DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Series Solutions of Ordinary Differential Equations.

Course Code: MAT2002 **Experiment: 4-B**
Course Name: Application of Differential and Difference Equations **Duration: 90 Minutes**

Series Solution when $x = 0$ ia an Ordinary Point of the Equation

$$P_0\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2y = 0 \tag{1}$$

where P's are polynomial in x and $P_0 \neq 0$ at $x = 0$.

1. Assume its solution to be of the form

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots \tag{2}$$

2. Calculate $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, from (2) and substitute the values of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ in (1).

3. Equate to zero the coefficients of the various powers of x and determine a_2, a_3, a_4, $\cdots$ in terms of a_0, a_1.

4. Substituting the values of a_2, a_3, a_4, $\cdots$ in (2), we get the desired series solution having a_0, a_1 as its arbitrary constants.

1. Solve in series the equation $\frac{d^2y}{dx^2} + y = 0$.

MATLAB CODE

```
clc
clear

syms x a0 a1 a2 a3
a = [a0 a1 a2 a3];
y = sum(a.*(x).^[0:3]);

dy = diff(y);
d2y = diff(dy);
gde = collect(d2y+y,x);
cof=coeffs(gde,x);

A2=solve(cof(1),a2);
A3=solve(cof(2),a3);

y=subs(y,a2,a3,A2,A3);
y=coeffs(y,[a1 a0]);
disp('Solution is')
disp(['y=A(',char(y(1)),'+ ...)+B(',char(y(2)),'+ ...)'])
```

OUTPUT

```
Solution is
y=A(1 - x^2/2+ ...)+B(x - x^3/6+ ...)
```

Exercise

2. Solve the following:
   (a) $\frac{d^2y}{dx^2} + xy = 0$
   (b) $\frac{d^2y}{dx^2} + x^2y = 0$
   (c) $y'' + xy' + y = 0.$
   (d) $(1 - x^2)y'' + 2y = 0; y(0) = 4, y'(0) = 5$

3. The half-life of radium is 1600 years, i.e., it takes 1600 years for half of any quantity to decay. If a sample initially contains 50 g, how long will it be until it contains 45 g by power series method?