Q1.(i)(a)

1.(i)(a). $p \wedge (q \vee r) \models (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

In all rows where both p and $(q \vee r)$ are true, $(p \wedge q) \vee (p \wedge r)$ are also true.
Therefore, $p \wedge (q \vee r) \models (p \wedge q) \vee (p \wedge r)$ is valid

(i)(b)

(i)(b). $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

CNF $\neg((p \wedge q) \vee (p \wedge r))$ (Negation of conclusion)

$\equiv \neg(p \wedge q) \wedge \neg(p \wedge r)$ (De Morgan)

$\equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$ (De Morgan)

Proof:

1. p (Hypothesis)
2. $q \vee r$ (Hypothesis)
3. $\neg p \vee \neg q$ (Negation of conclusion)
4. $\neg p \vee \neg r$ (Negation of conclusion)
5. $\neg q$ 1,3 Resolution
6. r 2,5 Resolution
7. $\neg p$ 4,6 Resolution
8. [] 1,7 Resolution

QED

(ii)(a)

(ii) (a). $\models p \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Last column is always true no matter what truth assignment to the atoms p and q. Therefore $p \rightarrow (q \rightarrow p)$ is a tautology.

(ii)(b)

(ii) (b). $\vdash p \rightarrow (q \rightarrow p)$

CNF $(\neg(p \rightarrow (q \rightarrow p)))$

$\equiv (\neg(\neg p \vee (q \vee p)))$ (Remove $\rightarrow$) ×2

$\equiv \neg\neg p \wedge \neg(\neg q \vee p)$

$\equiv \neg\neg p \wedge \neg\neg q \wedge \neg p$

$\equiv p \wedge q \wedge \neg p$

(ii) (b). $\vdash p \rightarrow (q \rightarrow p)$

CNF $\neg(p \rightarrow (q \rightarrow p))$

$\equiv \neg(\neg p \vee (\neg q \vee p))$ (Remove $\rightarrow$) ×2

$\equiv \neg\neg p \wedge \neg(\neg q \vee p)$ De Morgan

$\equiv \neg\neg p \wedge \neg\neg q \wedge \neg p$ De Morgan

$\equiv p \wedge q \wedge \neg p$ Double Negation

Proof
1. p Negation of Conclusion
2. q Negation of Conclusion
3. $\neg p$ Negation of Conclusion
4. [] 1, 3 Resolution

QED

(iii)(a)

iii) (a)

$KB = \exists x \forall y. Likes(x,y)$ $\quad \alpha = \forall x. \exists y. Likes(x,y)$

$D: \{0, 1\}$

$x, y \in D$

$I = \{ Likes(0,0), Likes(0,1), Likes(1,0), Likes(1,1) \}$

$\because KB = \exists x \forall y\ Likes(x,y)$

$\therefore \{Likes(0,0), Likes(0,1)\}$ are true or

$\{Likes(1,0), Likes(1,1)\}$ are true.

$\neg \alpha = \neg \forall x \exists y. Likes(x,y)$

$\equiv \exists x \forall y \neg likes(x,y)$.

Case 1 KB $\{Likes(0,0), Likes(0,1)\}$ are true

$\neg\alpha$ can find $\{Likes(1,0), Likes(1,1)\}$ are false

Case 2. KB $\{Likes(1,0), Likes(1,1)\}$ are true

$\neg\alpha$ can find $\{Likes(0,0), Likes(0,1)\}$ are false

Both cases are satisfiable

Thus, $\exists x \forall y. Likes(x,y) \models \forall x. \exists y. Likes(x,y)$ is invalid

(iii)(b)

(iii) (a). $\exists x. \forall y\ Likes(x,y) \vdash \forall x. \exists y\ Likes(x,y)$

$CNF(\exists x \forall y. Likes(x,y))$

$\equiv \forall y. Likes(a,y)$ (Skolemise) ~~a is a ...~~

$\equiv Likes(a,y)$ (Drop $\forall$)

$CNF(\neg \forall x. \exists y. Likes(x,y))$

$\equiv \exists x \neg \exists y\ Likes(x,y)$ Move $\neg$ inwards

$\equiv \exists x \forall y \neg likes(x,y)$ Move $\neg$ inwards

$\equiv \cancel{\exists x \forall}\ \forall y \neg likes(b,y)$ skolemise

$\equiv \neg likes(b,y)$ Drop $\forall$

Proof 1. $Likes(a,y)$

2 $\neg Likes(b,z)$

Can not ~~solve it~~ yet. [] Thus, $\exists x. \forall y. Likes(x,y) \vdash \forall x \exists y Likes(x,y)$

is ~~in...~~ invalid.

(iv)(a)

(iv) (a). $\neg p \to \neg q,\ p \to q \vDash p \leftrightarrow q$

p	q	$\neg p$	$\neg q$	$\neg p \to \neg q$	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	T	T

In all rows where both $\neg p \to \neg q$ and $p \to q$ are true, $p \leftrightarrow q$ is also true.

Therefore, $\neg p \to \neg q,\ p \to q \vDash p \leftrightarrow q$ is valid

(iv)(b)

(iv) (b). $\neg p \to \neg q,\ p \to q \vdash p \leftrightarrow q$

CNF$(\neg p \to \neg q)$

$\equiv \neg\neg p \vee \neg q$ (Remove $\to$)

$\equiv p \vee \neg q$ (Double Negation)

CNF$(p \to q)$

$\equiv \neg p \vee q$ (Remove $\to$)

CNF$(\neg(p \leftrightarrow q))$

$\equiv \neg((p \to q) \wedge (q \to p))$

$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$ (Remove $\to$)

$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$ (De Morgan)

$\equiv (\neg\neg p \wedge \neg q) \vee (\neg\neg q \wedge \neg p)$ (De Morgan)

$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$ (Double Negation)

$\equiv (p \vee (q \wedge \neg p)) \wedge (\neg q \vee (q \wedge \neg p))$ (Distributive)

$\equiv \cancel{(p \wedge q)}\ (p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p)$ (Distributive)

Proof:

1. $p \vee \neg q$ (Hypothesis)
2. $\neg p \vee q$ (Hypothesis)
3. $p \vee q$ (Negation of conclusion)
4. $p \vee \neg p$ (Negation of conclusion)
5. $\neg q \vee q$ (Negation of conclusion)
6. $\neg q \vee \neg p$ (Negation of conclusion)
7. p (1,3 Resolution)
8. $\neg p$ (2,6 Resolution)
9. [] (7,8 Resolution)

QED

(v)(a)

(v)(a). $\forall x.\ P(x) \rightarrow Q(x),\ \forall x\ Q(x) \rightarrow R(x),\ \neg R(a) \models \neg P(a)$

CNF$(\forall x.\ P(x) \rightarrow Q(x))$

$\equiv P(x) \rightarrow Q(x)$

CNF$(\forall x\ Q(x) \rightarrow R(x))$

$\equiv Q(x) \rightarrow R(x)$

CNF$(\neg R(a))$

$\equiv \neg R(x)$ $\quad (\cancel{\forall x}\ \{a/x\})$

CNF$(\neg P(a))$

$\equiv \neg P(x)$ $\quad (\{a/x\})$

$P(x)$	$Q(x)$	$R(x)$	$\neg R(x)$	$P(x) \rightarrow Q(x)$	$Q(x) \rightarrow R(x)$	$\neg P(x)$
T	T	T	F	T	T	F
T	T	F	T	T	F	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	T	F	T	T	F	T
F	F	T	F	T	T	T
F	F	F	T	T	T	T

In all rows where both $\neg R(x)$, $P(x) \rightarrow Q(x)$ and $Q(x) \rightarrow R(x)$ are true, $\neg P(x)$ is also true.

Therefore $\forall x.P(x) \rightarrow Q(x),\ \forall x.Q(x) \rightarrow R(x),\ \neg R(a) \models \neg P(a)$ is valid.

(v)(b)

$\forall x.P(x) \rightarrow Q(x),\ \forall x.Q(x) \rightarrow R(x),\ \neg R(a) \vdash \neg P(a)$

(v)(b). CNF$(\forall x\ (P(x) \rightarrow Q(x)))$

$\equiv \forall x(\neg P(x) \vee Q(x))$ $\quad$ (Remove $\rightarrow$)

$\equiv \neg P(x) \vee Q(x)$ $\quad$ (Drop $\forall$)

CNF$(\forall x\ (Q(x) \rightarrow R(x)))$

$\equiv \forall x(\neg Q(x) \vee R(x))$ $\quad$ (Remove $\rightarrow$)

$\equiv \neg Q(x) \vee R(x)$ $\quad$ (Drop $\forall$)

Proof

1. $\neg P(x) \vee Q(x)$ (Hypothesis)
2. $\neg Q(y) \vee R(y)$ (Hypothesis)
3. $\neg R(a)$ (Hypothesis)
4. $\neg(\neg P(a))$ (Negation of conclusion)
5. $P(a)$ (Double Negation)
6. $\neg P(a) \vee Q(a)$ (1. $\{x/a\}$)
7. $\neg Q(a) \vee R(a)$ (2. $\{y/a\}$)
8. $Q(a)$ 5,6 Resolution
9. $R(a)$ 7,8 Resolution
10. [] 3,9 Resolution

QED

Q2(i)

2.(i) 1. samefloor(Ivor, photographer)

2. above(Edwina, medical student)

3. study(Patrick, law) $\wedge$ above(Patrick, Ivor) $\wedge$ samefloor(Patrick, hostress)

4. Lives(store, flat 4)

5. Lives(Doris, flat 2)

6. Resident(Rodney) $\wedge$ Resident(Rosemary)

7. $\exists x$ job(x, clerk) $\wedge$ ($\neg \exists y \bullet$ job(y, clerk) $\wedge$ $x \neq y$)

(ii)

ii). From the fact the ~~Patrick~~ above(Patrick, Ivor)

Thus there are 4 possible solution

①. ~~P~~ Lives(Patrick, flat5) $\wedge$ ~~Ivor~~ Lives(Ivor, flat3)

②. Lives(Patrick, flat3) $\wedge$ Lives(Ivor, flat 1)

③. Lives(Patrick, flat 6) $\wedge$ Lives(~~Patrick~~ Ivor, flat 4)

④. Lives(Patrick, flat 4) $\wedge$ Lives(Ivor, flat 2)

Case ①. From samefloor(Ivor, photographer) $\wedge$ Lives(Ivor, flat 3)

we can get Lives(photographer, flat 4)

which is a contradiction to Lives(store, flat4)

Therefore, case ① is unsatisfiable

Case ②. From samefloor(Patrick, hostress) $\wedge$ live(Patrick, flat3)

we can get lives(hostress, flat 4)

which is a contradition to lives(store, flat 4)

Therefor case ② is unsatisfiable.

Case ③. From Lives(Ivor, Flat 4) $\wedge$ Lives(store, flat 4)

we can get job(Ivor, store)

From samefloor(Patrick, hostress) $\wedge$ lives(Patrick, flat 6)

we can get live(hostress, flat 5)

From samefloor(Ivor, photographer) $\wedge$ Lives(Ivor, flat 4)

we can get lives(photographer, flat 3)

~~Thus we have~~ ~~job~~ ~~study~~

Thus, just flat 1 and flat 2 have no job

From above(Edwina, medical student) $\wedge$

~~we get lives(medical student, flat 1) $\wedge$~~

above(flat 4, Flat 2) $\wedge$ live(Ivor, flat 4)

we get not lives(medical student, flat 2)

Thus, live(medical student, flat 1)

From above(Edwina, medical student) $\wedge$ live(flat 3, flat 1)

~~we get~~ $\wedge$ live(medical student, flat 1)

we get lives(Edwina, flat 3)

Thus, job(Edwina, photographer)

From $\exists x$ job(x, clerk) $\wedge$ ($\neg\exists y$ job(y, clerk) $\wedge$ $x \neq y$)

job(Doris, clerk)

Thus, we can not determine the job and flat for Rodney and Rosemary

Case ④. From lives(Ivor, flat 2) $\wedge$ lives(Doris, flat 2)

which is a contradition

Thus, case ④ is not satisfiable

As a result, we can not determine the name and situation

(iii)

(iii). Further sentence: female(Rosemary) $\wedge$ female(hostress) $\wedge$ male(Rodney)

~~In that case: we can get R lives(~~

If we apply these sentences to case ③.
we can get lives(Rosemary, flat 5) $\wedge$ job(Rosemary, hostress)

Then. lives(Rodney, flat 1) $\wedge$ job(Rodney, medical student)

As a result we cant get all the name and situation of resident of each flat.

hostress	5	Rosemary	Law	6	Patrick
photographer	3	Edwina	store	4	Ivor
medical student	1	Rodney	clerk	2	Doris

Q3

My code is written by python3, so please type ‘chmod u+x assn1q3’ before testing.

Q4(i). Intelligent reasoning: Applying both the logical view and the psychological view to represent knowledge.

(ii). For logical aspect, first order logic can be used to represent knowledge and various deduction is applied to reasoning.

For psychological view, goals, plans and other complex mental structure can be used to address problems. For example, modern manifestations include work on SOAR as a general mechanism for producing intelligent reasoning and knowledge based systems as a means of capturing human expert reasoning.