

UNITED COLLEGE OF ENGINEERING & RESEARCH		DEPARTMENT OF APPLIED SCIENCES & HUMANITIES			
I Sessional Exam		Semester : I	Section: A to M	Date: 16.01.21	
Time: 2hrs		Subject: Maths	Paper code: KA5-103	MM: 30	
READ ALL INSTRUCTIONS AND QUESTIONS CAREFULLY				Marks	
SECTION A (Attempt all questions)					
1	a	If $y=x^2e^{2x}$ determine $y_n(0)$ .	[6]	CO	BL
			1	2	L3
1	b	If $z = x^y$ find $\frac{\partial^2 z}{\partial x \partial y}$ .	1	3	L2
1	c	Find stationary points of $5x^2 + 10y^2 + 12xy - 4x - 6y + 1$ .	1	3	L1
1	d	If $y = x^3 \log x$ , show that $y_{4=\frac{6}{x}}$ .	1	2	L2
1	e	Define homogeneous function. Find degree of $f(x, y) = \frac{x^2 y}{x+y}$ .	1	3	L1
1	f	If $u = lx + my, v = mx - ly$ , then find $\frac{\partial(x,y)}{\partial(u,v)}$ .	1	3	L2
SECTION B (Attempt any three questions)					
			[9]		
2	If $u = \sec^{-1} \left( \frac{x^3+y^3}{x+y} \right)$ , prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$ . Also evaluate $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial y^2}$ .		3	3	L4
3	If $y = e^{a \sin^{-1} x}$ , prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$ . Deduce that $\lim_{x \rightarrow 0} \frac{y_{n+2}}{y_n} = n^2 + a^2$ .		3	2	L3
4	The torsional rigidity of a length of a wire is obtained from the formula $\frac{8\pi n L}{t^2 r^4}$ . If L is decreased by 2%, r increased by 2%, t increased by 1.5%, show that the value of N is diminished by 13% approximately.		3	3	L2
5	If $u = f(r)$ , where $r^2 = x^2 + y^2$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$ .		3	3	L2
SECTION C (Attempt any three questions)					
			[15]		
6	Expand $e^x \tan^{-1} y$ in powers of $(x - 2)$ and $(y - 1)$ up to the terms of degree 2.		5	3	L2
7	Use Jacobian to show that the functions $u = y + z, v = x + 2z^2, w = x - 2y^2 - 4yz$ are not independent of one another. Find the relation between them.		5	3	L3
8	If $y = (\sin^{-1} x)^2$ then show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$ and hence calculate $y_n$ when $x=0$ .		5	2	L3
9	Find the maximum and minimum distance of the point $(1, 2, -1)$ from the sphere $x^2 + y^2 + z^2 = 24$ .		5	3	L3
END OF PAPER					