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1. $\begin{pmatrix}\varepsilon_i \\ T_i\end{pmatrix} \sim N\left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}\right)$ ← joint distribution of $\begin{pmatrix}\varepsilon_i \\ T_i\end{pmatrix}$ unconditional of X_i

From this, $T_i \perp \varepsilon_i$ because $\mathrm{cov}(T_i, \varepsilon_i) = 0$ & this is a joint distribution independent of X_i. This necessarily also implies that $T_i \perp \varepsilon_i \mid X_i$ because they are independent, $\mathrm{cov}(t_i, \varepsilon_i) = 0$ $\forall$ values of x

2. $\mathrm{cov}(T_i, X_i) = \mathrm{Cov}(T_i, aT_i + u_i) = a\,\mathrm{Cov}(T_i, T_i) + \mathrm{cov}(T_i, U_i) = a$

$\mathrm{cov}(T_i, X_i) = \rho_2$ from Ω

$\therefore \rho_2 = a$

3. $E(y_i \mid T_i) = E(\gamma T_i + \beta X_i + \varepsilon_i \mid T_i) = \gamma E(T_i \mid T_i) + \beta E(X_i \mid T_i) + \cancel{E(\varepsilon_i \mid T_i)}$

$= \gamma T_i + \beta E(aT_i + \mu \mid T_i) = \gamma T_i + a\beta E(T_i \mid T_i) + \beta E(\mu \mid T_i)$

$= \gamma T_i + a\beta T_i = (\gamma + a\beta) T_i = (\gamma + \rho_2 \beta) T_i$

$\Rightarrow \clubsuit = \gamma + \rho_2 \beta$

4. In order for $E(\gamma + \beta\rho_2) = \gamma$, we need $\rho_2 = 0$ because $\beta > 0$. This implies T_i & X_i will be uncorrelated $\Rightarrow$ if T_i & X_i are uncorrelated, the coefficient of T_i from $E(y_i \mid T_i)$ is an unbiased estimate of γ.

5. $\rho_1 = \mathrm{cov}(X_i, \varepsilon_i) = \mathrm{cov}(X_i, b \cdot T_i + cX_i + v_i)$

$= b\,\mathrm{cov}(X_i, T_i) + c\,\mathrm{cov}(X_i, X_i) + \mathrm{cov}(X_i, V_i)$

$= b\rho_2 + c$

$\Rightarrow \rho_1 = b\rho_2 + c$

$0 = \mathrm{cov}(T_i, \varepsilon_i) = \mathrm{cov}(T_i, b \cdot T_i + cX_i + v_i)$

$= b\,\mathrm{cov}(T_i, T_i) + c\,\mathrm{cov}(T_i, X_i) + \mathrm{cov}(T_i, v_i)$

$= b + c\rho_2$

$\Rightarrow 0 = b + c\rho_2$

6. $E(y_i | X_i = x, T_i = t) = E(\gamma T_i + \beta X_i + \varepsilon_i | X_i = x, T_i = t)$

$= \gamma E(T_i | X_i = x, T_i = t) + \beta E(X_i | X_i = x, T_i = t) + E(\varepsilon_i | X_i = x, T_i = t)$

$= \gamma t + \beta x + \frac{\rho_1}{1-\rho_2^2} x - \frac{\rho_1 \rho_2}{1-\rho_2^2} t$

$= \left(\gamma - \frac{\rho_1 \rho_2}{1-\rho_2^2}\right) t + \left(\beta + \frac{\rho_1}{1-\rho_2^2}\right) x$

$\Rightarrow$ [redacted] $= \gamma - \frac{\rho_1 \rho_2}{1-\rho_2^2}$ & [redacted] $= \beta + \frac{\rho_1}{1-\rho_2^2}$

7. [redacted] will be unibase if $\frac{\rho_1 \rho_2}{1-\rho_2^2} = 0$, which happens if $\rho_1 = 0$ and/or $\rho_2 = 0$ and $\rho_2 \neq \pm 1$

This is essentially saying that if X_i is uncorrelated with ε_i and/or T_i while also not being perfectly correlated with ε_i, [redacted] is an unbiased estimata of γ

8. I would say that's ridiculous. This is possible when ρ_1 is large & ρ_2 is small.