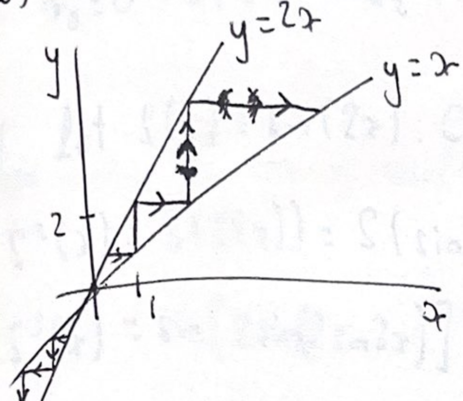
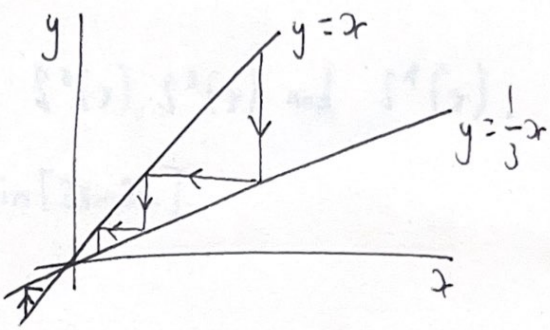
4.- Chapter 19/05

1. Use graphical analysis to describe the fate of all orbits for each of the following functions.

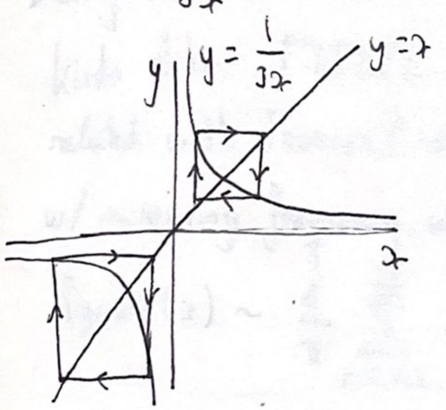
a) $F(x) = 2x$



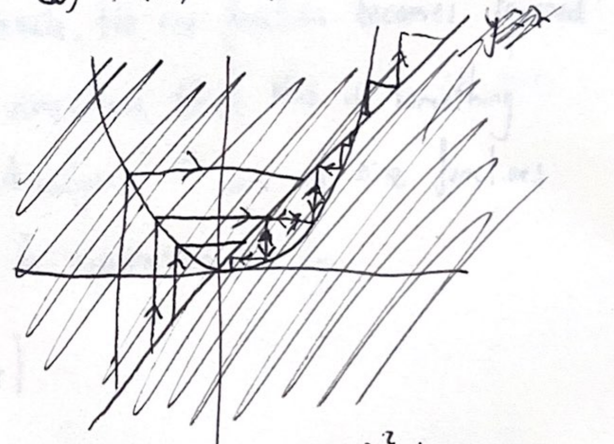
b) $F(x) = \frac{1}{3}x$

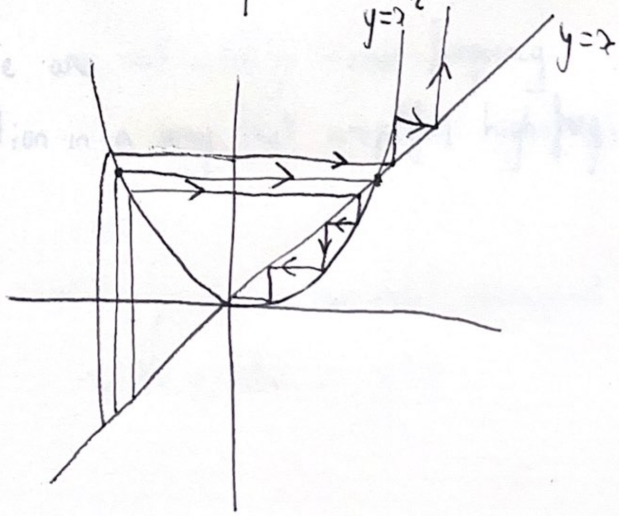


b) $Fx = \frac{1}{3x}$



d) $F(x) = x^2$ *





$\forall x > 0, F^n(x) > 0$

$\forall x < 0, F^n(x) < 0$

* i. if $x > 1$, $F^n(x) \to \infty$ as $n \to \infty$

ii. if $x = 1$, $F^n(x) = 1$

iii. if $0 < x < 1$, $F^n(x) \to 0$

iv. if $x < -1$, $F^n(x) \to \infty$

v. if $x = -1$, $F^n(x) = 1$, eventually fixed

vi. if $-1 < x < 0$, $F^n(x) \to 0$

20/05

Chapter 3

2. Let $F(x) = x^2+1$. Compute the first five points on the orbit of 0.

~~An.~~

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 2 \quad x_3 = 5 \quad x_4 = 26 \quad x_5 = 677$

4. Let $S(x) = \sin(2x)$. Compute $S^2(x), S^3(x)$ and $S^4(x)$.

$S^2(x) = S(S(x)) = S(\sin 2x) = \sin[2\sin 2x]$

$S^3(x) = \sin[2\sin(2\sin 2x)]$

$S^4(x) = \sin[2\sin(2\sin(2\sin 2x))]$

↳ Using desmos, I see that as we iterate, the sine function becomes "squared" kinda like . I read somewhere that this is something related with Fourier? → Fourier Analysis! A sum of sine functions w/ increasing frequency, which can be approximated by

$$\text{Square}(x) \sim \frac{4}{\pi} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin(nx).$$

However, with $F(x) = \sin 2x$, we are not adding higher-frequency ~~sines~~ sines, but nesting the function in a way that amplifies high-freq. behavior.

7. Find all real fixed points, if any:

d) $F(x) = x^3 - 3x \rightarrow x^3 - 3x = x \rightarrow x(x^2 - 4\cancel{x}) = 0$

$\therefore x = 0; \pm 2$, or, $\text{fix } F = \{0, \pm 2\}$

e) $F(x) = |x| \rightarrow$ ①: $x = x \quad \therefore \mathbb{R}$

②: $x = -x \rightarrow x = 0 \quad \therefore \text{fix } F = \{x : x \geq 0\}$

14. Discuss behaviour of resulting orbit under D for $x_0 = 1/16$.

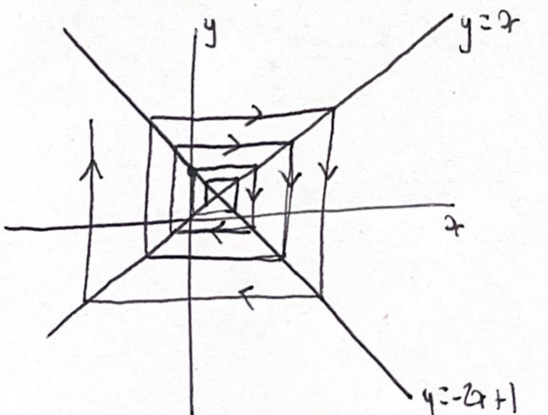
$x_0 = \frac{1}{16} \quad x_1 = \frac{1}{8} \quad x_2 = \frac{1}{4} \quad x_3 = \frac{1}{2} \quad x_4 = 0$

$\therefore$ eventually fixed

Chapter 4

1. Use graphical analysis to describe the fate of all orbits for each of the following functions.

c) $F(x) = -2x + 1 \rightarrow \text{fix } F = \{1/3\}$

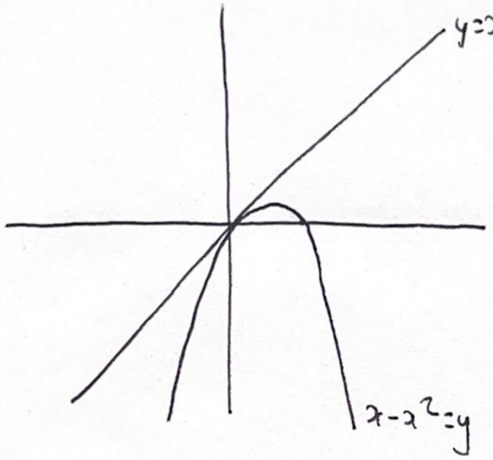


every two strokes, that is the next point on the orbit of x_0. *

Try to start on the first quadrant always

* The points on the orbit correspond to the x-value in $y = x$.

f) $F(x) = x - x^2 \to \text{fix } F = \{0\}$ and



i. $0 < x < 1$, $F^n(x) \to 0$

ii. $x > 1$, $F^n(x) \to -\infty$

iii. $x = 1, 0$, $F^n(x) \to 0$

iv. $x < 0$, $F^n(x) \to -\infty$

rmk: The fixed point is neutral, which we'll discuss in chapter 5.

2. Use graphical analysis to find $\{x_0 \mid F^n(x_0) \to \pm\infty\}$ for the following.

a) $F(x) = 2x(1-x) = 2x - 2x^2$

$2x - 2x^2 = x \to 2x^2 = x \to 2x = 1 \to \text{fix } F = \{1/2\}$

$-x(2x-2) = 0 \to x = 0, 1$

$\therefore$ vertex at $x = 1/2$

For $0 < x < 1$, $F^n(x) \to 1/2$. However

$\{x : F^n(x_0) \to \pm\infty\} = \{x : x < 0 \text{ or } x > 1\} \to -\infty$