at the same time instead of scheduling maintenance interventions for different components in different time instants, based on individual optimization recommendations [1]. In [2], the authors presented a proportional hazards model approach for CBM of multi-component systems. A CBM policy for multi-component systems with continuous stochastic deteriorations was proposed in [6].

This paper presents a maintenance cost optimization method for multiple components based on health condition information obtained from a PHM system. The proposed method considers the reduction in the overall maintenance cost obtained when some maintenance activities are grouped. The reduction in maintenance cost comes basically from a reduction in the fixed (or setup) cost.

II. PROBLEM DEFINITION

CBM optimization can be defined as finding the optimum maintenance schedule for a set of components so that the average maintenance cost per unit of time is minimized. The problem addressed in this paper is determining the optimum maintenance schedule for a group of similar components. It is assumed here that each component is monitored by a PHM system, and that inspections and maintenance interventions can only be performed at discrete times.

It is also assumed that the degradation accumulated by each monitored component between two consecutive inspections is a random variable and follows a known distribution. Many authors have used gamma processes to describe the degradation of systems [7]. A characteristic of this process is that it is monotonically increasing and positive, which makes it ideal for modeling deterioration processes. The degradation evolution process for a single component is illustrated in Fig. 1. In this paper, a gamma probability density function is considered, as described in Eq. (1).

$$f(x;k;\theta) = \frac{x^{k-1} \cdot \exp^{\frac{-x}{\theta}}}{\theta^k \cdot \Gamma(k)} \qquad (1)$$
$$for \quad x, k, \theta > 0$$

where x is the time instant, k is the shape parameter of the gamma distribution, θ is the scale parameter of the gamma distribution and $f(x,k,\theta)$ is the probability density function evaluated at x given k and θ. $\Gamma(k)$ is the Gamma function evaluated at k, as described in Eq. (2).

$$\Gamma(k) = \int_{t=0}^{\infty} t^{(k-1)} \cdot \exp^{-t} \, dt \qquad (2)$$

Moreover, the following assumptions are considered to be verified:

- The components are identical, and their degradation processes are independent from each other.
- The inspections are equally spaced in time.
- The time required for performing a maintenance activity is relatively small and can be ignored.
- The inspections are performed at zero cost.
- Maintenance interventions bring the component to "as good as new" state (degradation level returns to zero).
- A failure occurs whenever degradation is equal to or greater than a failure threshold level L.
- The increment in the degradation level is independent of the accumulated degradation level.
- During an inspection, all failed components must be repaired.
- To repair each failed component, a corrective maintenance cost C_{CM} is incurred.
- A degraded but still functional component can be repaired. In such a case, a preventive maintenance cost C_{PM} is incurred.
- A fixed maintenance cost C_F is incurred whenever a component is repaired. It does not matter the repair type (preventive or corrective).

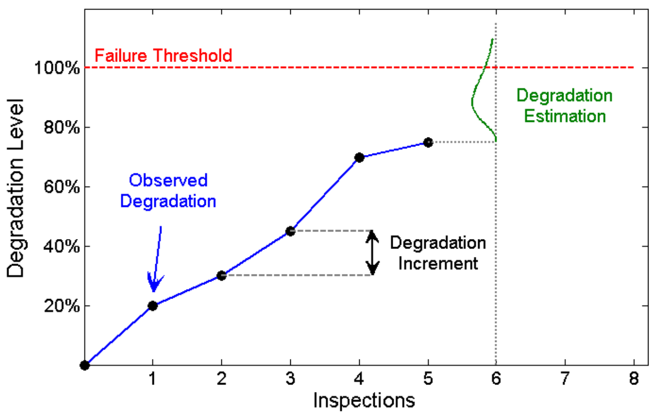

Fig. 1. Component degradation evolution

III. CBM Method for an Individual Component

In order to optimize the overall maintenance cost of a group of components, it is necessary to have a component level health condition estimation for each component. It means that at a certain inspection point, a failure probability distribution for each component must be generated. In this paper, the failure probability distribution for each component is obtained by Eq. (3).

$$P_{j+1} = P\left(F(x,k,\theta) \geq FT - D_j / D_j < FT\right) \qquad (3)$$

where $F(x,k,\theta)$ is the cumulative gamma distribution function, P_j is the probability that a failure will be detected in the *j-th* inspection, D_j is the degradation level measured in the *j-th* inspection and FT is the failure threshold.

When failed components are found during an inspection, for these components $P_0 = 1$ and $P_j = 0, \forall j > 0$. For the components that are not failed, $P_0 = 0$ and $P_j \ (j > 0)$ can be obtained using Eq.(4).

$$P_j = \left[1 - \sum_{y=0}^{j-1} P_y\right] \cdot P\left(F(x, j \cdot k, \theta) \geq FT - D_{j-1}\right) \qquad (4)$$

Fig. 2(A) shows an example of a failure probability distribution. In this example, the values of the shape parameter k and the scale parameter θ of the gamma distribution are 3.0 and 4.0, respectively. It was assumed a failure threshold of 100 and an initial degradation of 10.

The failure probability distribution is used to calculate the component expected maintenance cost in the *j-th* inspection, EMC_j. It can be obtained using Eq. (5).

$$EMC_j = \sum_{y=0}^{j} P_y \cdot C_{CM} + \left(1 - \sum_{y=0}^{j} P_y\right) \cdot C_{PM} \qquad (5)$$

Fig. 2(B) shows the corresponding expected maintenance cost for the failure probability distribution shown in Fig. 2(A). In this example, it was assumed a preventive maintenance cost C_{PM} of \$450 and a corrective maintenance cost C_{CM} of \$1,000.

The next step for the component level CBM is to calculate the expected maintenance cost per unit of time considering that the maintenance will be performed in the *j-th* inspection, $EMCT_j$. It can be obtained using Eq. (6).

$$EMCT_j = \frac{EMC_j}{j + TSI} \qquad (6)$$

where *TSI* is the time since installation, i.e., the number of inspections since the last maintenance activity performed in the component under consideration.

Fig. 2(C) shows the corresponding expected maintenance cost per unit of time for the example. The *TSI* used in this example is 2, i.e., it is assumed that the last maintenance activity in the component under consideration was performed two inspections ago.

Finally, the fixed (or setup) maintenance cost, C_F, is added and the expected total maintenance cost per unit of time in the *j-th* inspection, C_j, is calculated using Eq. (7).

$$C_j = EMCT_j + \frac{C_F}{j + TSI} \tag{7}$$

Fig. 2(D) shows the expected total maintenance cost per unit of time for the example. The setup cost used in this example is \$100.

In this example, the optimal maintenance recommendation for the component under consideration is to schedule the maintenance for the 5th inspection.

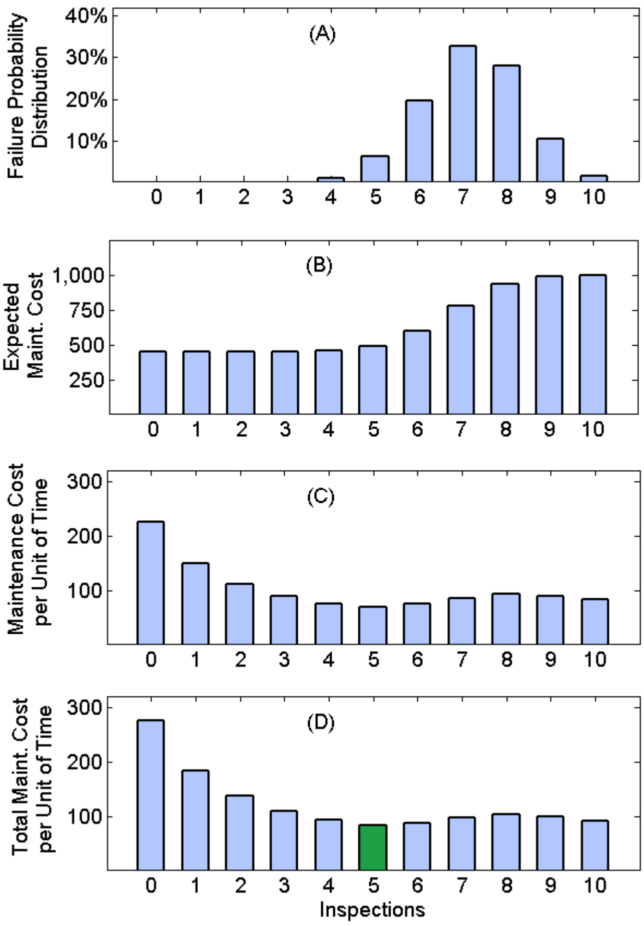

Fig. 2. Estimated maintenance cost per unit of time in future inspections

IV. CBM METHOD FOR MULTIPLE COMPONENTS

In this section, a method to optimize the overall maintenance cost of multiple components is presented.

Basically, the proposed method uses the optimization framework proposed in [8]. In this method, an algorithm that uses the principle of dynamic programming is used to find the optimal maintenance schedule for a group of components.

The information obtained from the PHM system is used to estimate the maintenance cost for each component as described in the previous section. Then, these individual component estimations are analyzed and may be processed in order to meet all the requirements for the application of the search space reduction approach presented in [8]. Finally, the dynamic programming algorithm is applied to find the final solution. The proposed model is described in more details in the next sections.

A. General Description of the Method

The proposed method is similar to other methods presented in the literature [8], [9], [10]. It is composed by the following steps:

- Estimation of the failure probability density function for each component.
- Individual maintenance cost optimization
- Reduction of the search space
- Maintenance activities grouping
- Maintenance scheduling and rolling-horizon update

B. Estimation of the Failure Probability Density Functions

In this step, the failure probability density function for each component is estimated using Eq. (3) and Eq. (4), as illustrated in Fig. 2(A). The proposed method differs from the other methods found in the literature because it takes into account the information regarding the health condition of the monitored component, generated by the PHM system, for building the failure probability density functions.

C. Individual Maintenance Cost Optimization

In this step, the maintenance cost per unit of time for component i if its maintenance is schedule for the *j-th* inspection, $C_j^{(i)}$, is obtained using Eq. (7). Then, the optimal maintenance schedule for component *i*, t_i , is the value that minimizes $C_j^{(i)}$.

The penalty function, $h_i(j)$, is the increment in maintenance cost per unit of time when the maintenance of component *j* is not performed in t_i. The following assumptions regarding the penalty functions are considered to be verified:

- **Assumption 1:** $h_i(\cdot)$ *has a unique minimum,* $i = 1,\ldots,n$.
- **Assumption 2:** $h_i(\cdot)$ *is decreasing left of its minimum,* $i = 1,\ldots,n$.
- **Assumption 3:** $h_i(\cdot)$ *is increasing right of its minimum,* $i = 1,\ldots,n$.
- **Assumption 4:** $h_i(\cdot) \geq 0$, $i = 1,\ldots,n$.
- **Assumption 5:** $h_i(0) = 0$, $i = 1,\ldots,n$.

D. Reduction of the Search Space

In this step, the expected maintenance cost for all components are analyzed together, and some rules are applied in order to reduce the search space. A more detailed explanation of the theorems cited in this section is presented in [8].

Assume that there are n components, and for each component there is an optimal maintenance execution time, t_i, $i = 1,\ldots,n$. Without loss of generality, assume that the components are sorted so that $t_1 \leq t_2 \leq \cdots \leq t_n$. Also, for each component, assume that there is an interval $I_i = \left[t_i - \Delta t_i^-, t_i + \Delta t_i^+\right]$, that defines the maximum allowable shift (backward and forward) for the execution time of maintenance in component *i*.

The theorems that are used in order to reduce the search space are now presented:

- **Theorem 1:** *A group G can only be part of an optimal solution if* $\bigcap_{i\in G} I_i \neq$ *Ø. Furthermore, the optimal execution time* t_G *of group G is in* $\bigcap_{i\in G} I_i$.
- **Theorem 2:** *A group F cannot be part of an optimal solution if it contains a cluster G that can be split more efficiently into two clusters of activities.*

After reducing the search space using theorems 1 and 2, the remaining possible solutions can still be reduced if there is an optimal solution in which every group has consecutive maintenance activities. To do this, we must guarantee that the following property is verified:

- **Property 1:** *For all* $i = 1,\ldots,n-1$, $h_i(\cdot)$ *dominates* $h_{i+1}(\cdot)$ *for* $j > t_{i+1}$ *and for all* $i = 2,\ldots,n$, $h_i(\cdot)$ *dominates* $h_{i-1}(\cdot)$ *for* $j < t_{i-1}$, *i.e.,* $h_i(t_{i+1} - t_i + \Delta t) > h_{i+1}(\Delta t)$ *for* $\Delta t > 0$ *and* $h_i(t_{i-1} - t_i - \Delta t) > h_{i-1}(-\Delta t)$ *for* $\Delta t > 0$.

For each component *i*, property 1 only needs to be satisfied on its interval I_i. The application of theorems 3 and 4 below completes the reduction of the search space.

- **Theorem 3:** *If property 1 holds, there is an optimal solution with consecutive activities.*
- **Theorem 4:** *If in an optimal solution of the first s activities the activity s is executed in another group than an activity p (1 ≤ p < s), then for any r > s there is an optimal solution of the first r activities in which activity s is also executed in another group than activity p.*

E. Maintenance Activities Grouping

In this step, a variation of the algorithm presented in [8] is used in order to find the optimum maintenance schedule. This algorithm is based on the principle of dynamic programming and uses the theorems presented above.

In the first iteration, the algorithm finds the group with minimum cost among all groups of consecutive maintenance activities that have activity *n* as the last activity, i. e., the best group among groups $\{1,\ldots,n\},\ldots,\{n-1,\ldots,n\},\{n\}$.

Assume that in the first iteration the algorithm found the group $\{i,\ldots,n\}$. In the next iteration, the algorithm will find the group with minimum cost among all groups of consecutive maintenance activities that have activity *i*-1 as the last activity, i. e., the best group among groups $\{1,\ldots,i-1\},\ldots,\{i-2,\ldots,i-1\},\{i-1\}$.

The algorithm stops when all maintenance activities have been scheduled.

F. Maintenance Scheduling and Rolling Horizon Update

The result obtained with the algorithm is used to schedule the maintenance activities for all the components. This schedule will be updated when new information on the health condition of any component becomes available.

V. Numerical Example

In this section, a numerical example is presented in order to illustrate the application of the proposed method.

In the example, there are ten identical components. The current degradation level and the *TSI* (Time Since Installation) of each component are listed in Table I. For simplicity, the components are sorted so that $t_1 \leq t_2 \leq \cdots \leq t_n$. The preventive maintenance cost, the corrective maintenance cost and the fixed cost are, respectively, \$300, \$800 and \$100. A failure occurs whenever the degradation level is 100 or higher. The increase in the degradation level of each component between two consecutive inspections follows a gamma distribution. The values of the shape parameter and the scale parameter of the gamma distribution are 3 and 4, respectively.

TABLE I. COMPONENT DATA

Component	TSI	Current Degradation Level
1	7	90
2	9	70
3	9	65
4	4	55
5	4	50
6	6	35
7	3	35
8	3	30
9	1	15
10	1	10

Table II shows the expected maintenance cost for each component, according to Eq. (6). Theorems for reduction of the search space were already applied. A "-" indicates that scheduling the maintenance for the *j-th* inspection is out of the allowable interval for the component under consideration. Fig. 3 illustrates the application of the dynamic programming algorithm to this example.

In the **first iteration**, the algorithm finds the best group of consecutive activities that has activity 10 as its last activity. In the example, the best result is obtained scheduling the maintenance activities for group {6-7-8-9-10} in *j*=5. According to the theorems for reduction of the search space, the group {6-7-8-9-10} is part of an optimal solution.

In the **second iteration**, since the schedule for group {6-7-8-9-10} is already defined, the algorithm finds the best group of consecutive activities that has activity 5 as its last activity. In the example, the best result is obtained scheduling the maintenance activities for group {2-3-4-5} in *j*=2.

Finally, in the **third iteration**, the algorithm would find the best group of consecutive activities that has activity 1 as its last activity. Since it is not possible to split this group into two groups, the maintenance of component 1 is scheduled for the individual optimal maintenance date of this component (*j=0*), and the algorithm has found the final solution, shown in Fig. 4. For comparison purposes, in Fig. 4, the individual optimal maintenance execution time for each component is shown in parenthesis.

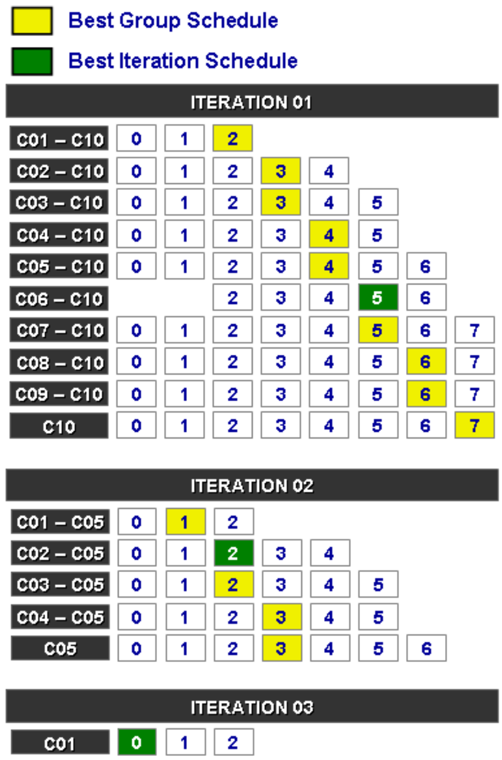

Fig. 3. Algorithm iterations

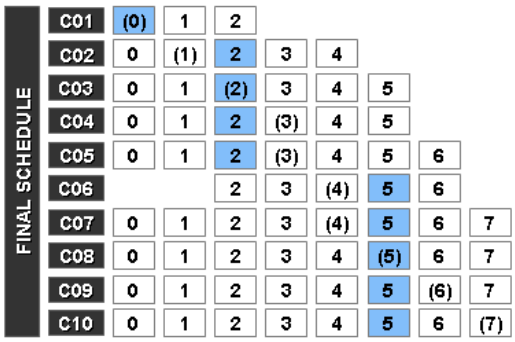

Fig. 4. Final solution

TABLE II. EXPECTED MAINTENANCE COST

Component	TSI	j = 0	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7
1	7	300.0	508.0	768.5	-	-	-	-	-
2	9	300.0	$305.2	$370.5	$564.4	$747.3	-	-	-
3	9	300.0	$301.8	$333.7	$469.1	$674.7	$783.3	-	-
4	4	300.0	$300.2	$306.6	$353.7	$494.2	$680.3	-	-
5	4	300.0	$300.1	$302.7	$327.6	$422.8	$597.3	$744.4	-
6	6	-	-	$300.2	$302.9	$322.0	$391.1	$532.7	-
7	3	$300.0	$300.0	$300.2	$302.9	$322.0	$391.1	$532.7	$691.3
8	3	$300.0	$300.0	$300.1	$301.3	$311.3	$354.5	$462.7	$620.8
9	1	$300.0	$300.0	$300.0	$300.1	$301.3	$309.2	$341.0	$423.4
10	1	$300.0	$300.0	$300.0	$300.0	$300.6	$304.8	$323.9	$381.1

In this example, the maintenance of only three components (1, 3 and 8) will coincide with their individual optimal maintenance dates. Six components (2, 4, 5, 6, 7 and 9) will have their maintenance dates shifted (backward or forward) one period in comparison with their individual optimal maintenance dates, while component 10 will have its individual optimal maintenance date anticipated two periods.

The expected maintenance cost per unit of time considering the maintenance schedule suggested by the proposed method is \$45.8. If maintenance grouping was not considered and the maintenance activity of each component was performed in its optimal individual date, the expected cost would be \$50.5. In this example, the application of the proposed method would cause a reduction of 9.3% in the maintenance cost.

VI. CONCLUSIONS

A method to optimize the maintenance cost of multiple components was presented. The method uses health condition information of each monitored component in order to define the optimal maintenance schedule for the whole set of components. It also takes into consideration the economic dependency among the components, and reduces the overall maintenance cost by scheduling preventive maintenance activities for multiple components simultaneously, instead of scheduling maintenance interventions for different components in different time instants, based on individual optimizations.

In this paper, inspections were considered to be equality distributed in time, but it is not a restriction of the method. It is only necessary to know how the degradation level of each component increases between two consecutive inspections.

The presented method may easily incorporate the occurrence of maintenance opportunities and also allows the maintenance manager to include additional restrictions. Maintenance opportunities may occur, for example, due to a failure of one component. In this case, a maintenance intervention in the failed component must take place, and preventive maintenance activities can be carried out without incurring an additional fixed cost.

One example of manager decision that can be easily incorporated in the presented method is the decision to perform maintenance activities in two or more components as a group. In this case, a new activity that combines the costs of all components the manager wants to group is defined and the method is applied in the same way.

Future research may extend the presented model by considering that the inspections are not equally distributed in time. Another opportunity for future development is to consider the inspections dates as the optimization variables.

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