
Quiz-4 CS618

Duration: 45 Minutes Max Marks: 50

- Write your name and roll number on the question paper and the answer book.
- No explanations will be provided. In case of a doubt, make suitable assumptions and justify.
- There are 2 questions on 2 pages.

Useful Definitions

- Simply Typed λ-terms

$$\begin{array}{rcll} \mathtt{t} & := & x & \text{– } \textit{Variable} \\ & | & \lambda x : T.\ \mathtt{t} & \text{– } \textit{Abstraction} \\ & | & \mathtt{t}\ \mathtt{t} & \text{– } \textit{Application} \end{array}$$

- The Set of Values

$$v \quad := \quad \lambda x : T.\ \mathtt{t} \qquad \text{– } \textit{Abstraction Value}$$

- The Evaluation Rules

$$\frac{\mathtt{t}_1 \to \mathtt{t}_1'}{\mathtt{t}_1\ \mathtt{t}_2 \to \mathtt{t}_1'\ \mathtt{t}_2} \qquad \text{(E-App1)}$$

$$\frac{\mathtt{t}_2 \to \mathtt{t}_2'}{v\ \mathtt{t}_2 \to v\ \mathtt{t}_2'} \qquad \text{(E-App2)}$$

$$(\lambda x : T_1.\ \mathtt{t}_1) v_2 \to [x \mapsto v_2]\mathtt{t}_1 \qquad \text{(E-AppAbs)}$$

- The Typing Rules

$$\frac{\Gamma, x : T_1 \vdash \mathtt{t}_2 : T_2}{\Gamma \vdash \lambda x : T_1.\ \mathtt{t}_2 : T_1 \to T_2} \qquad \text{(T-Abs)}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \qquad \text{(T-Var)}$$

$$\frac{\Gamma \vdash \mathtt{t}_1 : T_1 \to T_2 \qquad \Gamma \vdash \mathtt{t}_2 : T_1}{\Gamma \vdash \mathtt{t}_1\ \mathtt{t}_2 : T_2} \qquad \text{(T-App)}$$

(P.T.O.)

1. (**20 Marks**) For each of the term t below, find the types T_1, T_2, etc. such that t has a valid type T. If such a type can not be found, show why.[1]

   - $\lambda x : T_1 \; y : T_2. \; x \; y$
   - $\lambda x : T_1 \; y : T_2. \; x \; y \; y$
   - $(\lambda x : T_1 \; y : T_2. \; x \; y \; y) \; (\lambda x : T_3 \; y : T_4. \; x \; y)$

2. (**30 Marks**) We define a λ-calculus program to be **COMPLETE** if it does not contain any free variables (variables that are not bound by any λ in the program).

   (a) [10] Complete the following analysis that checks for complete λ-calculus programs.

$$\frac{x : T \in \Gamma}{\Gamma \vdash \text{complete}(x)} \quad \text{(Comp-Var)}$$

$$\frac{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}{\Gamma \vdash \text{complete}(\lambda x : T_1. \; \mathtt{t}_2)} \quad \text{(Comp-Abs)}$$

$$\frac{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}{\Gamma \vdash \text{complete}(\mathtt{t}_1 \; \mathtt{t}_2)} \quad \text{(Comp-App)}$$

   (b) [10 + 10] Define *Progress* and *Preservation* specific for the analysis rules above. Explain the intuition behind the definitions.

[1] You might want to use α-renaming to avoid issues with variable name reuse.