

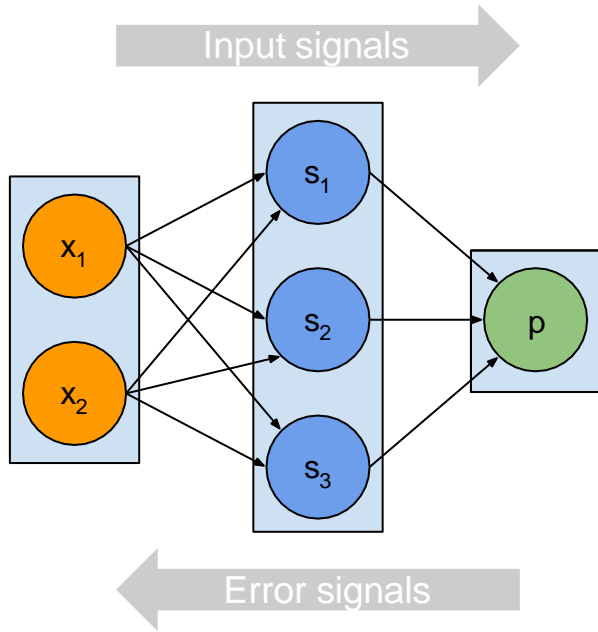
Training a neural network: Backward propagation and gradient descent

by
Mausam
Chouksey

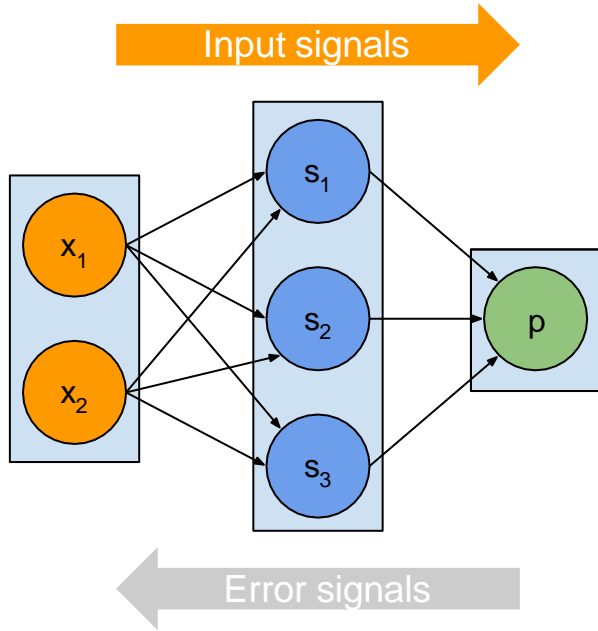
Training a neural network

- Tweak weights of the connections
- Feed training data (input + target) to the network
- Iterative adjustments

Training a neural network

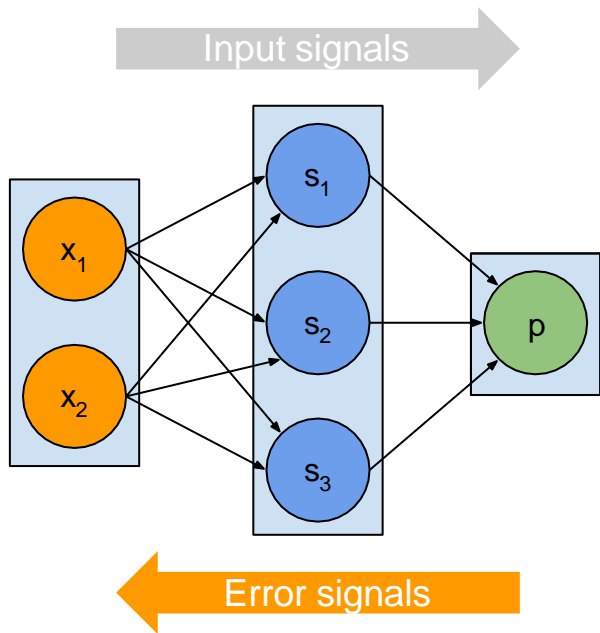


Training a neural network



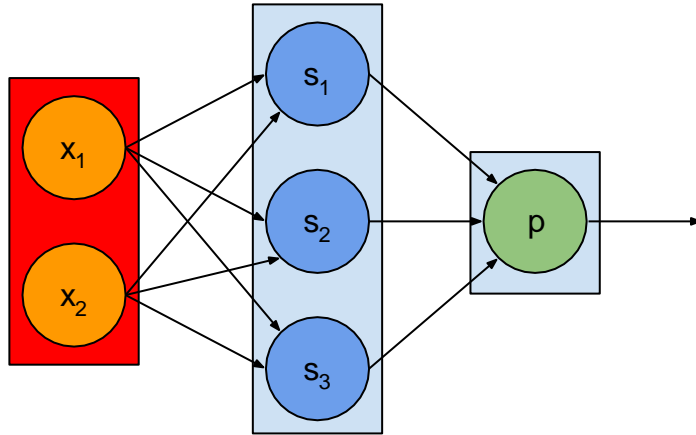
1. Get prediction
2. Calculate error
3. Calculate gradient of error function over the weights
4. Update parameters

Training a neural network

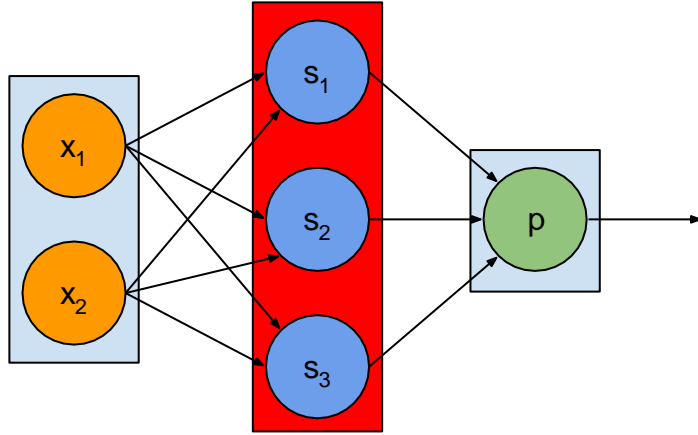


1. Get prediction
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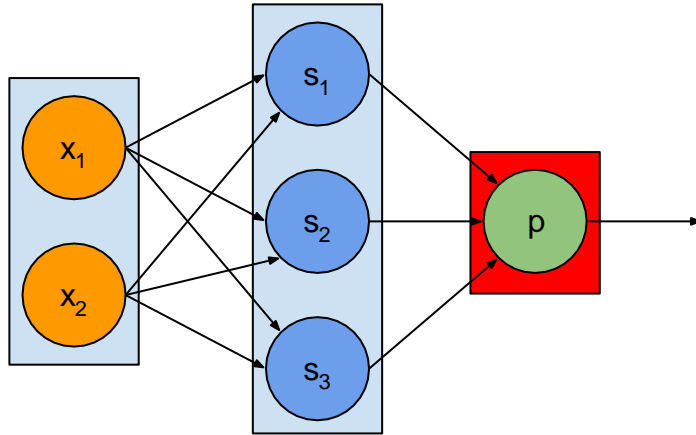
Get prediction



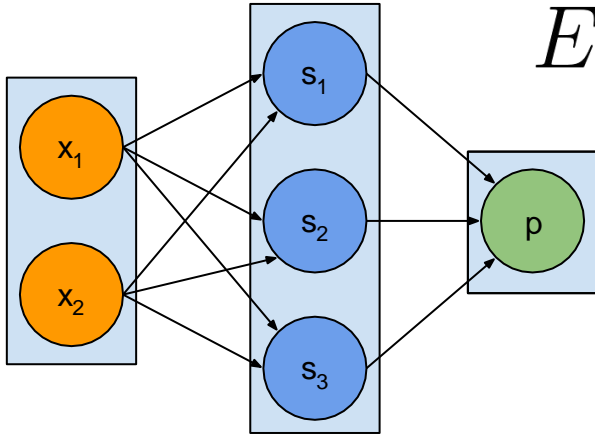
Get prediction



Get prediction

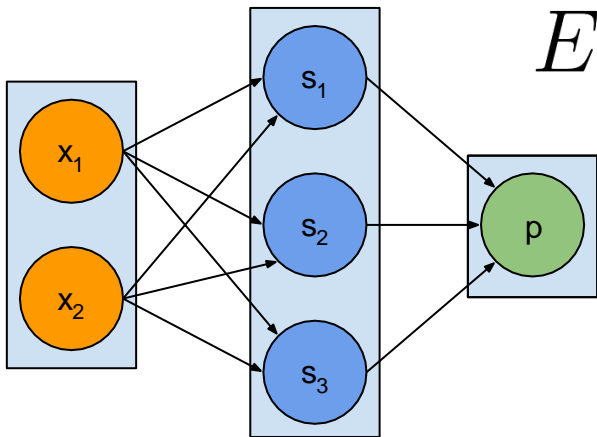


Calculate error



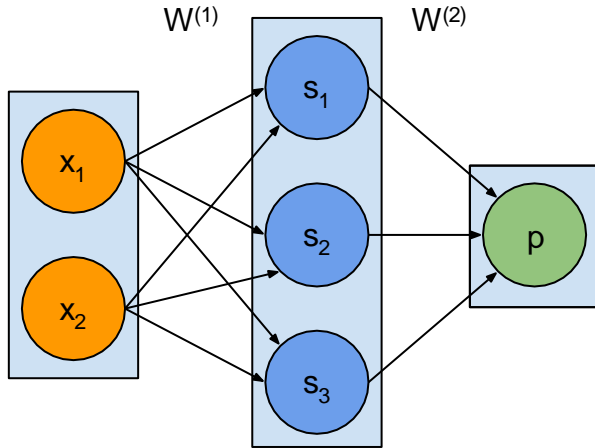
$$E = E(\mathbf{p}, \mathbf{y})$$

Calculate error



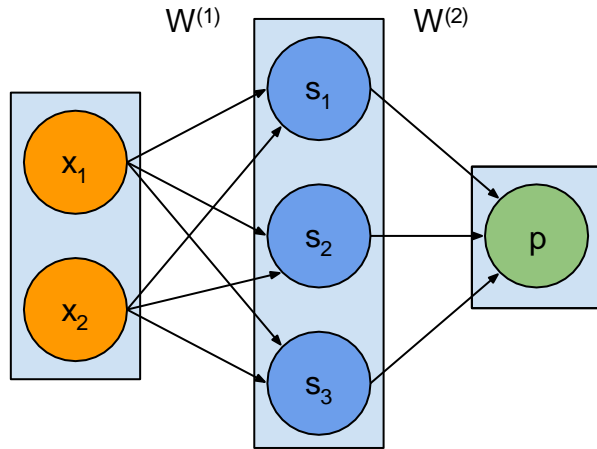
$$E = E(\mathbf{p}, \mathbf{y}) = \frac{1}{2}(\mathbf{p} - \mathbf{y})^2$$

Calculate gradient of error function



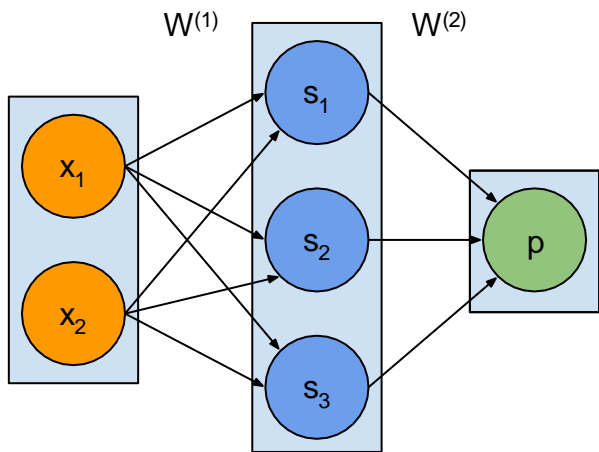
$$\frac{\partial E}{\partial W^{(n)}}$$

Calculate gradient of error function



$$F = F(\mathbf{x}, W)$$

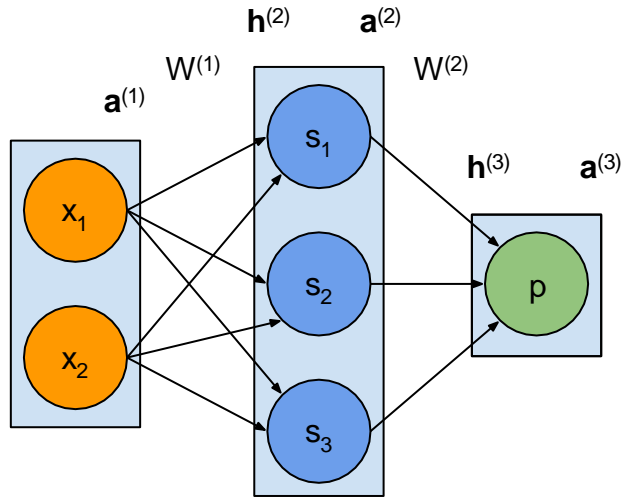
Calculate gradient of error function



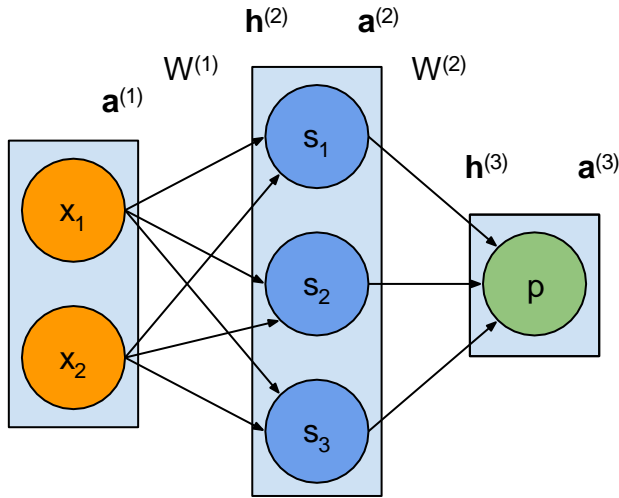
$$F = F(\mathbf{x}, W)$$

$$E = E(\mathbf{p}, \mathbf{y}) = E(F(\mathbf{x}, W), \mathbf{y})$$

Calculate gradient of error function

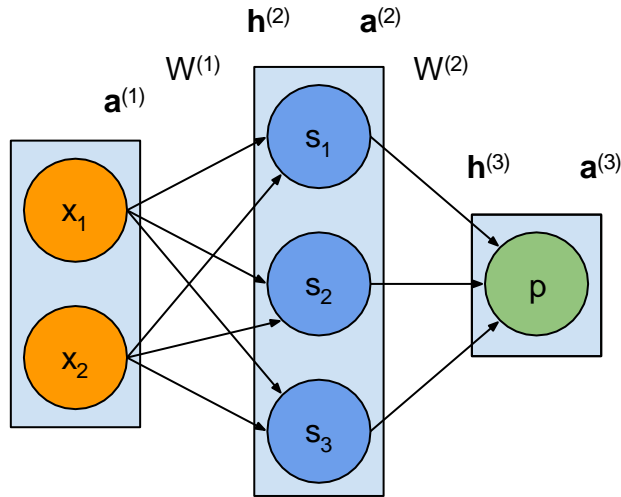


Calculate gradient of error function



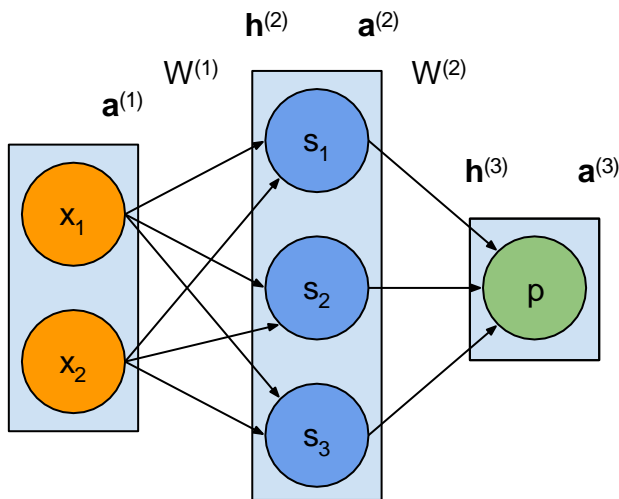
$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \boxed{\frac{\partial E}{\partial a^{(3)}}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

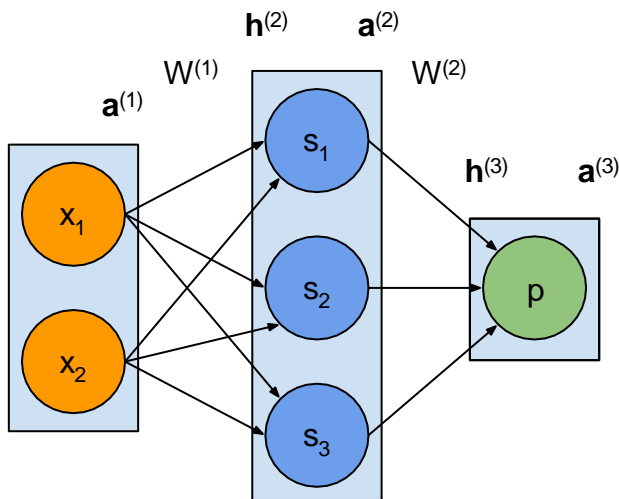
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial \boxed{E}}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$\boxed{E = \frac{1}{2}(a^{(3)} - y)^2}$$

Calculate gradient of error function

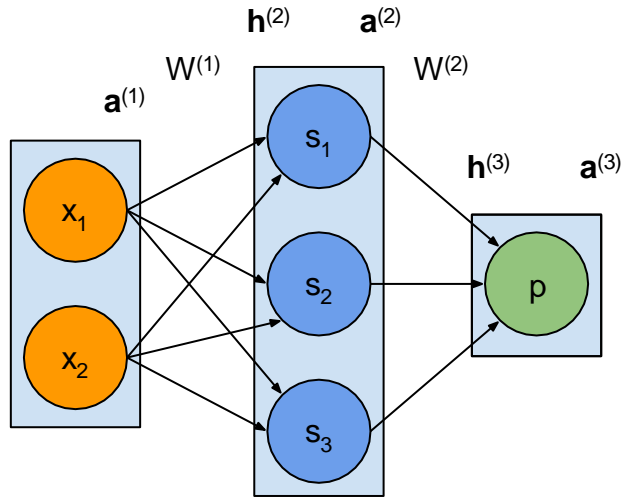


$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$E = \frac{1}{2}(a^{(3)} - y)^2$$

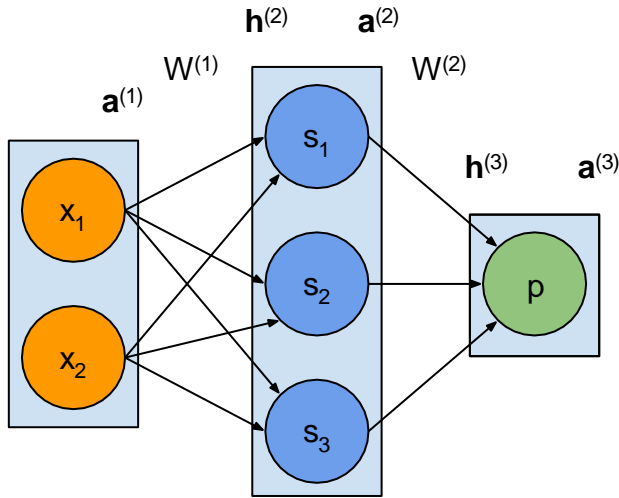
$$\frac{\partial E}{\partial a^{(3)}} = 2 \cdot \frac{1}{2}(a^{(3)} - y) \cdot 1 = a^{(3)} - y$$

Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

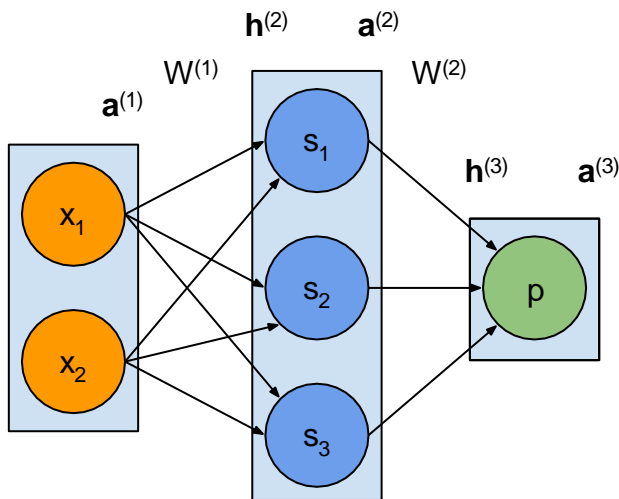
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$a^{(3)} = \frac{1}{1 + e^{-h^{(3)}}}$$

Calculate gradient of error function

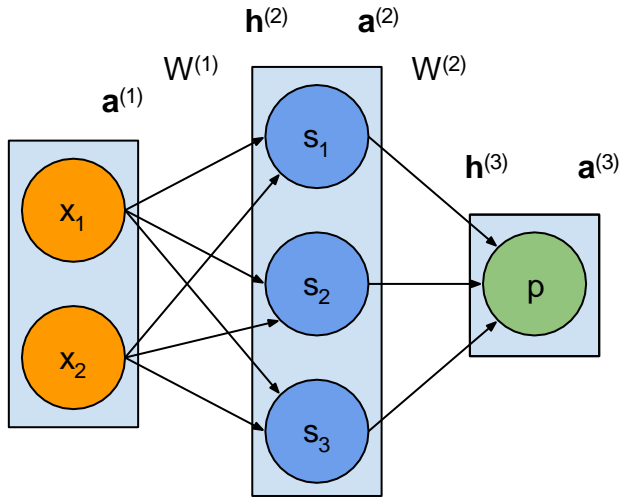


$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \boxed{\frac{\partial a^{(3)}}{\partial h^{(3)}}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$a^{(3)} = \frac{1}{1 + e^{-h^{(3)}}}$$

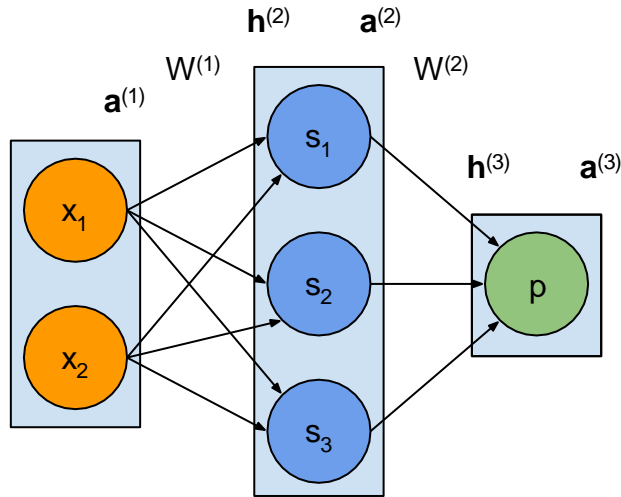
$$\boxed{\frac{\partial a^{(3)}}{\partial h^{(3)}} = \sigma'(h^{(3)}) = \sigma(h^{(3)})(1 - \sigma(h^{(3)}))}$$

Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial W^{(2)}}}$$

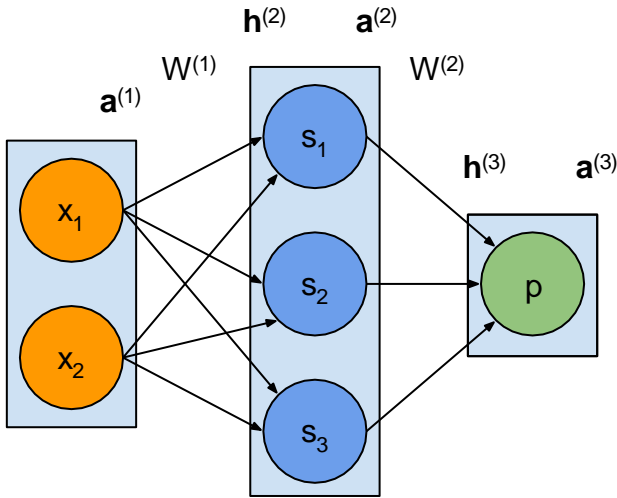
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$h^{(3)} = a^{(2)} W^{(2)}$$

Calculate gradient of error function

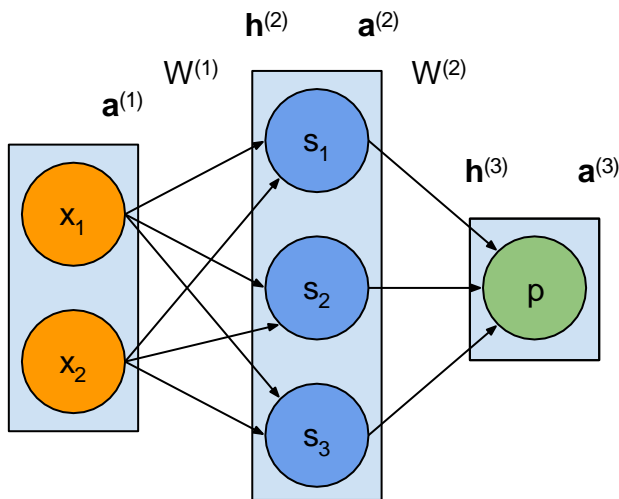


$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial W^{(2)}}}$$

$$h^{(3)} = a^{(2)} W^{(2)}$$

$$\boxed{\frac{\partial h^{(3)}}{\partial W^{(2)}} = a^{(2)}}$$

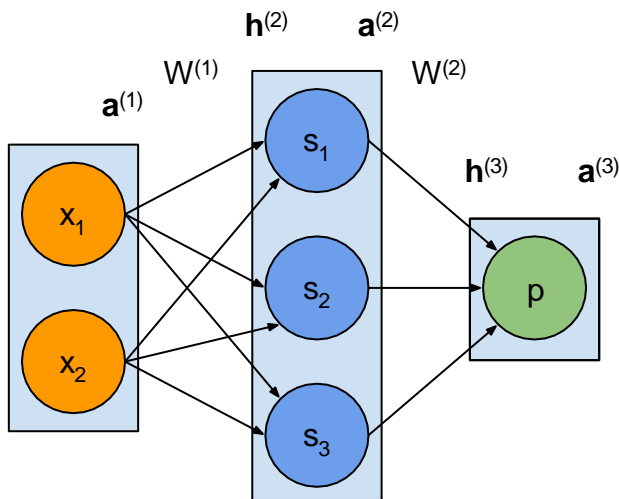
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$\frac{\partial E}{\partial W^{(2)}} =$$

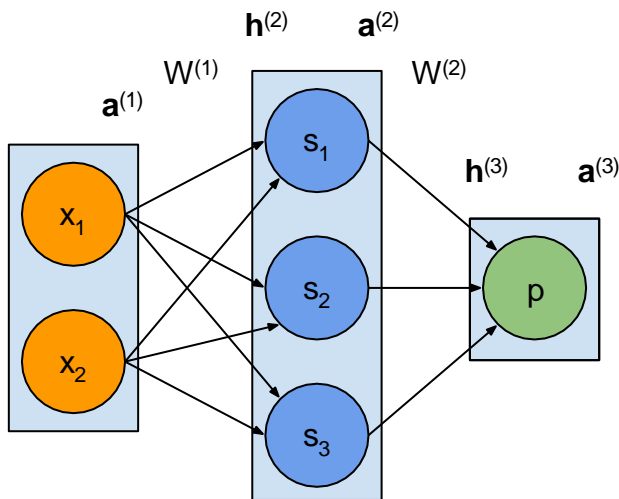
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \boxed{\frac{\partial E}{\partial a^{(3)}}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$\frac{\partial E}{\partial W^{(2)}} = \boxed{(a^{(3)} - y)}$$

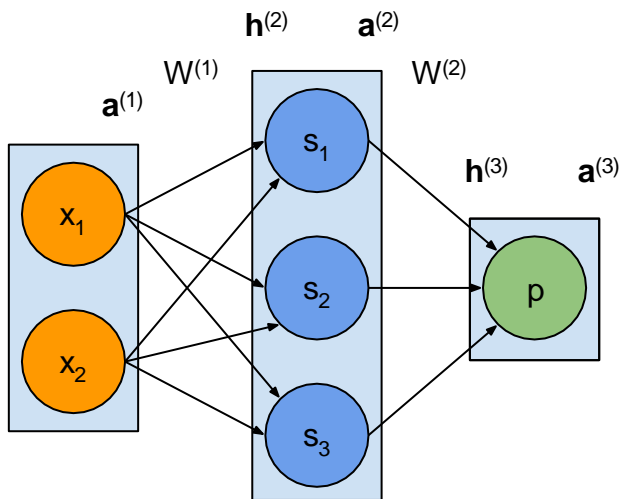
Calculate gradient of error function



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial W^{(2)}}$$

$$\frac{\partial E}{\partial W^{(2)}} = (a^{(3)} - y) \sigma'(h^{(3)})$$

Calculate gradient of error function

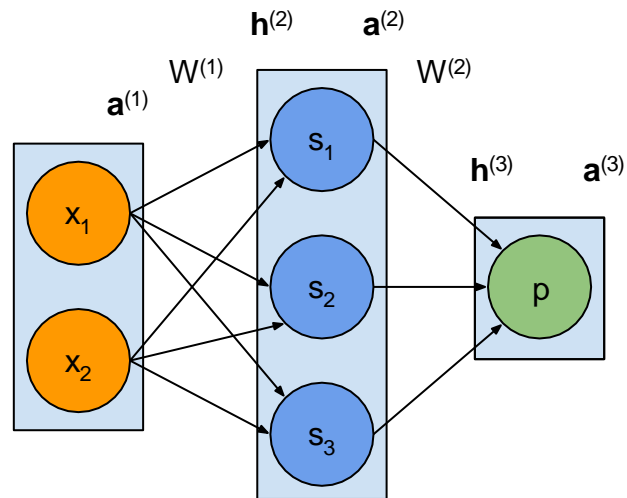


$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial W^{(2)}}}$$

$$\frac{\partial E}{\partial W^{(2)}} = (a^{(3)} - y) \sigma'(h^{(3)}) \boxed{a^{(2)}}$$

Calculate gradient of error function

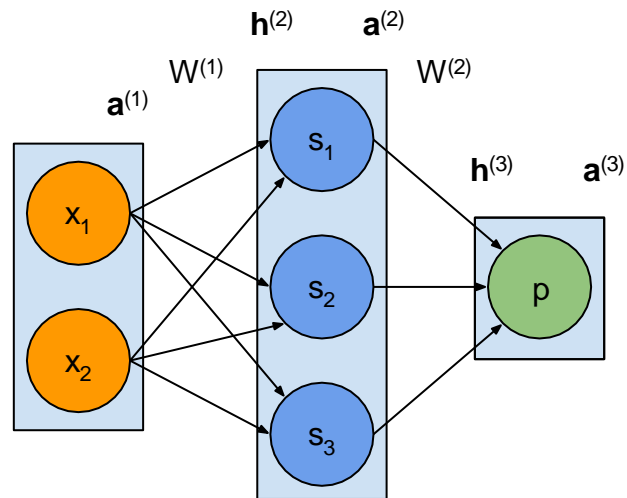
$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$\frac{\partial E}{\partial a^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}}$$

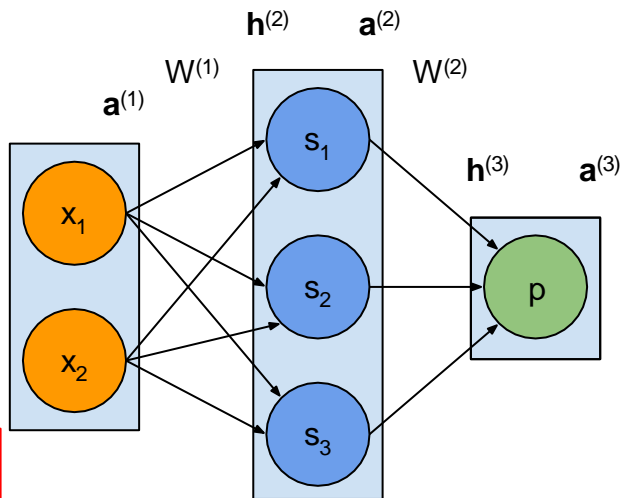


Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

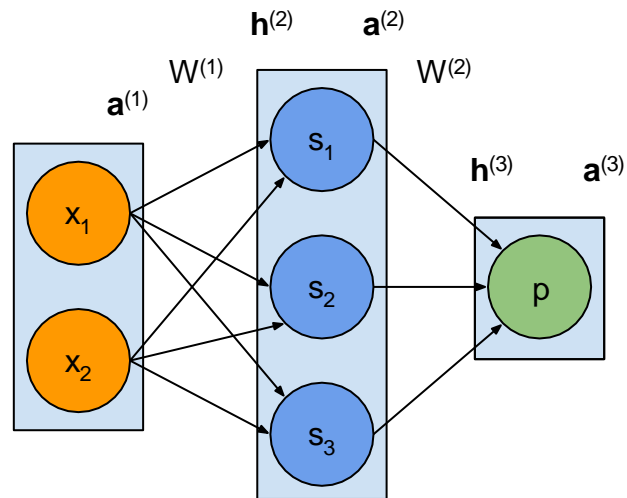
$$\frac{\partial E}{\partial a^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}}$$

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$



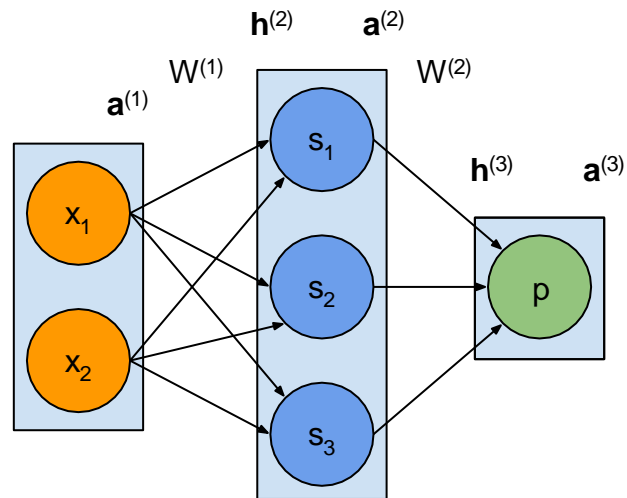
Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}}$$



Calculate gradient of error function

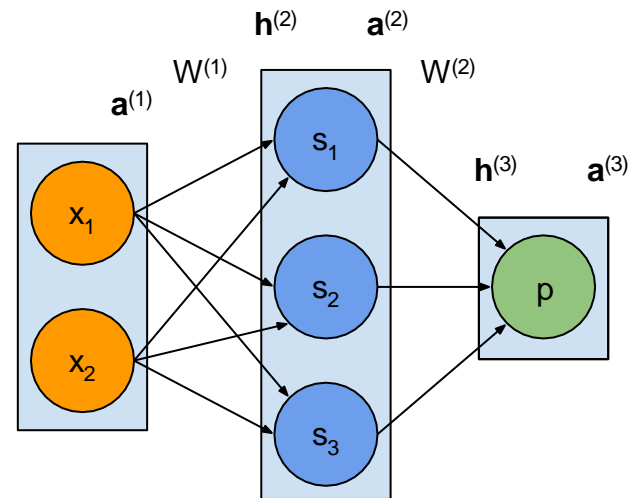
$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$h^{(3)} = a^{(2)} W^{(2)}$$

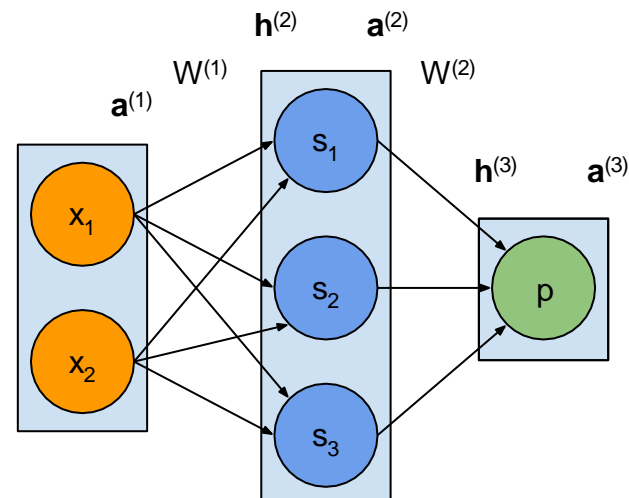


Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial a^{(2)}}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

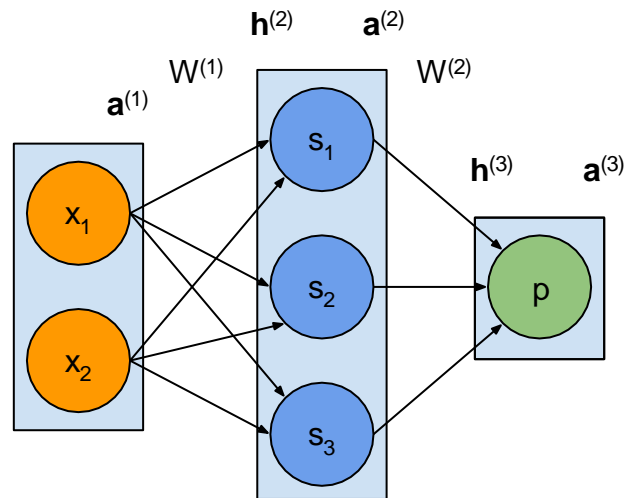
$$h^{(3)} = a^{(2)} W^{(2)}$$

$$\boxed{\frac{\partial h^{(3)}}{\partial a^{(2)}} = W^{(2)}}$$



Calculate gradient of error function

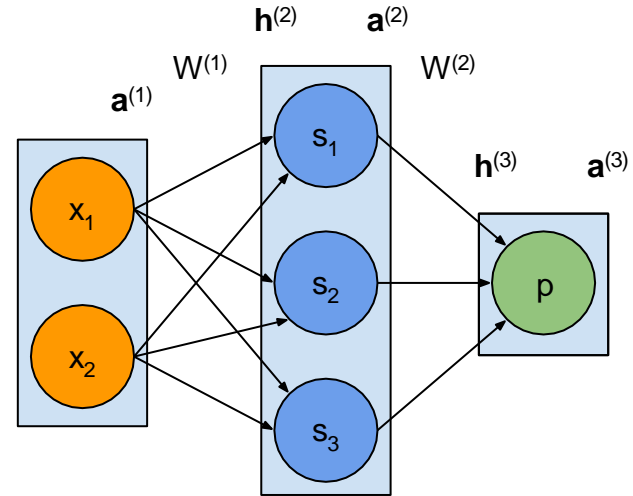
$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$a^{(2)} = \frac{1}{1 + e^{-h^{(2)}}}$$

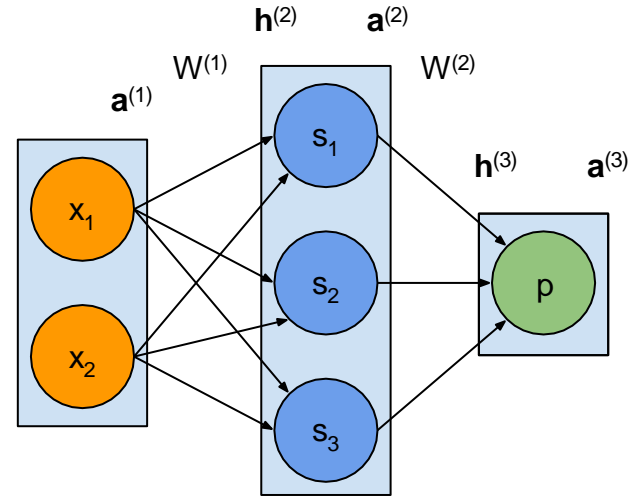


Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

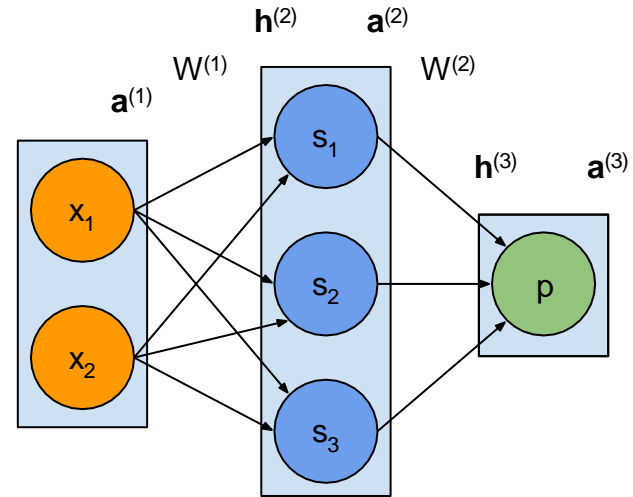
$$a^{(2)} = \frac{1}{1 + e^{-h^{(2)}}}$$

$$\frac{\partial a^{(2)}}{\partial h^{(2)}} = \sigma'(h^{(2)}) = \sigma(h^{(2)})(1 - \sigma(h^{(2)}))$$



Calculate gradient of error function

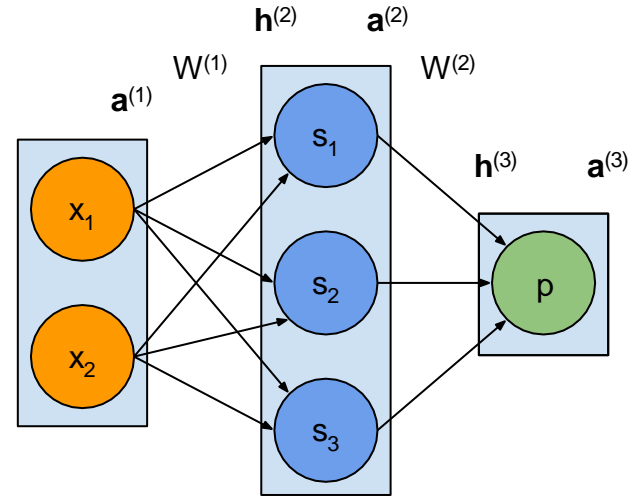
$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$h^{(2)} = xW^{(1)}$$

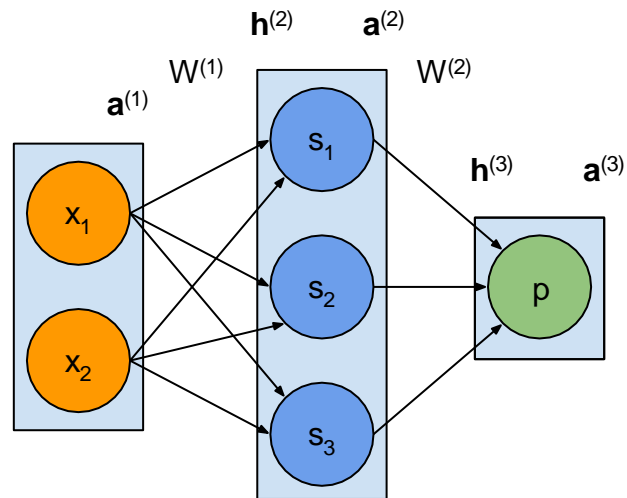


Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$h^{(2)} = xW^{(1)}$$

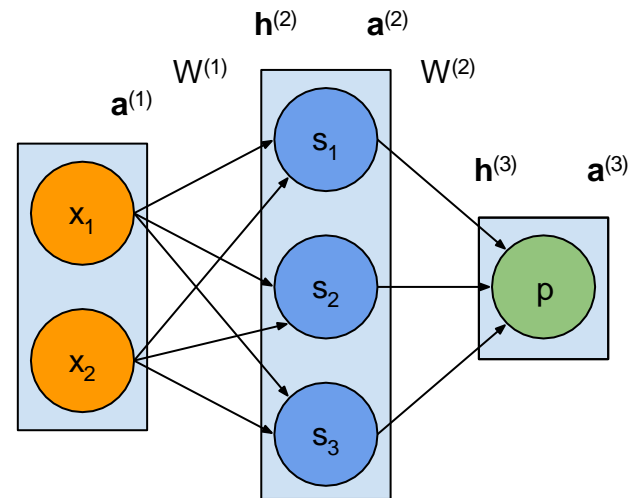
$$\frac{\partial h^{(2)}}{\partial W^{(1)}} = x$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

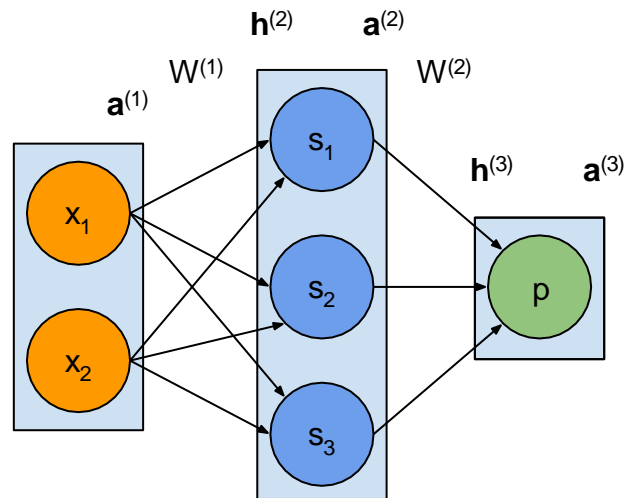
$$\frac{\partial E}{\partial W^{(1)}} =$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

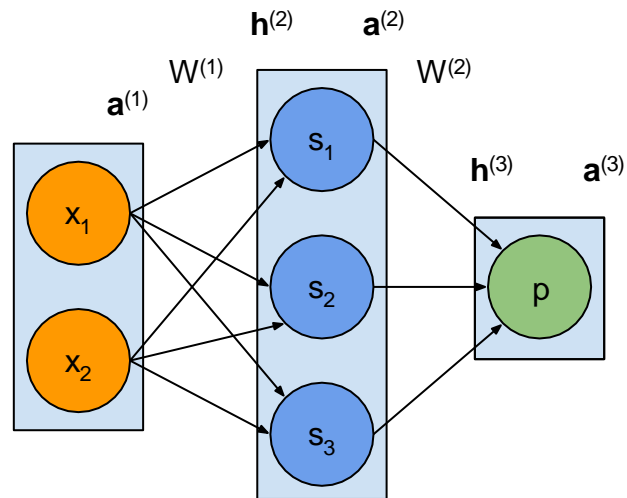
$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y)$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

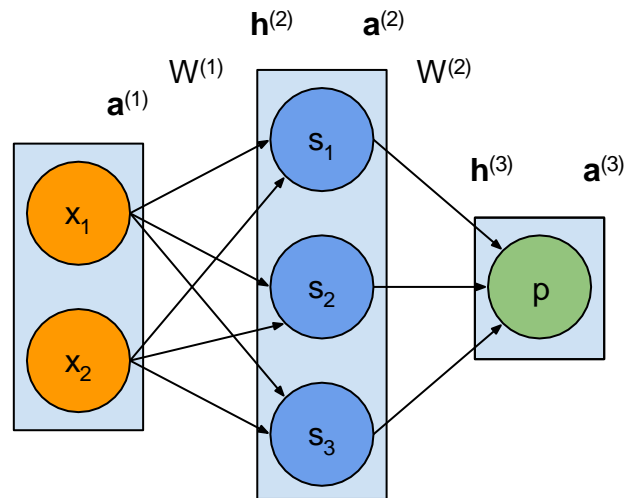
$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y) \sigma'(h^{(3)})$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

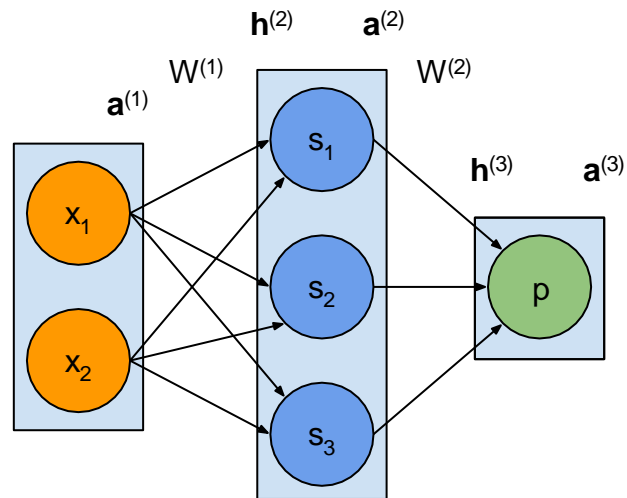
$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y) \sigma'(h^{(3)}) W^{(2)}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \boxed{\frac{\partial a^{(2)}}{\partial h^{(2)}}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

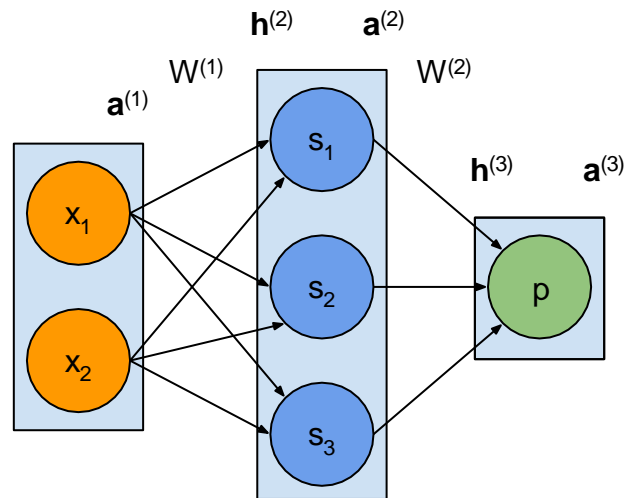
$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y) \sigma'(h^{(3)}) W^{(2)} \boxed{\sigma'(h^{(2)})}$$



Calculate gradient of error function

$$\frac{\partial E}{\partial W^{(1)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \frac{\partial h^{(3)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial W^{(1)}}$$

$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y) \sigma'(h^{(3)}) W^{(2)} \sigma'(h^{(2)}) x$$

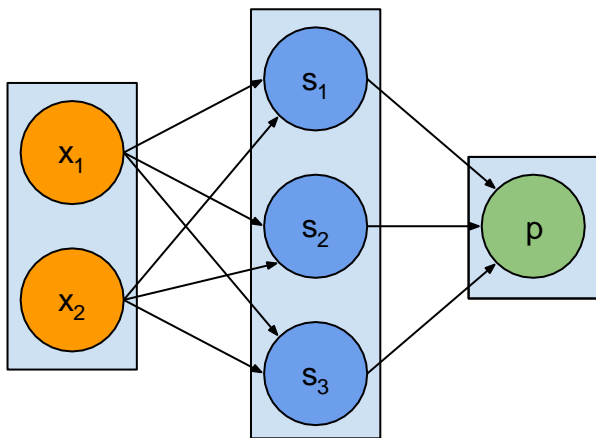


Calculate gradient of error function

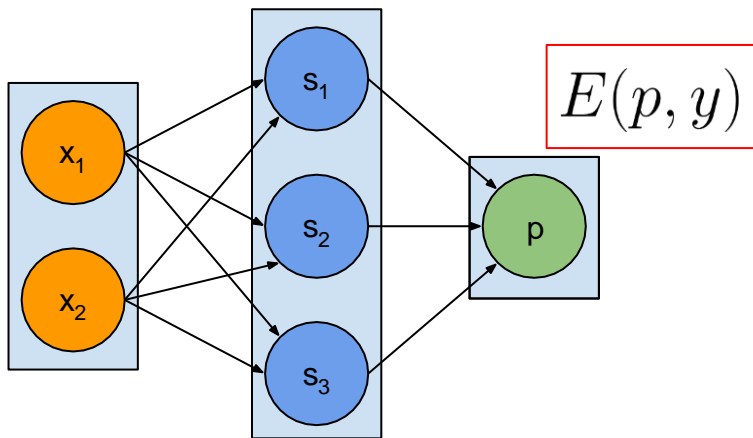
$$\frac{\partial E}{\partial W^{(2)}} = (a^{(3)} - y)\sigma'(h^{(3)})a^{(2)}$$

$$\frac{\partial E}{\partial W^{(1)}} = (a^{(3)} - y)\sigma'(h^{(3)})W^{(2)}\sigma'(h^{(2)})x$$

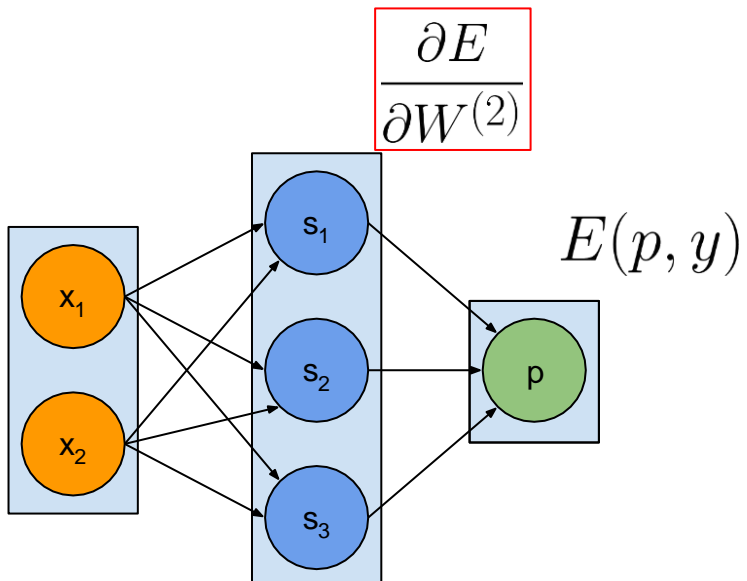
Backpropagation



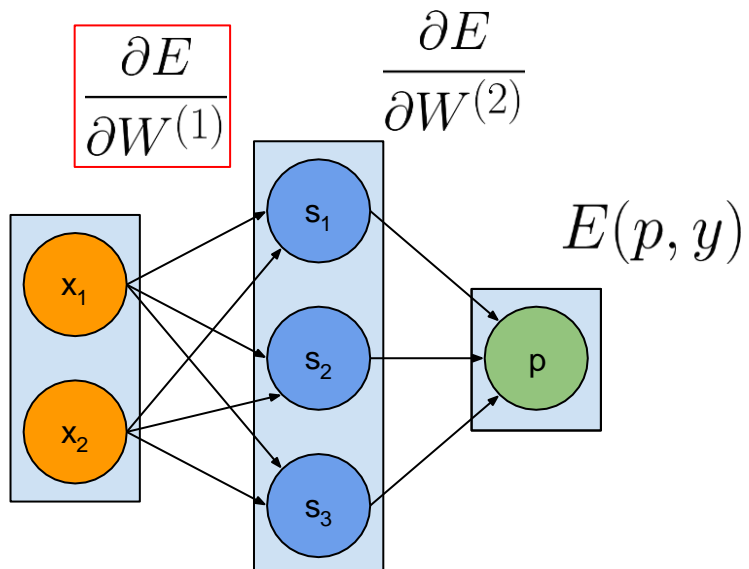
Backpropagation



Backpropagation



Backpropagation



Training steps

1. Get prediction
2. Calculate error
3. Calculate gradient of error function over the weights
4. Update parameters

Gradient descent

- Take a step in opposite direction to gradient
- Step = Learning rate

