CONTINUES WEEK 3

Electric Potential

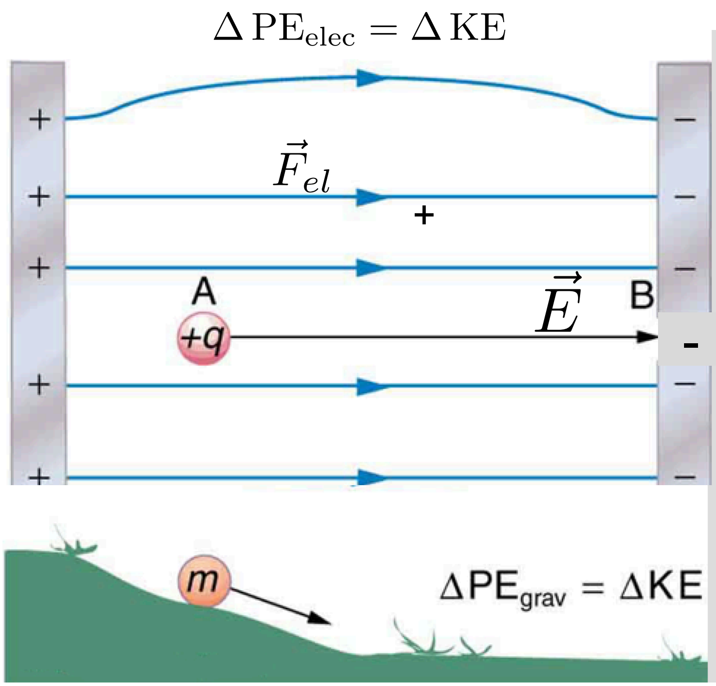
Chapter 17

- Electric Potential Energy and Electrical Potential
  - In a uniform Electric Field
  - In an electric field due to Point Charges
- Storing Charge:
  - Capacitance
  - Capacitors with Dielectrics
- Stored Electrical Potential Energy

Electric Potential Energy and Electric Potential (17-1,2)

A charge in an electrical field can undergo changes in its kinetic energy and in its *electrical potential energy*. This is because it feels a force, which implies work can be done on it.

In the Figure below: the free charge $+q$ **converts electrical potential energy it had on the left to kinetic energy** on the right.



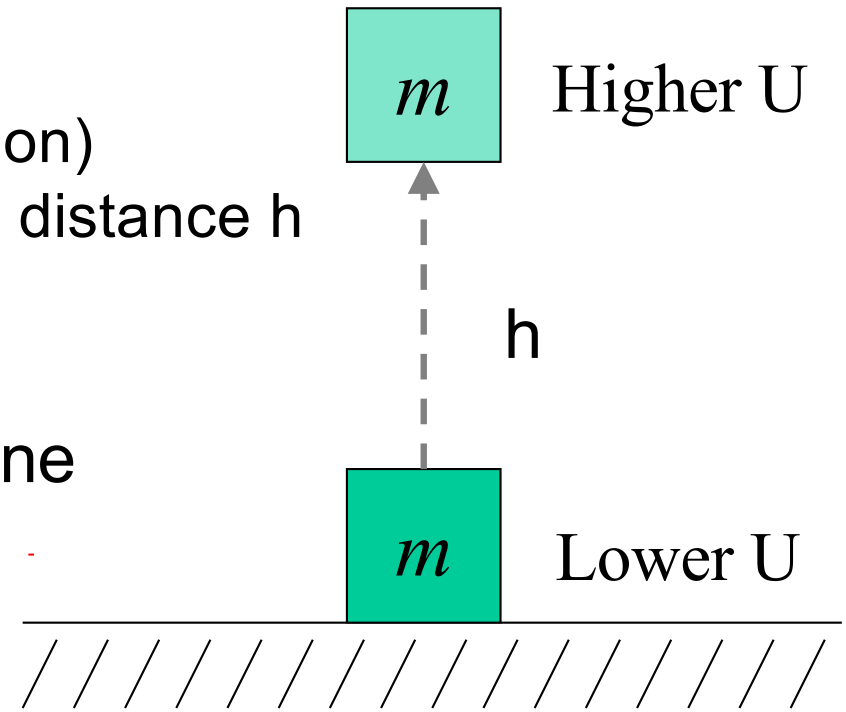
The charge has a *higher* electrical potential energy near the positive plate

The charge has a *lower* electrical potential energy near the negative plate

These sections study changes of electrical potential in a *uniform* electric field.

Analogy with Gravity

Begin by analogy with gravity (see Chap. 6 in Text)



Gravitational Potential Energy

Work required to raise (without acceleration) an object of mass m from the ground to a distance h

Work-Energy Theorem:

Change in potential energy = work done

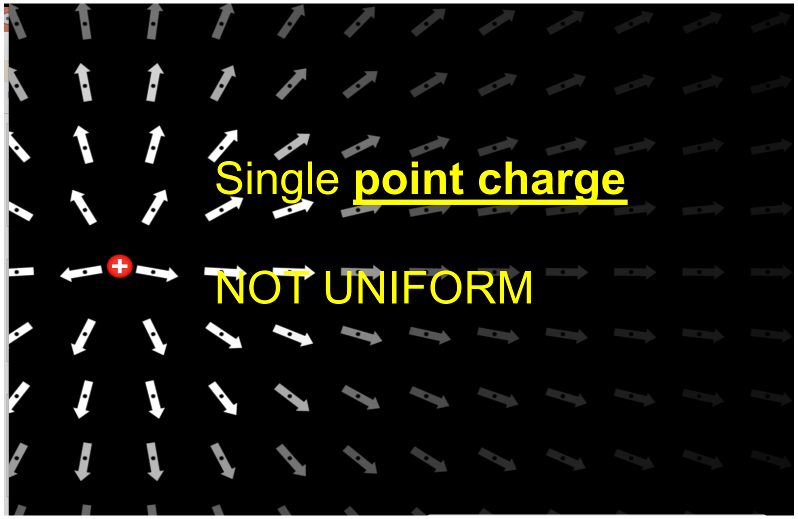
$\Delta U = \Delta W_{\text{ext}} =$ External force X Displacement

Equivalent

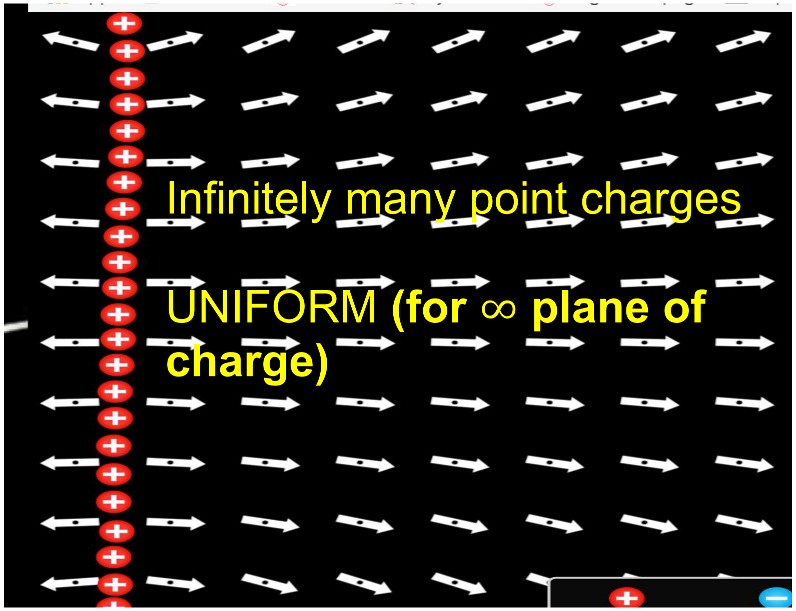
$=$ - *(*gravitational force*)* X Displacement

Ch16 Review: Uniform Vs. Non-Uniform Electric Fields

The figure below shows an electric field from a point charge is NOT constant, it changes in space.



Evenly distributed charges on the surface of a plane give a nearly constant electric field near the centre. For an infinite plane covered uniformly with charges, it becomes exactly constant everywhere in space. That is called ***a uniform electric field***.



Results presented in 17.1-2 only hold when the electric field is constant everywhere in space.

Change of Electrical ***Potential Energy*** in a ***Uniform*** E Field

Definition (in words)

Negative of the **work done by electric field** during the displacement of charge q from A to B

Definition for **a Uniform Electric Field** (formula)

$$\Delta U_{AB} = -\vec{F}_{el} \cdot \vec{d}$$

displacement vector of q

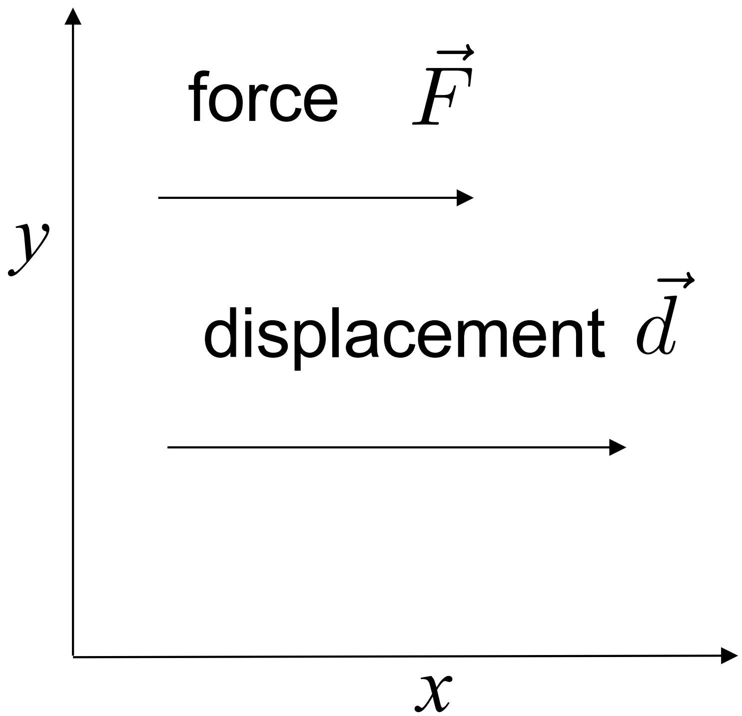
change of electric **potential energy** between A and B

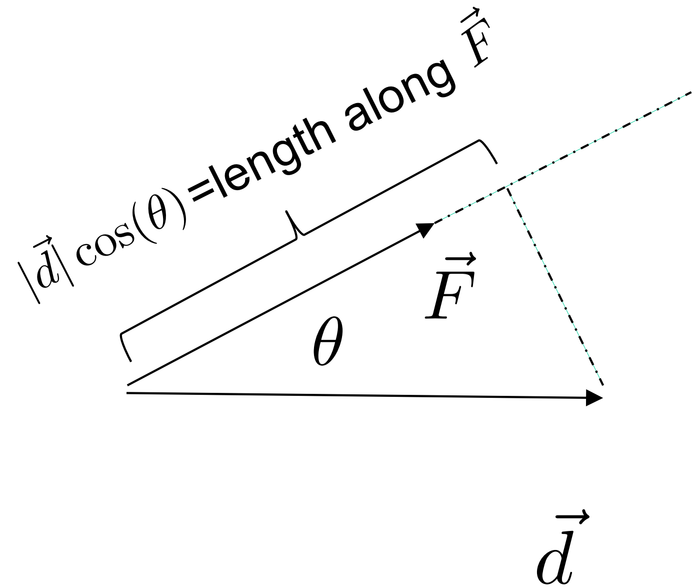
"negative makes it the work of an external force"

electric force vector on charge on q

Vector Math Digression: Review of "*Dot Product*"

$$\boxed{\vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}| \cos(\theta)}$$





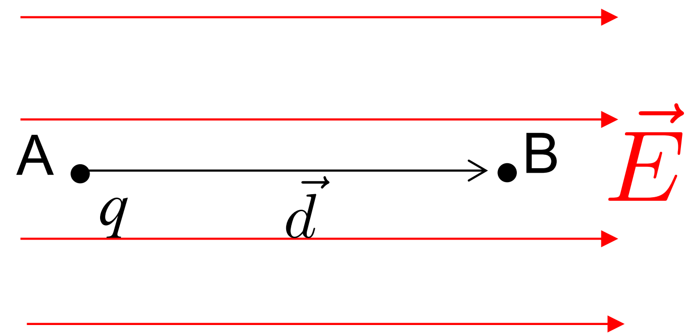
$\theta = 0, 180$

$$\Longrightarrow \; \vec{F} \cdot \vec{d} = \underbrace{(\pm F)\,(\pm d)}_{\text{+/- means relative to x-axis}}$$

$$\Longrightarrow \vec{F} \cdot \vec{d} = F \times \underbrace{\text{displacement parallel to } \vec{F}}_{\substack{\text{positive in direction of F } \theta < 90 \\ \text{negative opposite to F } \theta > 90}}$$

Illustrating Change of Electric **Potential Energy**

Calculate the change in electric potential energy when a q is displaced slowly from A → B as in figure to the right



***Uniform* electric field** with magnitude E

Positive direction of electric field

Assume q is ***positive*** with magnitude q:

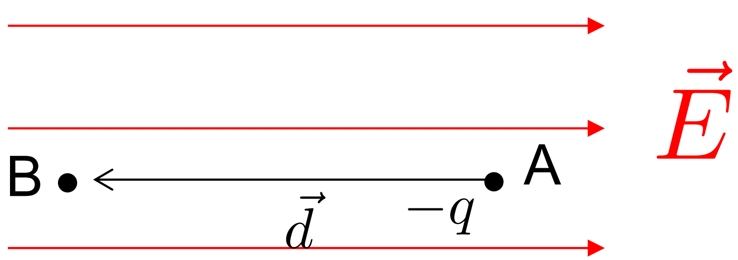
$$\Delta U_{AB} = -\vec{F}_{el} \cdot \vec{d} = -(+qE)d = -qEd$$

Assume q is ***negative*** with magnitude q:

$$\Delta U_{AB} = -\vec{F}_{el} \cdot \vec{d} = -(-qE)d = qEd$$

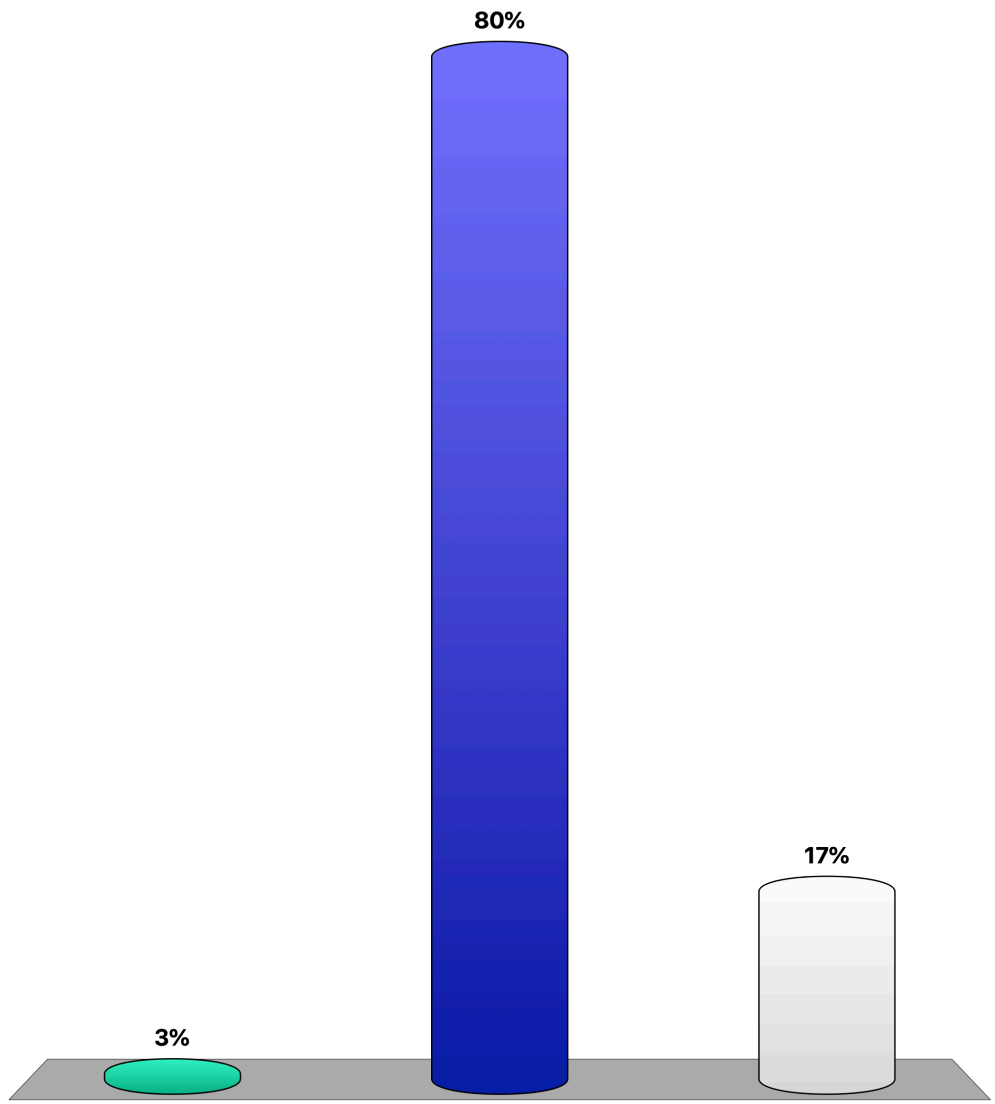
Example 1

Calculate the potential energy difference (ΔU_{BA}) for a particle with **negative charge** of *magnitude* q displaced from A to B in a uniform electric field as shown.

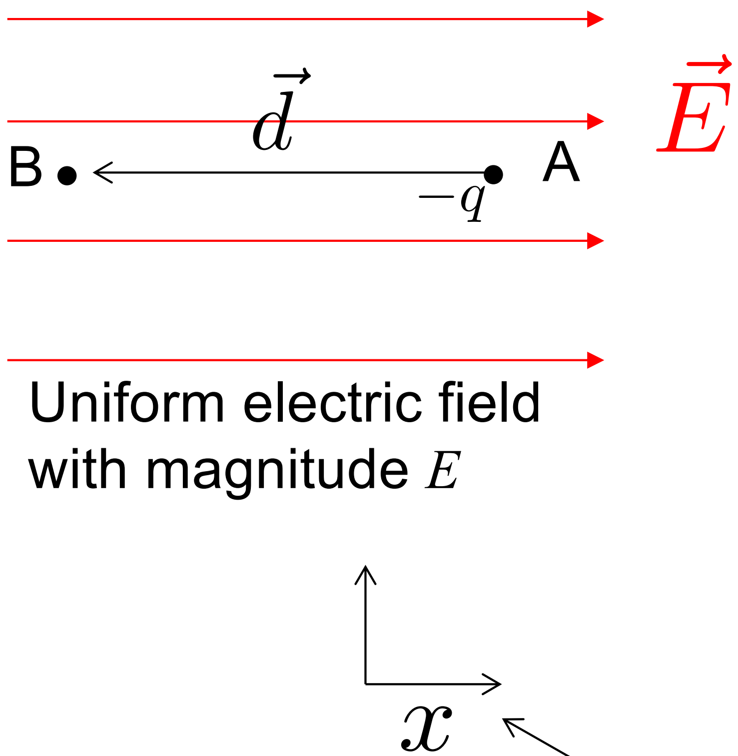


Uniform electric field with magnitude E

1. Ed
2. $-qEd$ ←
3. qEd



Example 1 (solution)



$$\Delta U = -\vec{F}_{el} \cdot \vec{d}$$

$$= -(-qE)(-d)$$

$$= -qEd$$

Mind the signs of force and displacement relative to positive x-axis shown.

Can also do this using the dot product definition between force and displacement:
- |qE|*|d| cos(0)=-qEd

Electrical *Potential* Difference in a *Uniform* Electric Field

General Definition (in words)

Potential energy change per unit charge

Definition for a Uniform Electric Field (formula)

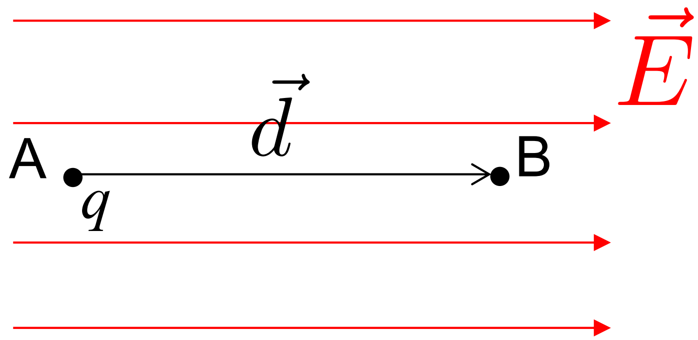
$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = \frac{-\vec{F}_{\mathrm{el}} \cdot \vec{d}}{q}$$

Divide out test charge (include sign)

Convenient trick: ΔV_{AB} is just the potential *energy* change of a *unit positive charge (q=+1)*

Electric ***Potential*** is a Property of the Electric Field

Calculate the potential energy change from A to B using a positive test charge q in the figure below



Uniform electric field with magnitude E

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = \frac{-\vec{F}_{\mathrm{el}} \cdot \vec{d}}{q} = -\frac{(+qE)d}{q} = -Ed$$

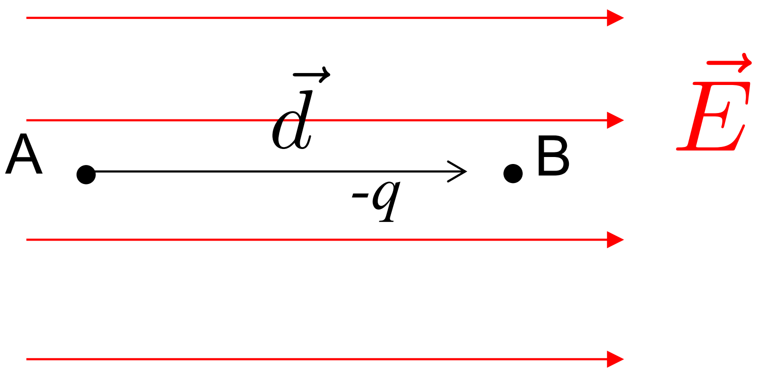
Test charge

ΔV_{AB} is only a property of electric field:

- **Negative when displacement is in direction of E-field**
- **Positive when displacement is opposite to E-field**

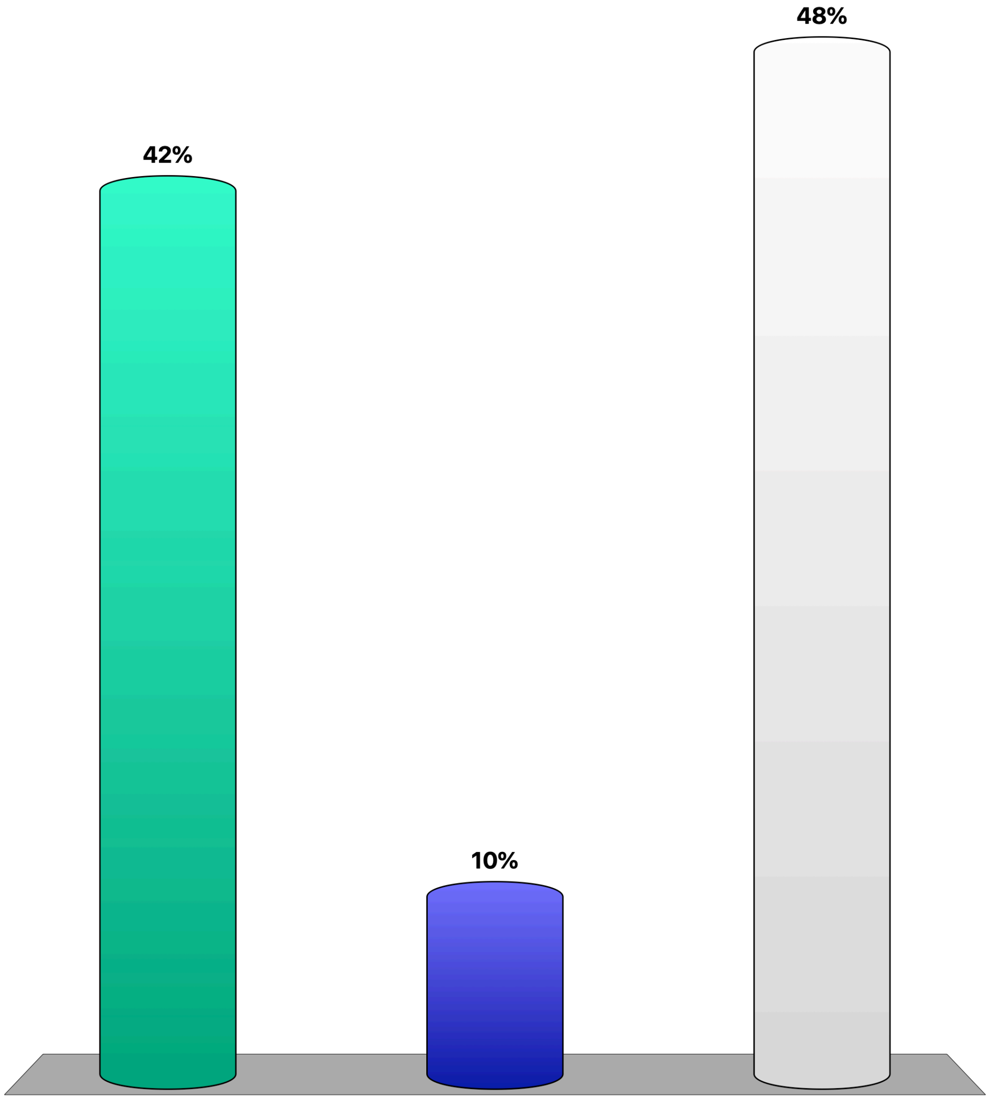
Example 2

Calculate the potential difference *using a p*article with **negative charge** of *magnitude* q displaced from A to B in a uniform electric field as shown.

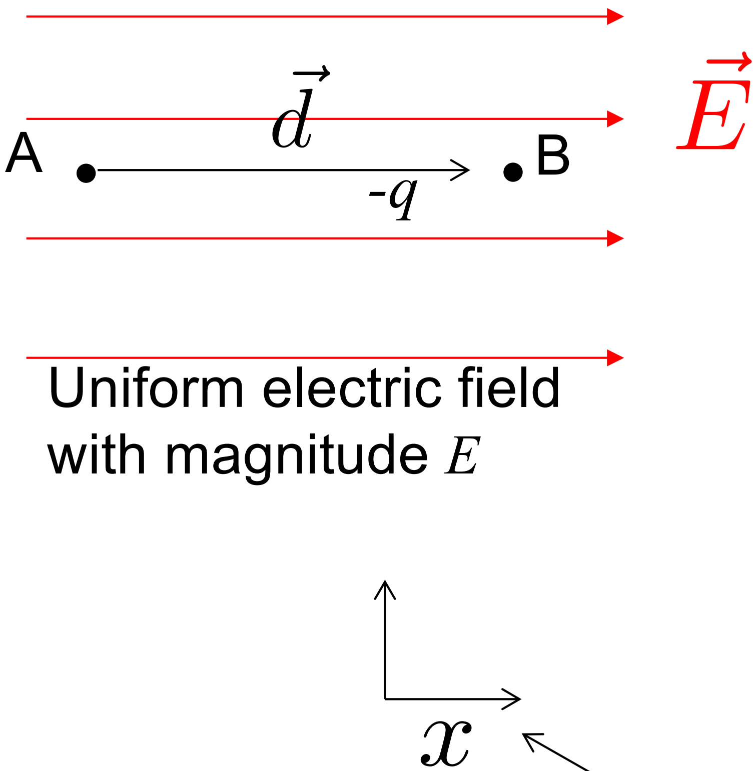


Uniform electric field
with magnitude E

1. Ed
2. $-qEd$
3. $-Ed$ ←



Example 2 (solution)



$$\Delta V_{AB} = -\frac{\vec{F}_{el} \cdot \vec{d}}{q}$$

$$= -\frac{(-qE)(d)}{-q}$$

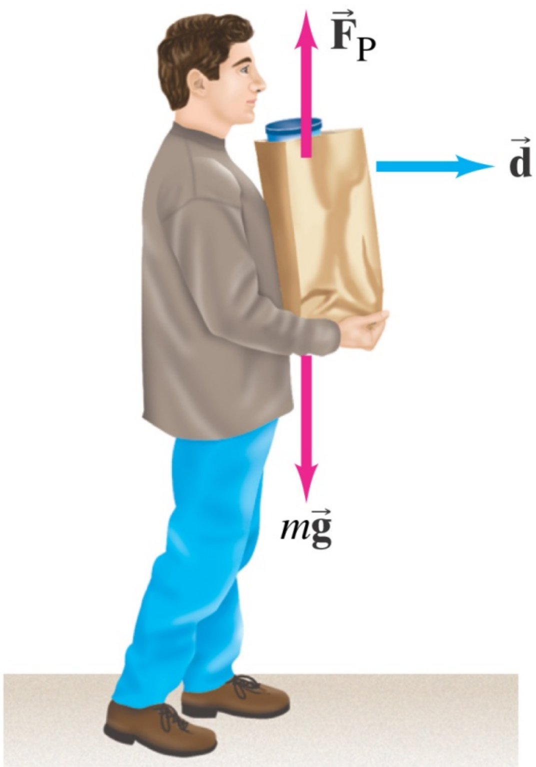
$$= -Ed$$

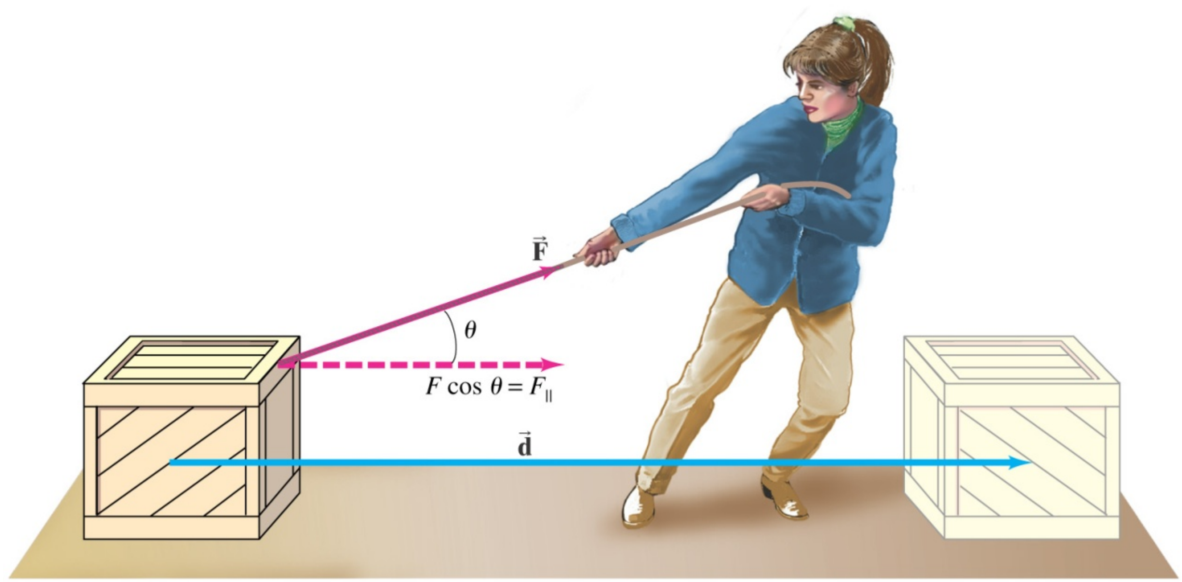
Mind the signs of force and displacement relative to positive x-axis shown.

Can also do this using the dot product definition between force and displacement:
- |qE|*|d| cos(180)/(-q)=-Ed

Work When Displacement is at an Angle to Force

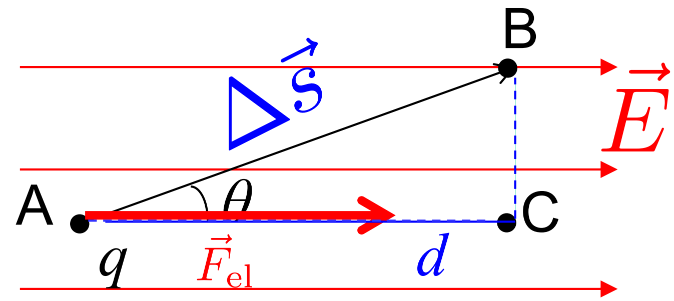
Only the component of the force **parallel to the displacement** contributes to the work.





Displacement of Charge at an Angle to ***Uniform*** E-Field

What is the change in the electric potential energy change of q when displaced from A-B?



Uniform electric field with magnitude E

$$\Delta U_{AB} = -\vec{F}_{\rm el}\cdot\Delta\vec{s} = -\overbrace{\left|q\vec{E}\right|}^{\text{Electric force magnitude}}\underbrace{\left|\Delta\vec{s}\right|\cos\theta}_{d}$$

Definition of electric potential energy change

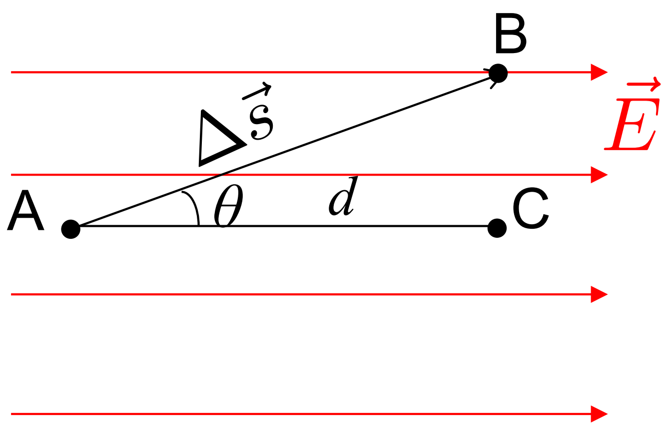
Displacement magnitude

d=displacement parallel to $\boldsymbol{E}$-field (positive when $\theta > 90$, negative when $\theta < 90$)

$$= -qEd$$ same as ΔU_{AC} !

Example 3

Calculate the **electric potential difference** ***(ΔV)*** in the paths A-B **and** A-C, respectively.

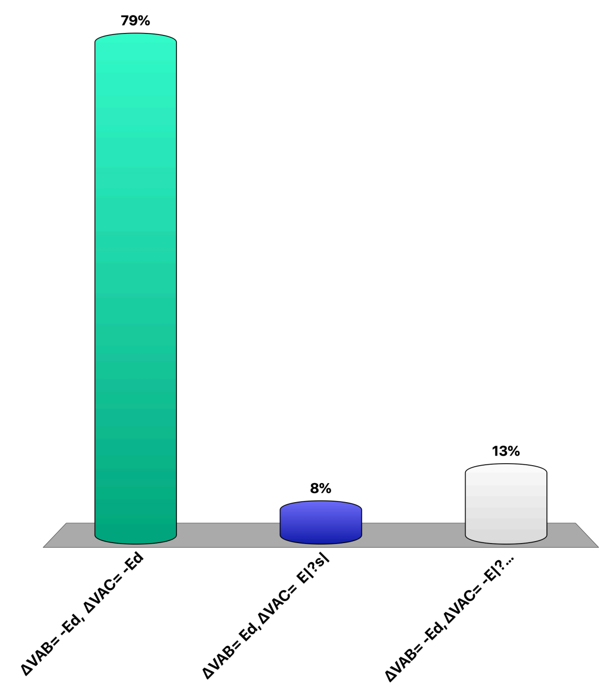


Uniform electric field
with magnitude E

A. $\Delta V_{AB}= -Ed,\ \ \Delta V_{AC}= -Ed$ $\longleftarrow$

B. $\Delta V_{AB}= Ed,\ \Delta V_{AC}=\ E|\Delta s|$

C. $\Delta VAB= -Ed,\ \Delta VAC= -E|\Delta s/$



Example 3 (solution)

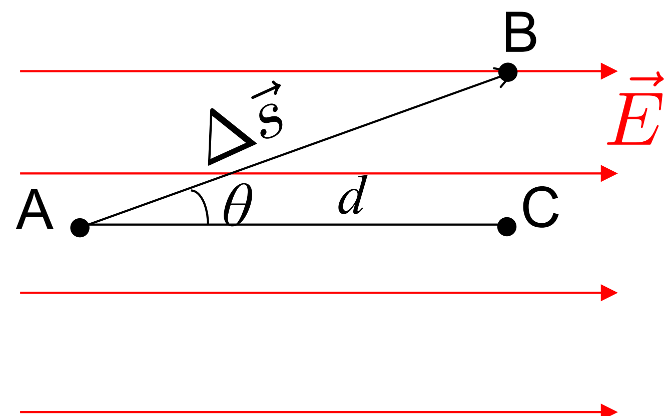
To calculate the **potential d**ifference in the paths A-B **Versus** A-C, lets use a positive test charge q

Path 1: A-B

$$\Delta V_{AB} = -\frac{\vec{F}_{el} \cdot \Delta\vec{s}}{q}$$

$$= -|q\vec{E}|\underbrace{|\Delta\vec{s}|\cos\theta}_{d}/q$$

$$= -Ed$$



Uniform electric field with magnitude E

Path 2: A-C (done already on a previous slide)

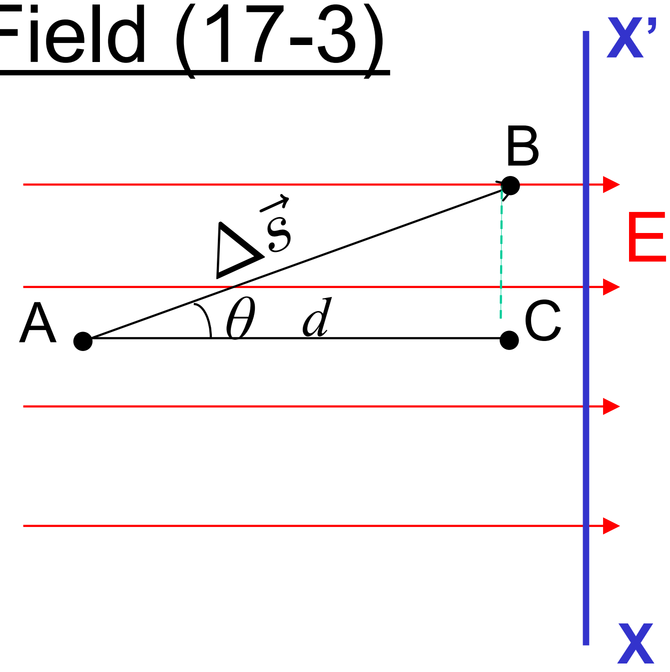
$$\Delta V_{AC} = -Ed$$

Only distance moved in direction parallel to E changes the electrical potential

Equipotential Surfaces in an Electric Field (17-3)

$$\Delta V = V_B - V_A = V_C - V_A$$

$$\Longrightarrow \ V_C = V_B$$



For a uniform electric field: points on any plane perpendicular to E-field are at the same potential (relative to a reference point)

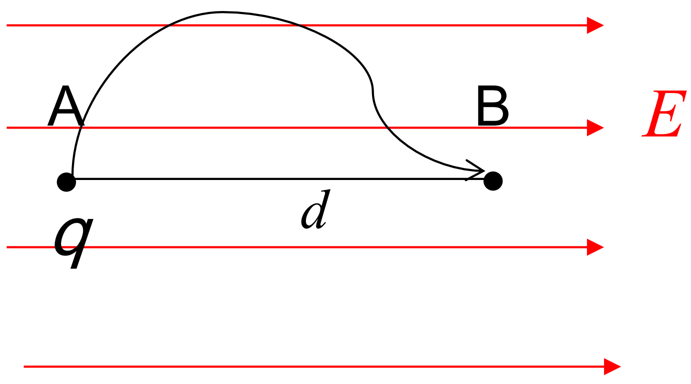
Points having the same potential are called *equipotential*

*Analogous to the gravitational case, where points at the same height have the same gravitational potential or potential energy

Electric Potential Energy and Conservative Forces

Potential energy change depends only on final and initial positions

$$\Delta U = U_B - U_A$$



Similarly for potential difference:

$$\Delta V = V_B - V_A$$

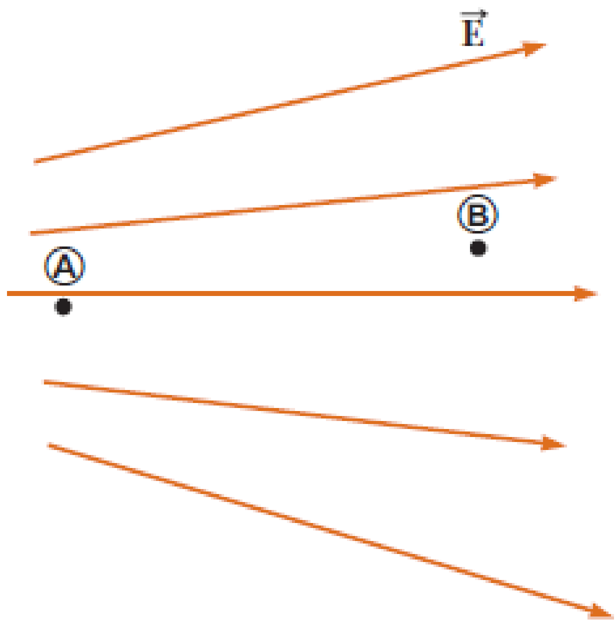
In general: ΔU (or potential difference ΔV) depends **only** on the location of the points A and B and not on the path from A to B**

Electric force is a *Conservative force*

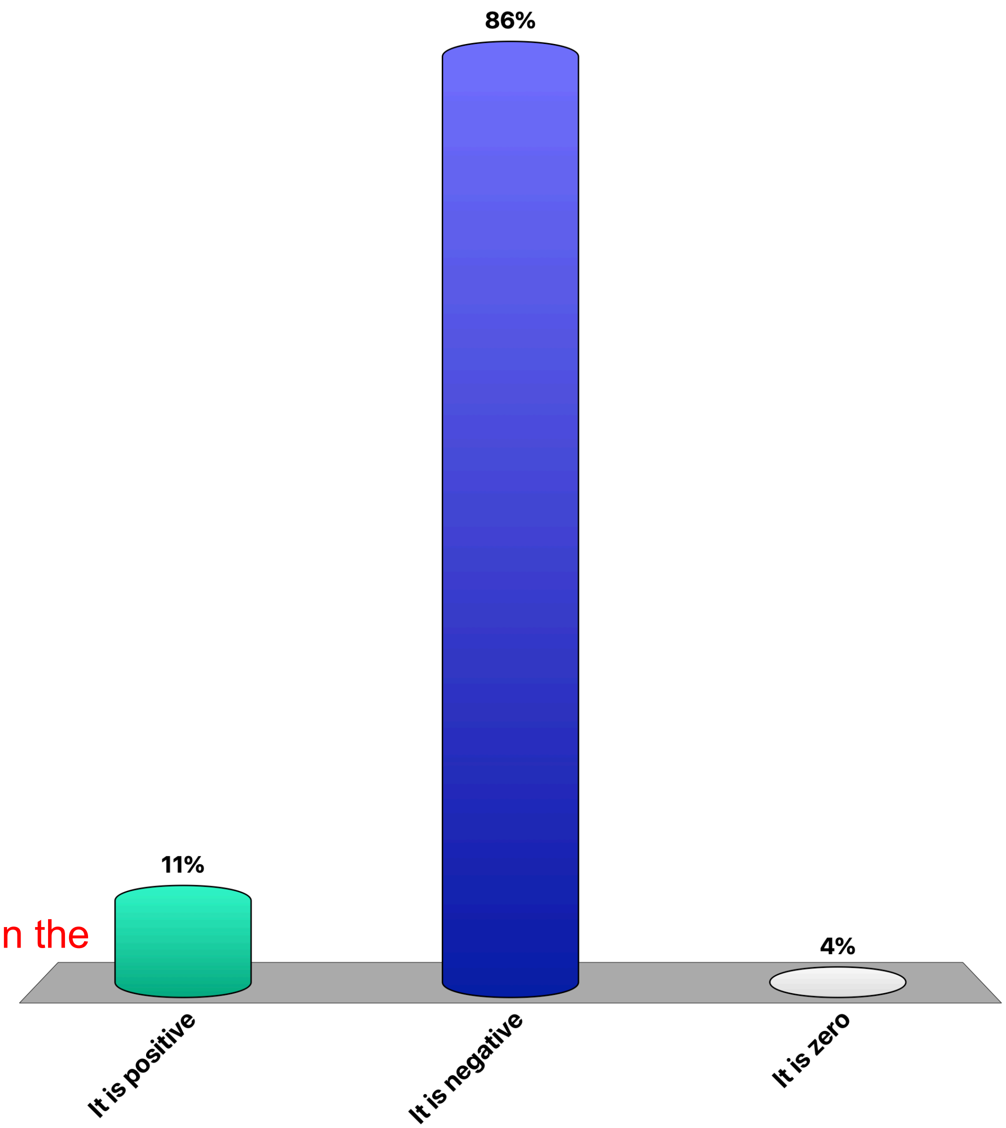
**Not true for time-dependent electric fields!

Question 1

Two points A and B are located within a region in which there is an electric field. How would you describe the potential difference $\Delta V = V_B - V_A$?



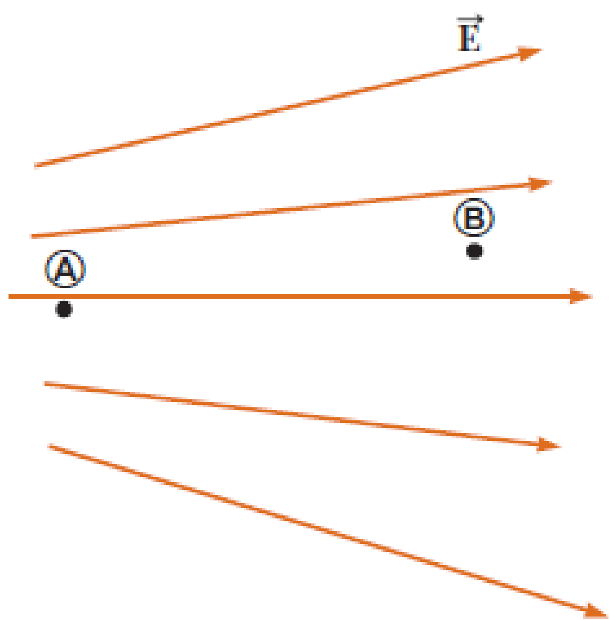
1. It is positive
2. It is negative ← ΔV negative when displacement component in the same direction of E. See Example 3
3. It is zero



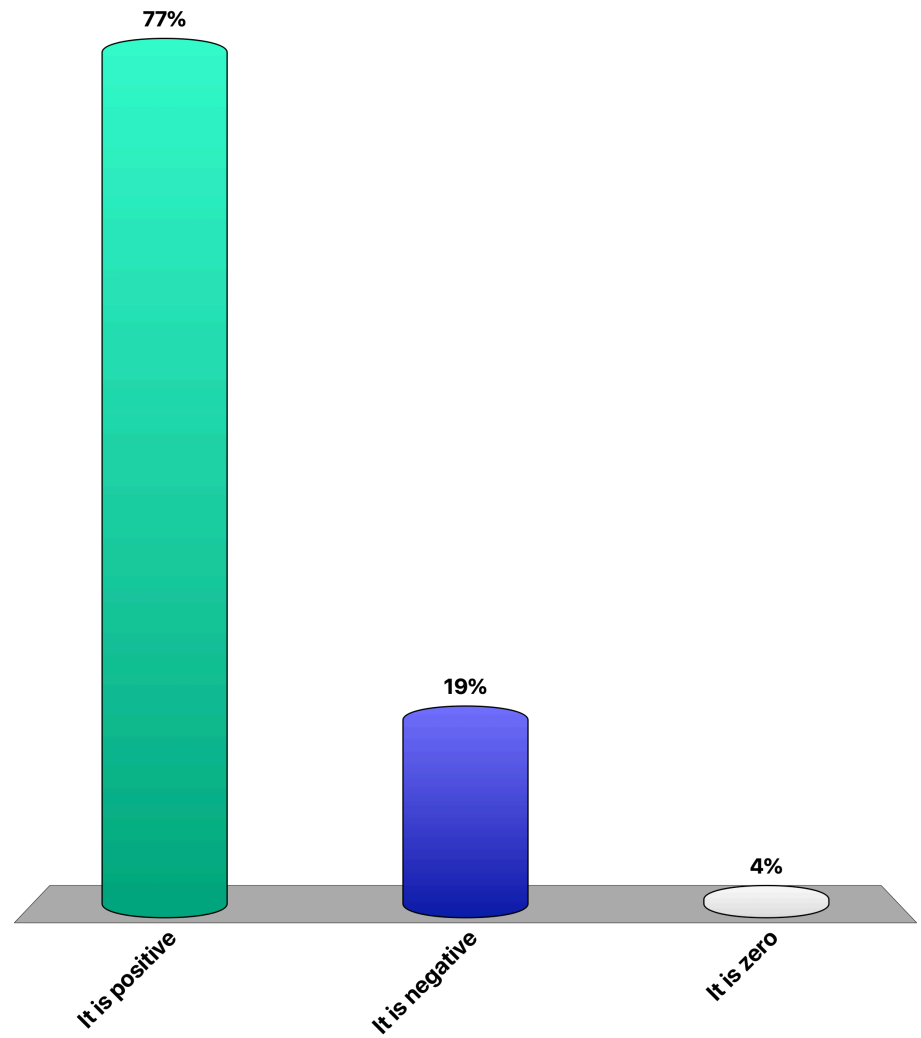
Question 2

Two points A and B are located within a region in which there is an electric field. A negative charge is placed at A and then moved to B. How would you describe the change in potential energy difference $\Delta U = U_B - U_A$? from A to B?

Hint: $\Delta U = q\ \Delta V$



1. It is positive ←Change of U= -charge* X negative ΔV *in Q1*
2. It is negative
3. It is zero



Electric Potential Units (17-4)

Similarly to the E-field, electric potential at a point is defined as the electric potential energy per unit charge:

$$\Delta V = \frac{\Delta U}{q} = V_B - V_A$$

Units: (J/C).
also Volt (V)

The potential is a scalar quantity (a number, NO direction)

Electron Volt (eV)

In atomic physics, the energies involved are much smaller than a joule. One can use the energy gained by a proton or electron when moved through a potential difference of 1 V.

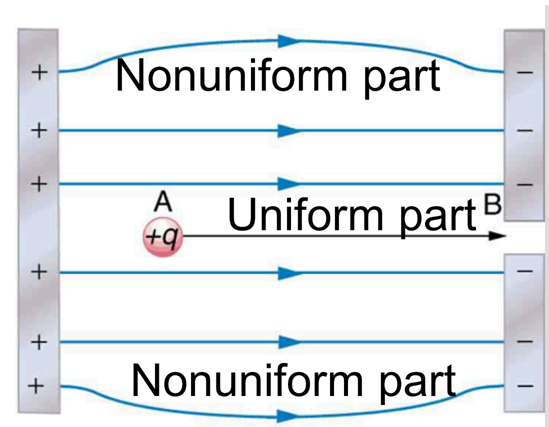
$$\Delta U = q\Delta V = (1.6\times10^{-19}\,\mathrm{C})(1\ \mathrm{V}) = 1.6\times10^{-19}\,\mathrm{J}$$

$$\therefore\ 1\ \mathrm{eV} = 1.6\times10^{-19}\,\mathrm{J}$$

Chapter 17 Week 3 Summary: Important Formulas/Concepts

An electric field carries with it an **electrical potential**, which means that a change displaced between two points will undergo a change in its stored **electrical potential energy**, which that can be converted into useful work in the form of kinetic energy, heat in a resistor, and others. In this week, we consider electrical potential in **a uniform electric field**.

A uniform electric field $\vec{E}$ (and by association $\vec{F}_{el}$) is constant at all points in space. The figure below right shows example where the field on the fringes is non-uniform, while the field in the middle field is uniform.



Nonuniform electric **varies** at different positions

Uniform electric is **constant** value at different positions

The Electric field is conservative: As with gravity, the electric force is a conservative force, as a result, the sum of a particle's kinetic and electrical potential is constant (more on this later).

Chapter 17 Week 3 Summary: Important Formulas/Concepts

$$\Delta U_{AB} = -\vec{F}_{el} \cdot \vec{d}$$

Electric potential energy change of a charge q displaced by $\vec{d}$ from A to B **in a uniform electric field $\vec{E}$**

$$\Delta V_{AB} = \frac{\Delta U}{q} = \frac{-\vec{F}_{\rm el} \cdot \vec{d}}{q} = -\vec{E} \cdot \vec{d}$$

Electric potential difference across the displacement $\vec{d}$ from the points A to B, in a **uniform electric field**. This is a property of the electric field, not the test charge q used to measure it.

1. Specialization of the dot product in potential energy change formula: displacement from A and B that is parallel to the **uniform electric field**:

$$\Delta U_{AB} = -\vec{F}_{\rm el} \cdot \vec{d} = -(\pm F_{\rm el})(\pm d)$$

Magnitudes with appropriate signs to indicate directions relative to x-axis (assume this is the common axis of displacement and electric field)

Sign indicates that force that does the work, to change the potential energy, i.e. is *opposite* to the force exerted on q by the *electric field*

Chapter 17 Week 3 Summary: Important Formulas/Concepts

2. Specialization of the dot product in potential energy change formula: displacement from A and B is ***not parallel*** to the uniform electric field:

$$\Delta U_{AB} = -\vec{F}_{\text{el}} \cdot d = -|\vec{F}_{\text{el}}||\vec{d}| \cos(\theta)$$

Sign indicates same thing as before

$= |q||\vec{E}|$

angle of displacement vector from A to B and electric force on charge q

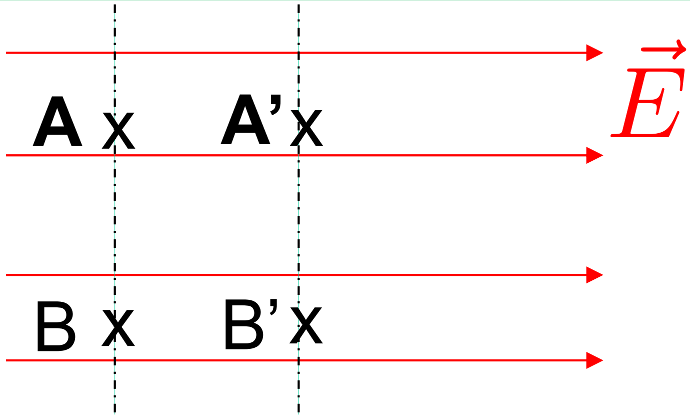
3. Specialization of the dot product in the potential difference formula: displacement from A and B that is ***parallel*** to the electric field:

$$\Delta V_{AB} = -\vec{E} \cdot \vec{d} = -\left(\pm E\right)\left(\pm d\right)$$

4. Specialization of the dot product in the potential difference formula: displacement from A and B that is ***not parallel*** to the electric field:

$$\Delta V_{AB} = -\vec{E} \cdot \vec{d} = -|\vec{E}||\vec{d}| \cos(\theta)$$

Chapter 17 Week 4 Summary: Important Formulas/Concepts



Equipotential lines in a uniform electric field ($V_A = V_B \neq V_{A'} = V_{B'}$)

A line (or plane in 3D) that is perpendicular to the electric field is called an equipotential surface, meaning all points on this surface are at the same electrical potential (relative to some reference point). See the figure to the left for a visual illustration.

Independence of Path: The change of electrical potential difference ΔV_{AB} is only a function of the initial and final points in any path between A and B. Similarly for the change in electrical potential energy ΔU_{AB} of a charge q displaced from A to B along any such path.

END OF WEEK 3

START OF WEEK 4

Electric Potential

Chapter 17

- Electric Potential Energy and Electrical Potential
  - In a uniform Electric Field
  - In electric fields due to Point Charges
- Storing Charge:
  - Capacitance
  - Capacitors with Dielectrics
- Stored Electrical Potential Energy

Electrical Potential of ***Point Charges*** (17-5)

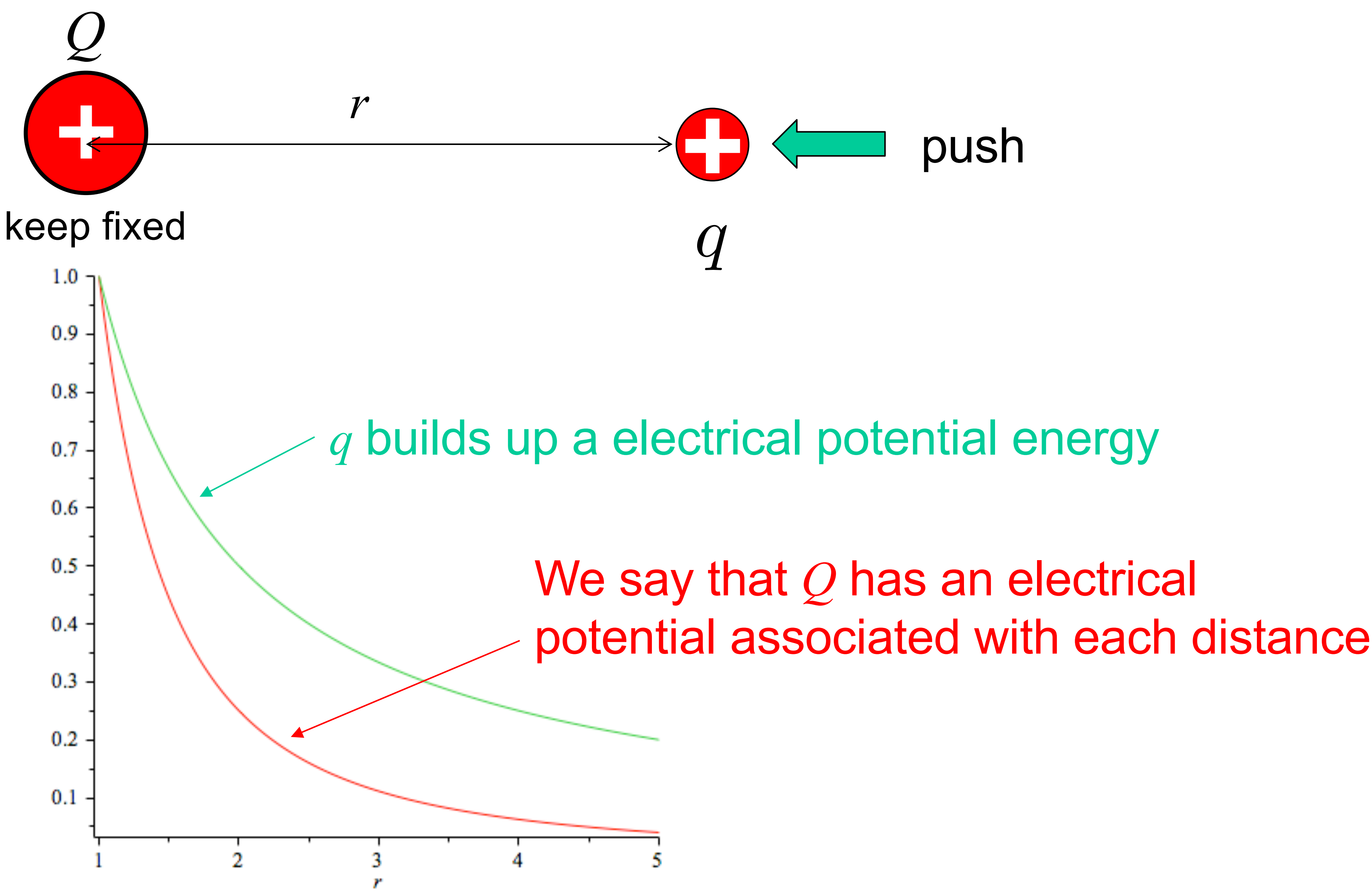
Charge q feels the the electric field of the charge Q and gains or looses electrical potential energy as it move closer or father from Q

There is thus an electrical potential associated with the charge Q, which is different at each distance from it.

Here, the electric force felt by q is not constant, so we cannot use the any of the formulae we used in Section 17.1-2..

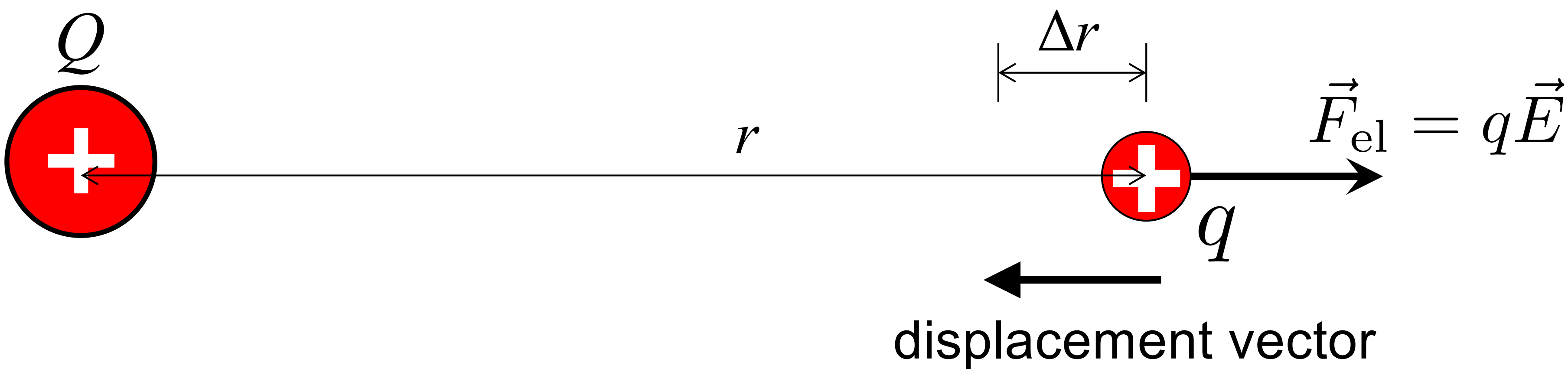
This section presents new formulae specifically for the change of electrical potential/potential energy in systems of point charges.

Illustration of Potential Energy of a Point Charge



Electrical Potential *Energy* from Coulomb's Law

Consider the **electrical potential *energy* difference** of a charge q displaced from *r* to *r*-Δr, *in the* electric field of charge Q (ΔU):



Small change of potential energy in moving +q:

$$\Delta U = -\vec{F}_{\mathrm{el}} \cdot \Delta\vec{S} = -(+|qE|)(-|\Delta r|)$$

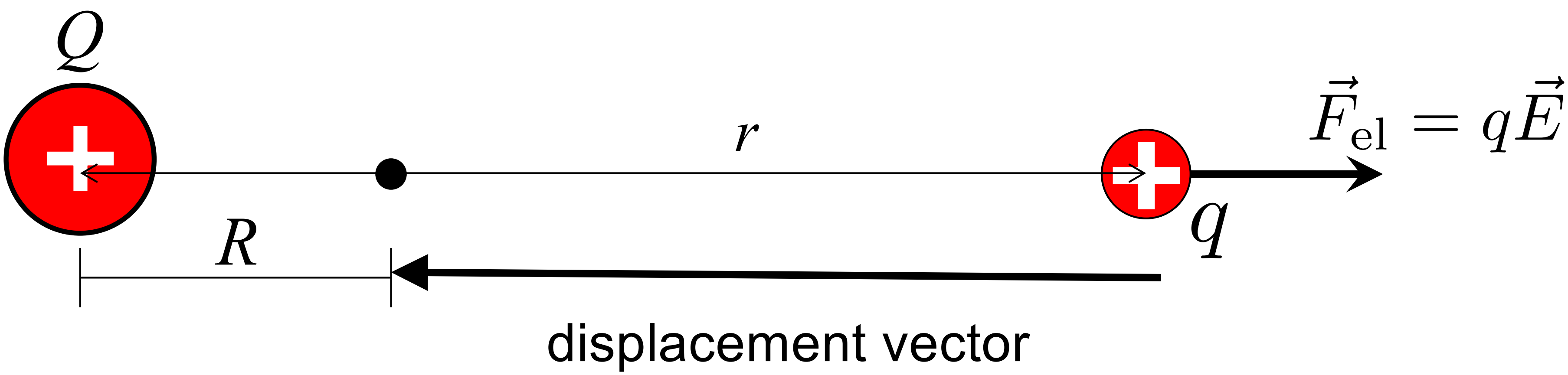
$$= q\left(\frac{kQ}{r^2}\right)|\Delta r|$$

Coulomb's law: E is not constant, need calculus→

Note: define Δr and force as negative/positive toward/away from Q

Change of Electrical Potential *Energy* of a Point Charge

Define: **electrical potential *energy difference* of a charge q** displaced from r **to *another*** R *in the* electric field of charge Q *(ΔU*$_{\mathrm{rR}}$*):*



$$\boxed{\Delta U_{rR} = U_R - U_r = \frac{kQq}{R} - \frac{kQq}{r}}$$

Change of *electrical potential energy* of q is *the work* required for an external force to displace a charge q from r to R

(signs of charges matter here!)

Electrical Potential *Energy* of a Point Charge q

Define: **Electrical *potential energy* of a charge q** is ΔU_{rR} with initial $r=\infty$ and final $r=R$ (distances relative to the point charge Q):

$$\boxed{U = \frac{kQq}{R}}$$ ← **signs of charges matter!**

Electrical Potential energy is the work required for an external force to displace the charge q to a separation of R from Q, when starting from infinite separation.

Electrical *Potential Difference of* a Point Charge Q

Define: **Electrical potential difference** (ΔV_{AB}) a charge Q between two positions r_A and r_B from the charge Q is the electrical potential *energy* difference required to displace a ***unit positive test charge*** from r_A to r_B:

$$\boxed{\Delta V_{AB} = V_B - V_A = \frac{U_B - U_A}{q} = \frac{kQ}{r_B} - \frac{kQ}{r_A}}$$

Define: **Electrical *potential*** is ΔV_{rR} with initial $\boldsymbol{r_A}=\infty$ and final $r_B=R$ (distances relative to the point charge Q*):*

$$\boxed{\Longrightarrow V = \frac{kQ}{R}}$$

*****Electrical potential is relative to infinite separation ($r_A=\infty$) where $V=0$, so we drop the word "difference"***

Relation of ΔU_{AB} Vs. ΔV_{AB} & U Vs. V For Point Charges

To calculate the change in **electrical potential *energy*** of a charge q displaced from $r_A \rightarrow r_B$ from knowledge of the **electrical potential difference** between $r_A \rightarrow r_B$ from a point charge Q:

$$\Delta U_{AB} = q\Delta V_{AB}$$

Potential Simulator

To converting potential (V) at distance R from Q into potential energy (U) of a charge q at a distance R from Q use:

$$U = qV$$

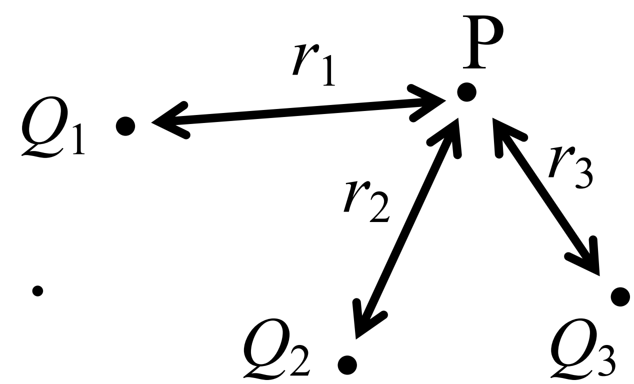
signs of charges matter in these formulae!

Electric Potential Due to Multiple Point Charges

The potential at a point P due many charges, Q_1, Q_2, Q_3, ... is given by the superposition principle (**scalar** sum of numbers!)

$$V_p = V_1 + V_2 + V_3 + \cdots$$

$$= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} + \cdots$$



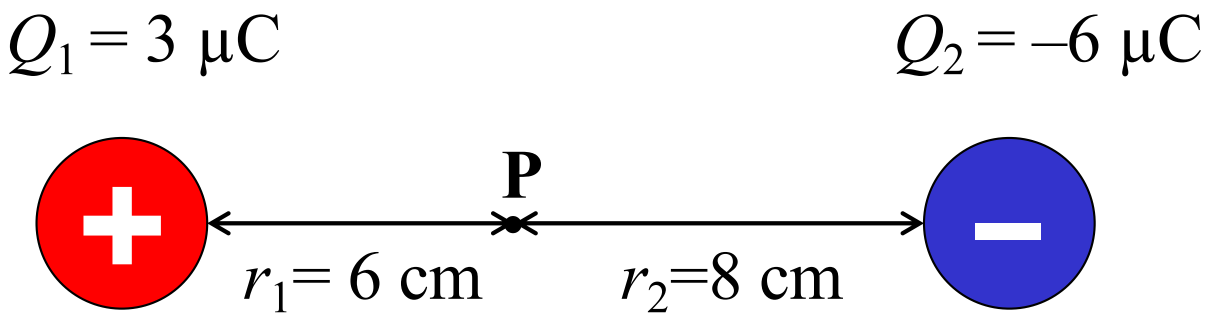
**Remember to include the sign of the charges Q_1, Q_2, Q_3, ... , when calculating V_p.

Electrical potential energy of a charge q displaced to P

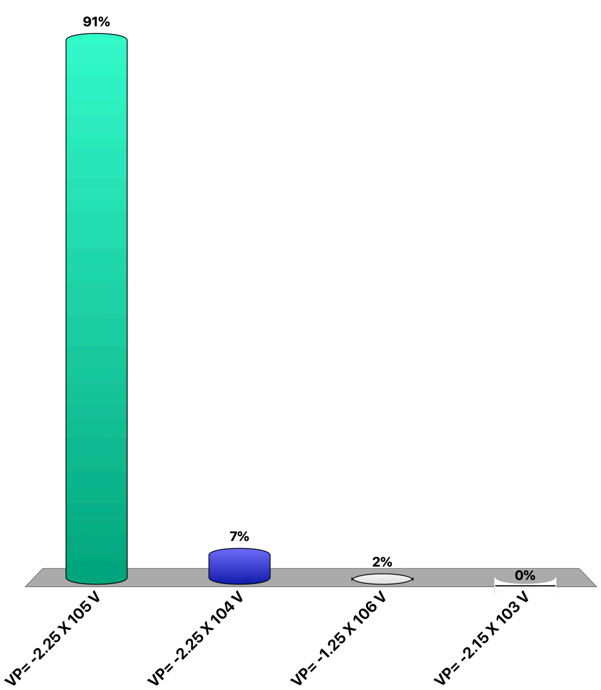
$$\boxed{U = qV_p}$$

Example 4a

Calculate the potential at point P in the figure;

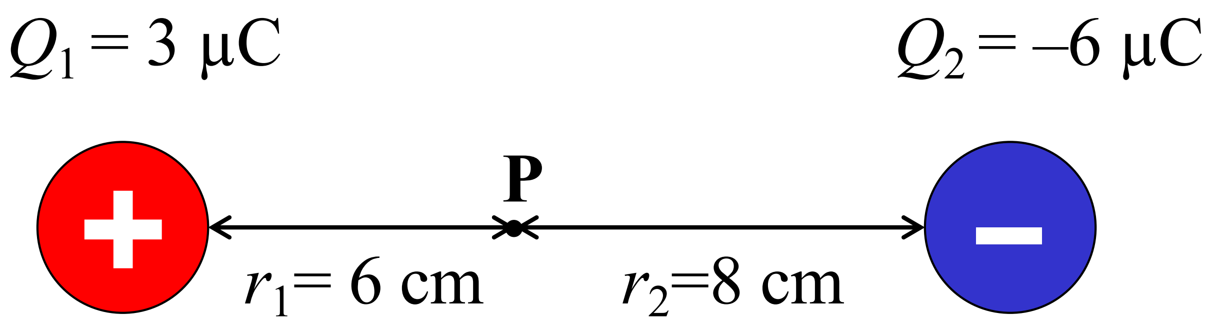


A. $V_P = -2.25 \times 10^5$ V ←
B. $V_P = -2.25 \times 10^4$ V
C. $V_P = -1.25 \times 10^6$ V
D. $V_P = -2.15 \times 10^3$ V



Example 4a (solution) :

Calculate the potential at point P in the figure;



$$V_P = V_1 + V_2$$

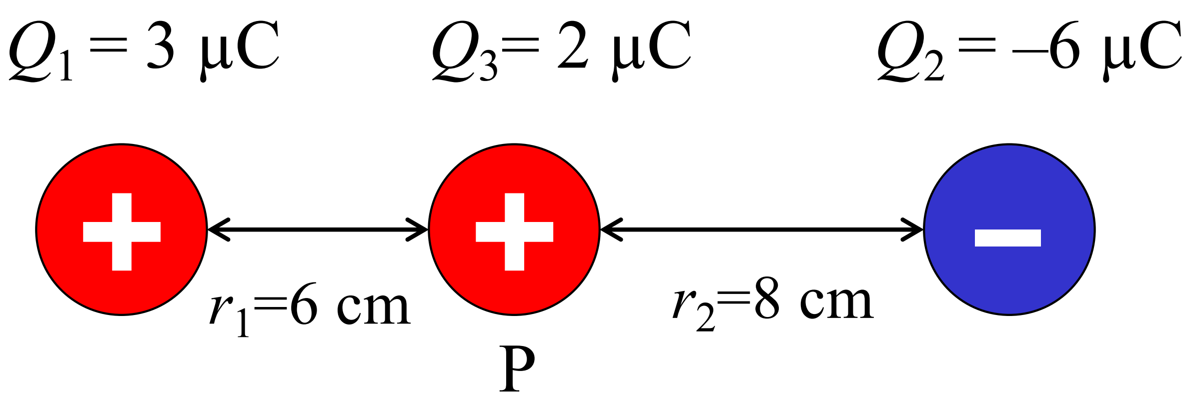
$$= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$= \frac{\left(9\times10^9 Nm^2/C^2\right)\left(3\times10^{-6}C\right)}{0.06m} + \frac{\left(9\times10^9 Nm^2/C^2\right)\left(-6\times10^{-6}C\right)}{0.08m}$$

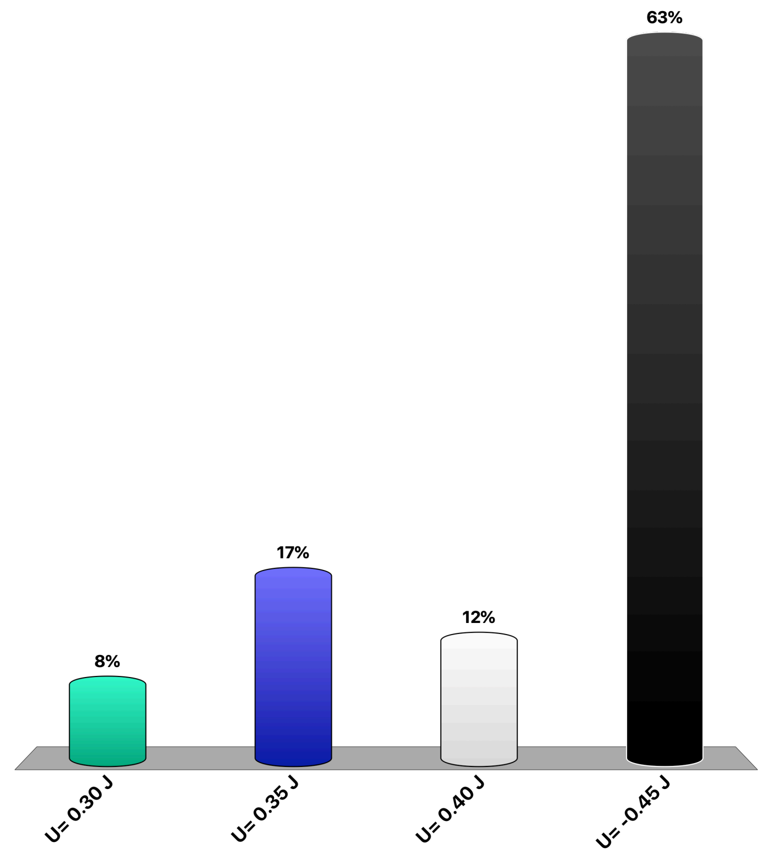
$$= -2.25 \times 10^5 V$$

Example 4b

What is the potential energy of a charge Q_3=2 μC at point P?

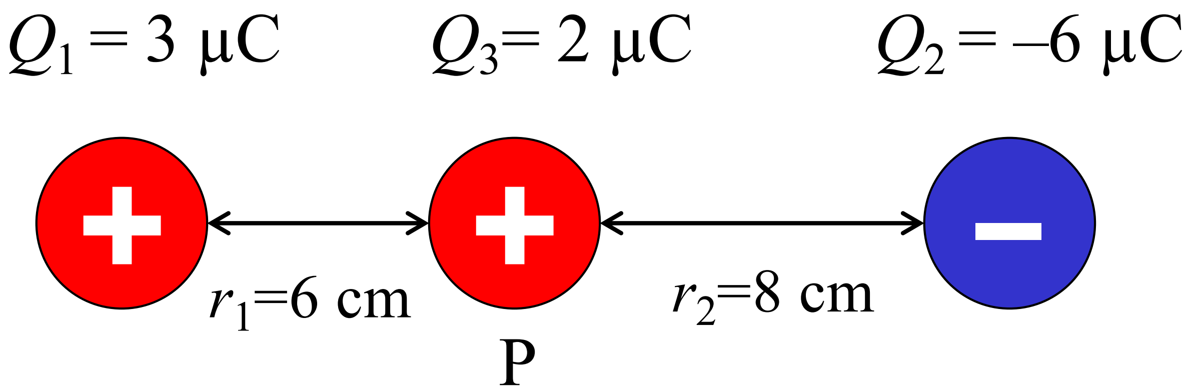


A. U= 0.30 J
B. U= 0.35 J
C. U= 0.40 J
D. U= -0.45 J ←



Example 4b (solution) :

What is the potential energy of a charge Q_3=2 μC at point P?



$$U = Q3\ V_P$$

$$= (2 \times 10^{-6} C)(-2.25 \times 10^5 V)$$

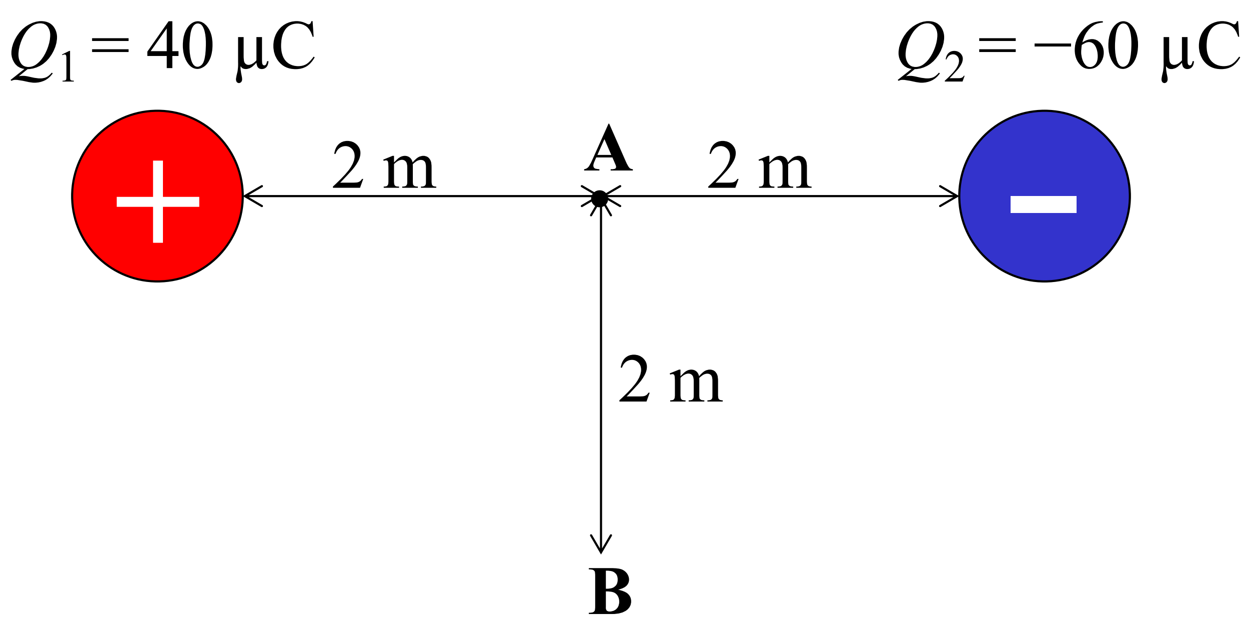
$$= -0.45 J$$

Note 1: Charges do not contribute to their own potential)

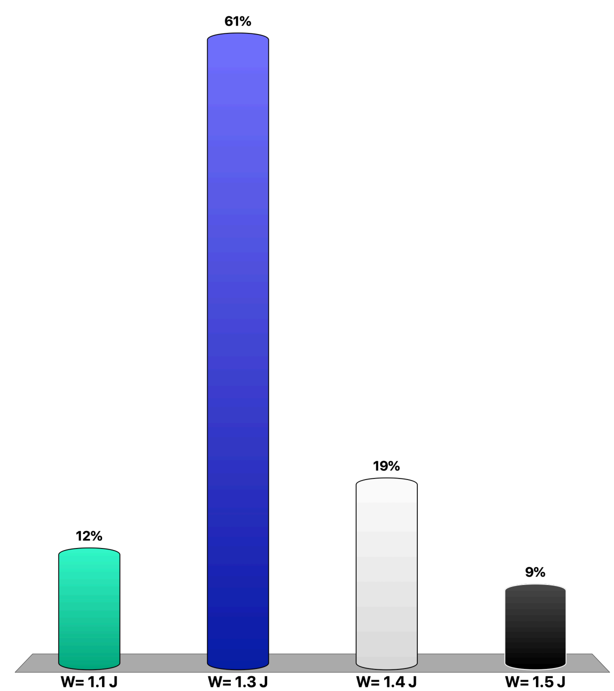
Note 2: If free to move, a charge will always try to lower its U.

Example 5

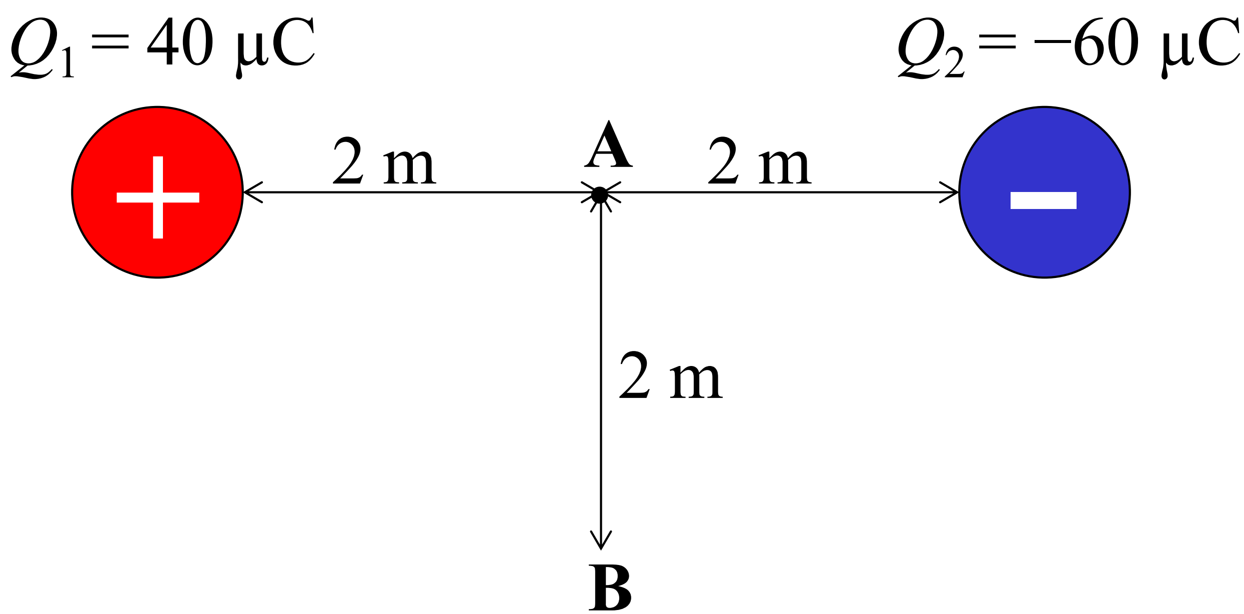
Two charges, Q_1=40 μC and Q_2=−60 μC are 4 m apart. How much work is done to move a charge Q =50 μC from point **A** to point **B**?



A. W= 1.1 J
B. W= 1.3 J
C. W= 1.4 J
D. W= 1.5 J



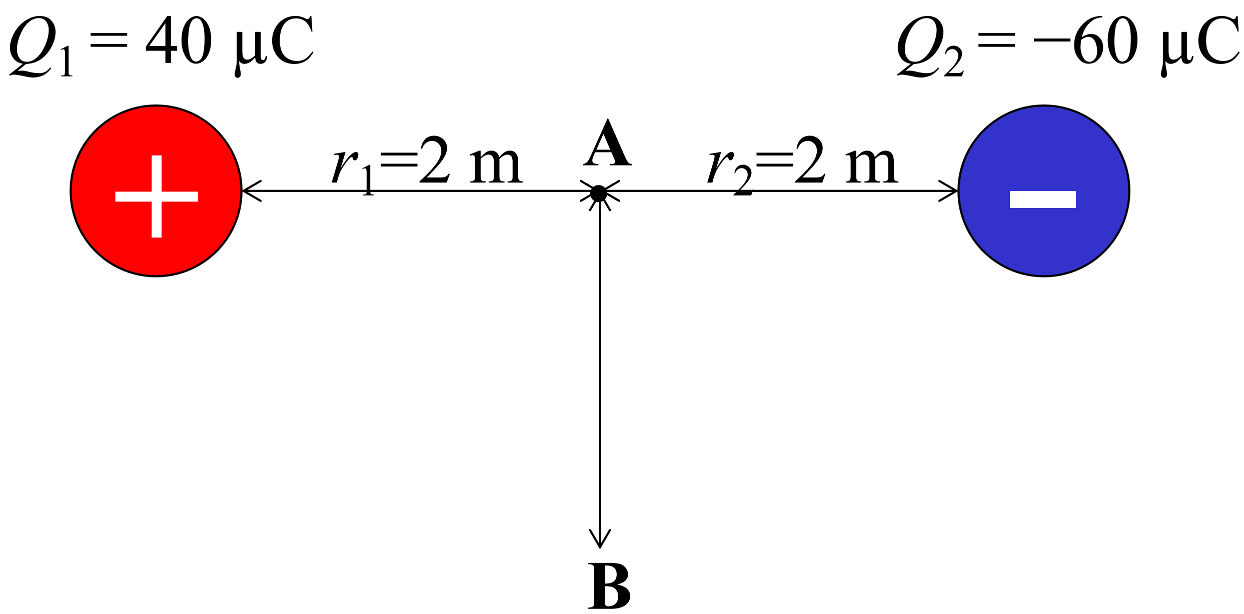
Example 5 (solution) :



Work done = Δ(electrical potential energy) from A to B

$$\Delta U = QV_B - QV_A = Q(V_B - V_A)$$

Example 5 (solution) :

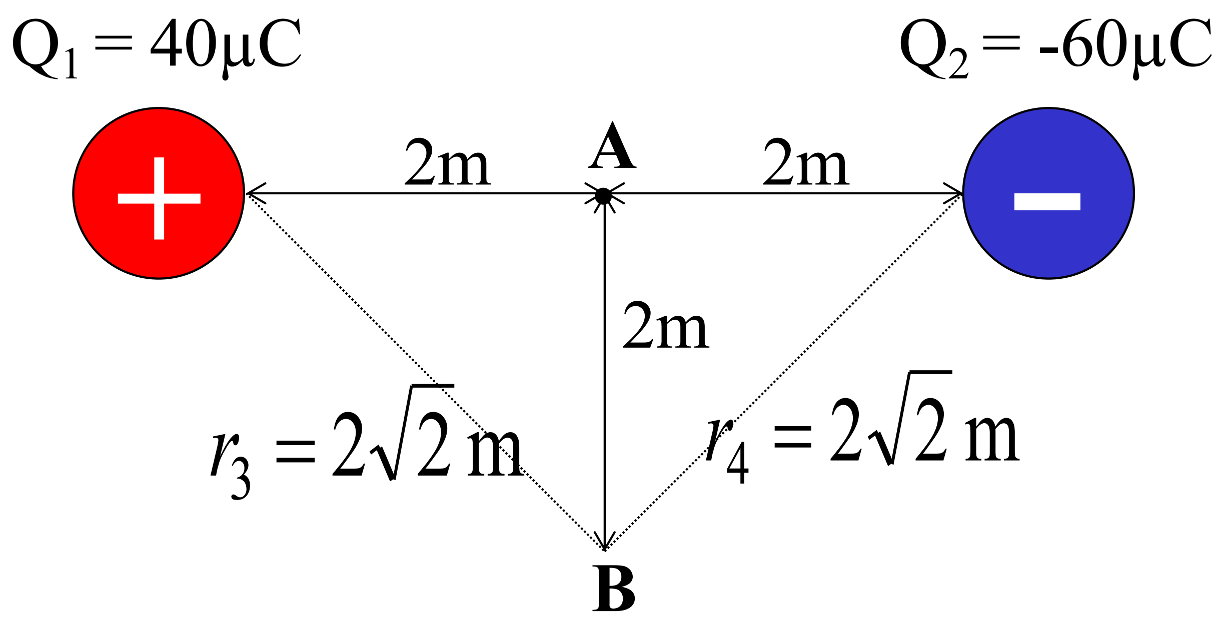


$$V_A = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$= \frac{(9\times10^9 Nm^2/C^2)(40\times10^{-6}C)}{2m} - \frac{(9\times10^9 Nm^2/C^2)(60\times10^{-6}C)}{2m}$$

$$= -9 \times 10^4 J/C$$

Example 5 (solution) :



$$V_B = \frac{kQ_1}{r_3} + \frac{kQ_2}{r_4}$$

$$= \frac{(9\times10^9 Nm^2/C^2)(40\times10^{-6}C)}{2\sqrt{2}m} - \frac{(9\times10^9 Nm^2/C^2)(60\times10^{-6}C)}{2\sqrt{2}m}$$

$$= -6.39 \times 10^4 J/C$$

Example 5 (solution) :

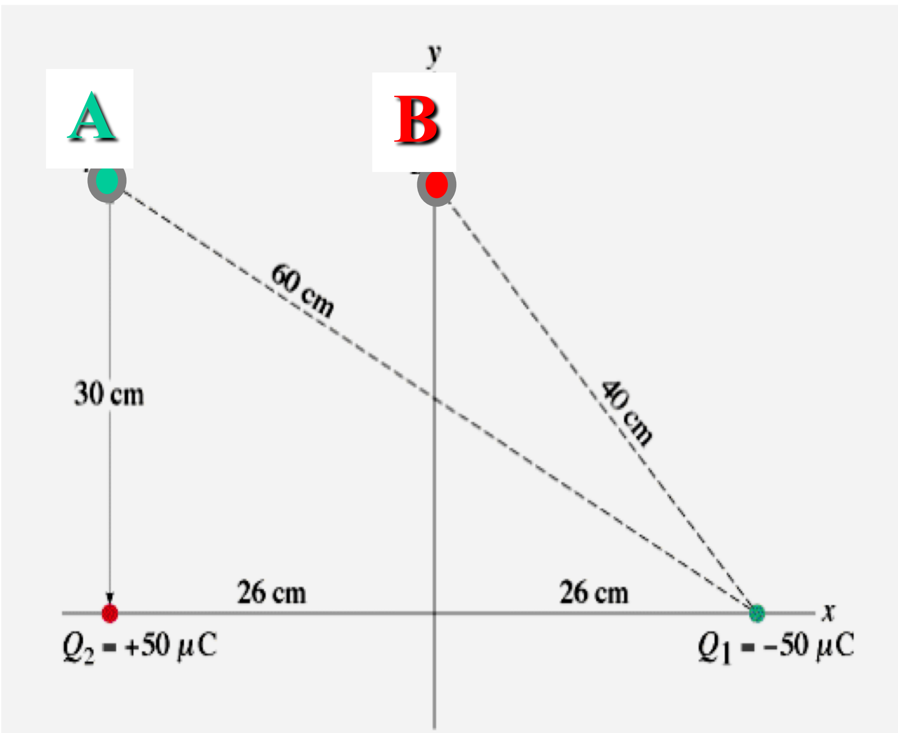
Work done $= Q(V_B - V_A)$

$$= 50 \times 10^{-6} C \left(-6.39 \times 10^4 J/C - (-9 \times 10^4 J/C)\right)$$

$$= 1.3 J$$

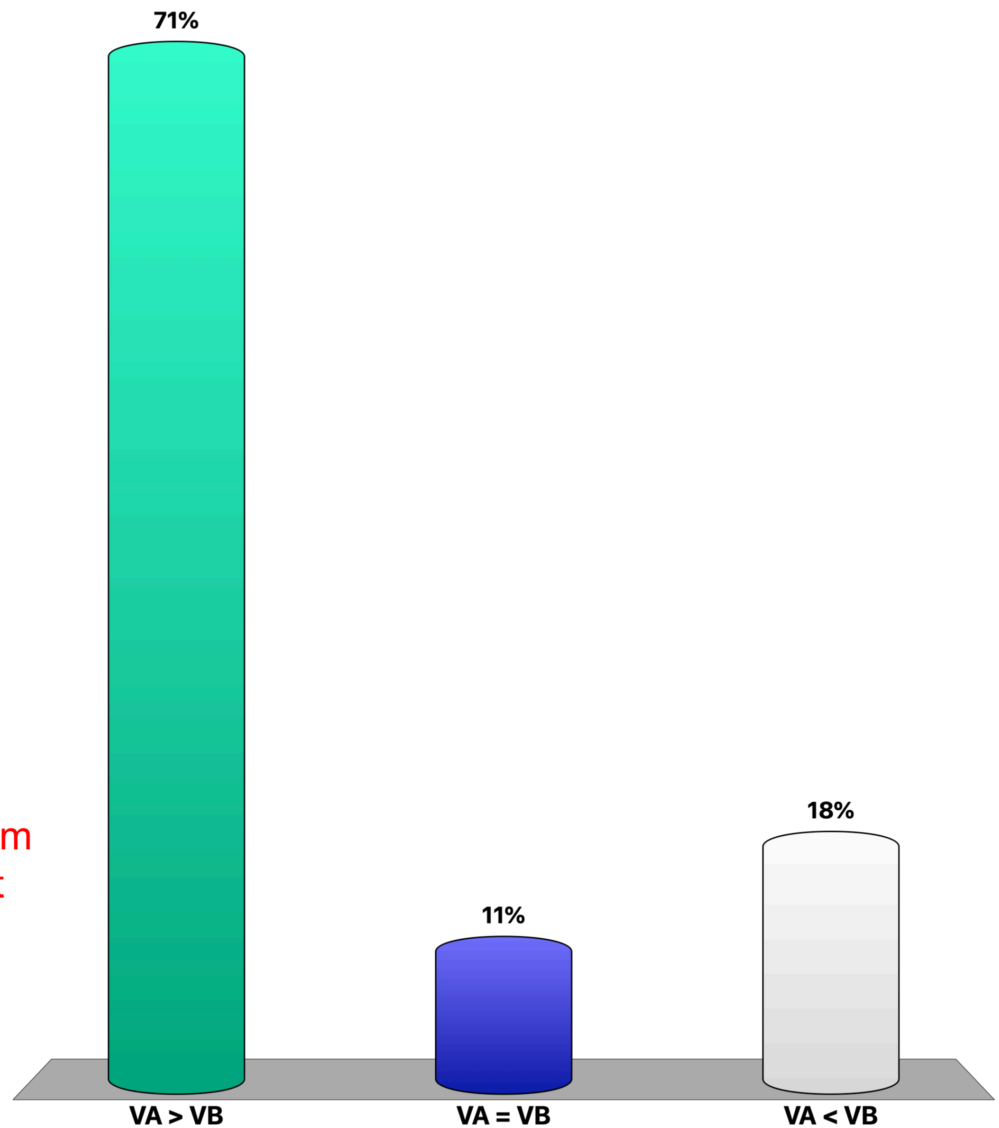
Question 3

Compare the electric potential at the point A and B.



1. $V_A > V_B$
2. $V_A = V_B$
3. $V_A < V_B$

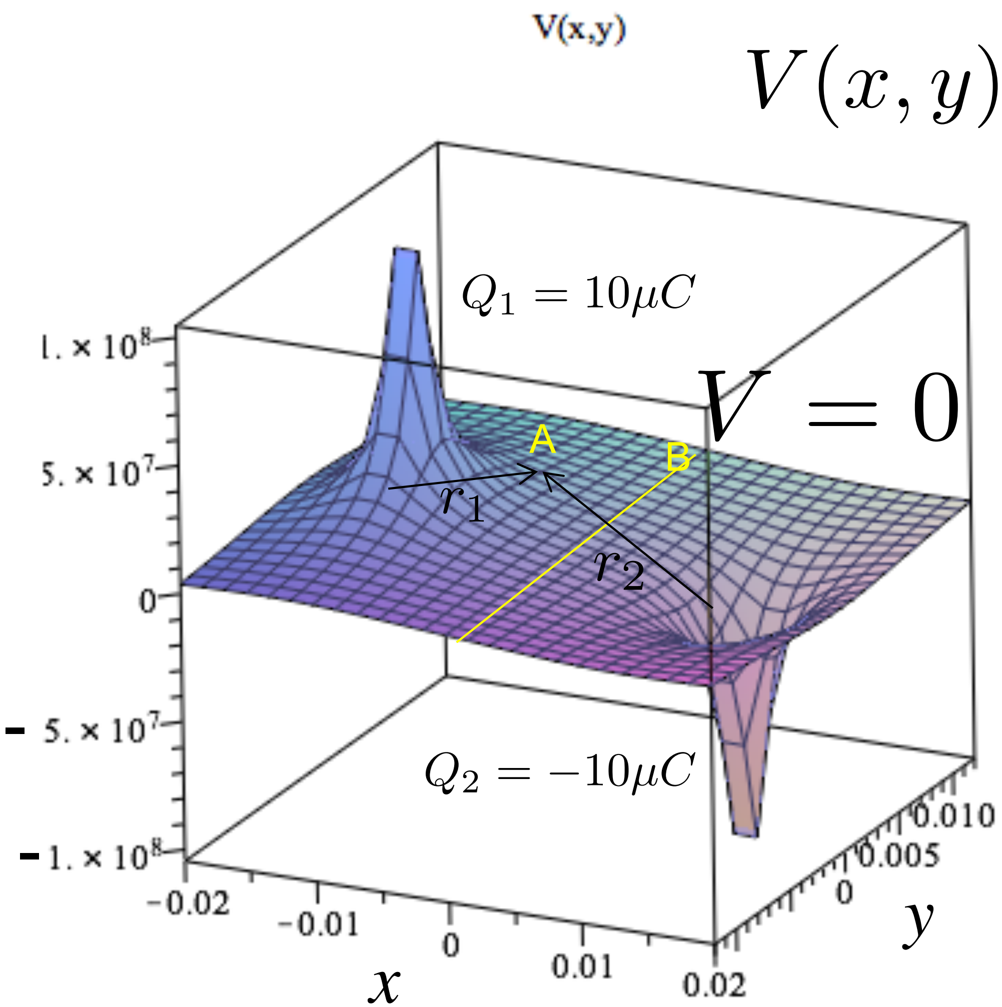
← V_A >0 V_B=0 since the sum of potentials exactly cancels at B while it does not at A (see formula)



Question 3 (solution)

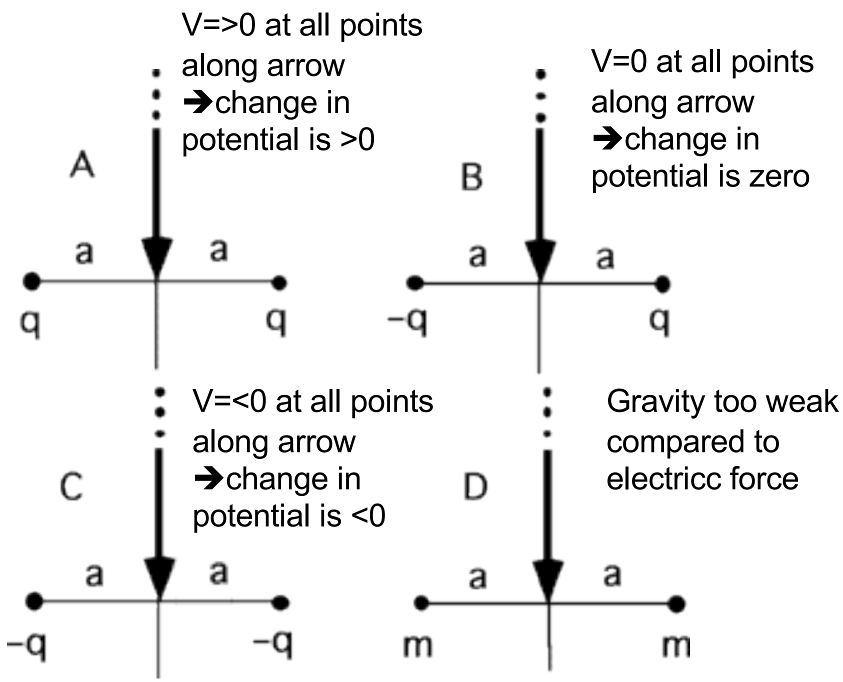
Computer-generated potential at different locations in space

$$V(x,y) = \frac{kQ_1}{r_2} + \frac{kQ_2}{r_2}$$

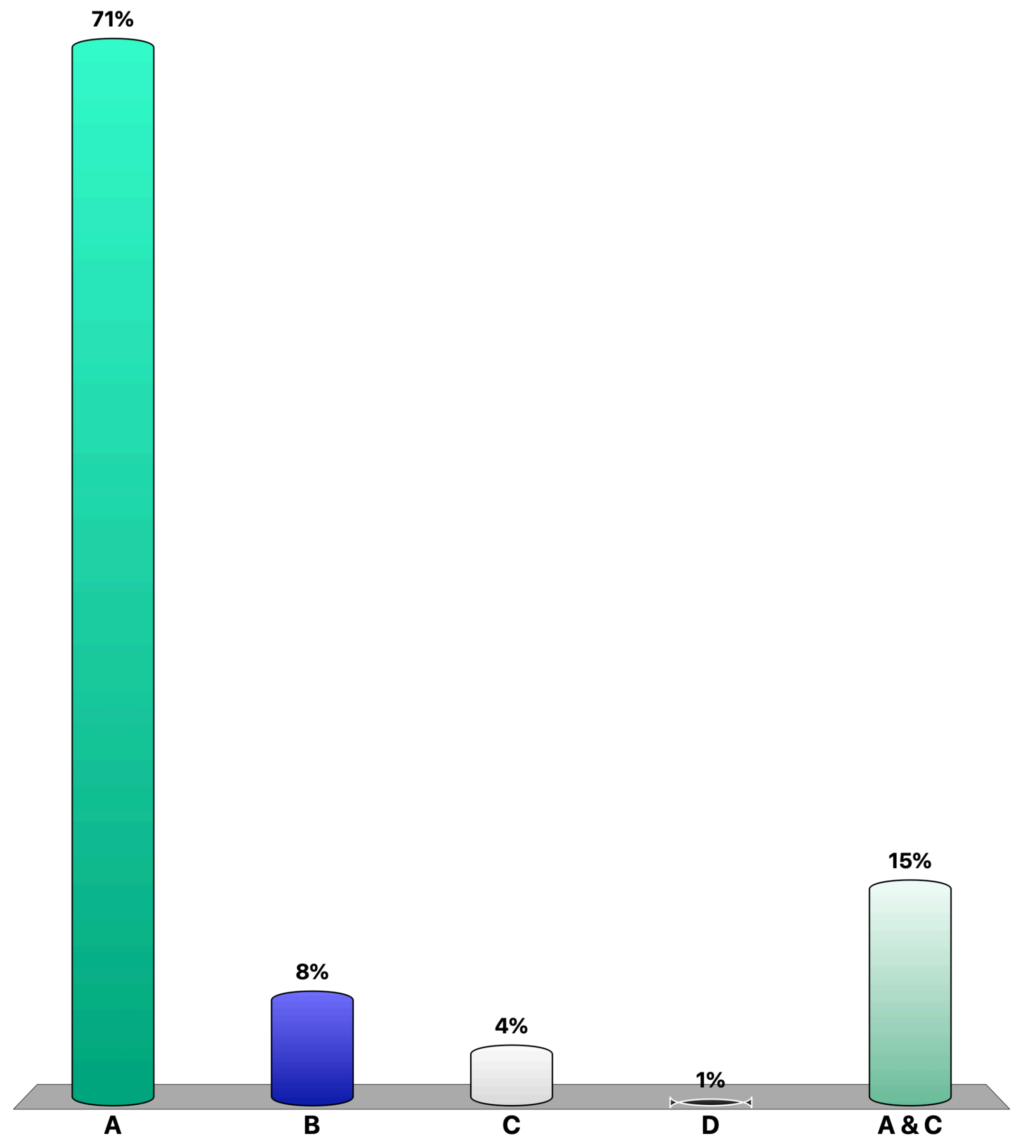


Question 4

For the following situations consider moving a positive charge of mass m from very far away to the origin along the *y*-axis. For which situation would an external force do the most work?

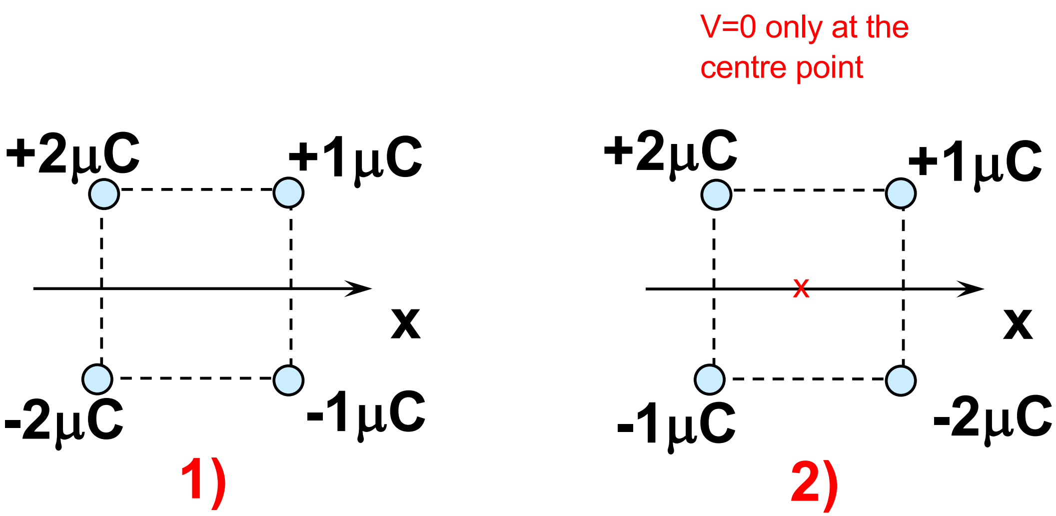


1. A ← V highest value + value
2. B
3. C
4. D
5. A & C

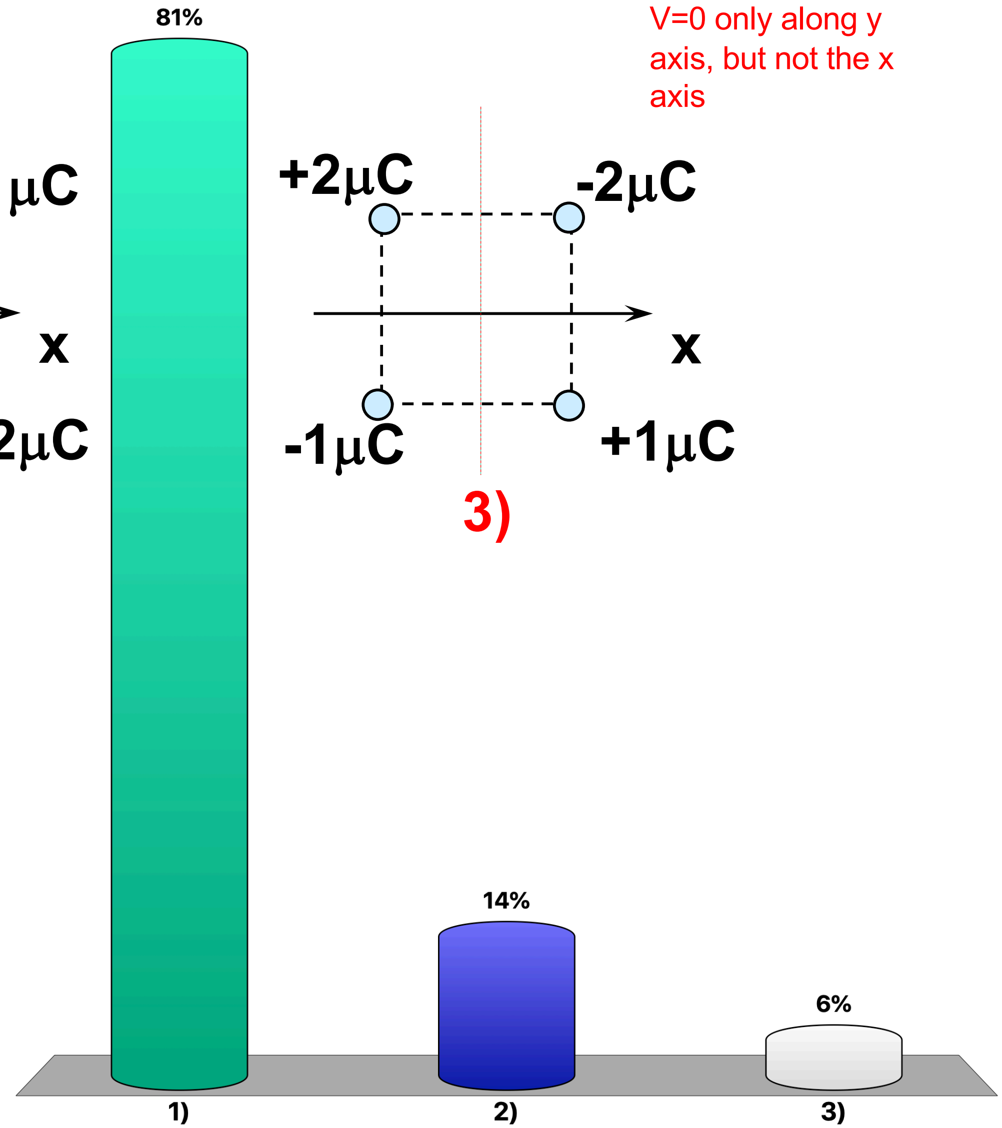


Question 5

For which of these configurations is $V = 0$ at all points on the x-axis?



1. 1) ← Sum of potentials cancel at all points along x axis
2. 2)
3. 3)



Conservation of Kinetic & Electrical Potential Energy (17-5b)

Electrical field is conservative →Charge moves in such a way that electrical potential and kinetic energy are always conserved. The situation here is exactly analogous to gravity, except with electrical potential energy.

$$KE_A + U_A = KE_B + U_B$$

In terms of explicit formulae

$$\boxed{\frac{1}{2}mv_A^2 + QV_A = \frac{1}{2}mv_B^2 + QV_B}$$

** These are electrical potentials due to an electric field at positions A and B

E-Field Simulator