Problem 1

(a). 8 possible functions

f(a) = 0, f(b) = 0 ,f(c) = 0
f(a) = 0, f(b) = 0 ,f(c) = 1
f(a) = 0, f(b) = 1 ,f(c) = 0
f(a) = 0, f(b) = 1 ,f(c) = 1
f(a) = 1, f(b) = 0 ,f(c) = 0
f(a) = 1, f(b) = 1 ,f(c) = 0
f(a) = 1, f(b) = 0 ,f(c) = 1
f(a) = 1, f(b) = 1 ,f(c) = 1

(b). The domain of answer for (a) belong to Pow({a,b,c})

(c). (i). There are n^m functions from A to B.
(ii). There are m*n relations between A and B.
(iii).There are m^2 symmetric relations on A.

Problem 2

(a). $S_{2,-3}$ = {-1,-2,-3,-4,-5} when (m = n = 1,2,3,4,5)

(b). $S_{12,16}$ = {28,56,84,112,140} when (m = n = 1,2,3,4,5)

(c). Assume Sx,y ⊆ {n : n ∈ Z and d|n} is invalid.
Thus, there must be a element k in $S_{x,y}$ but not in {n : n∈ Z and d|n}.
k = mx + ny
∵ d = gcd(x,y) and x,y ∈ Z
∴ we can get x = ad and y = bd (a,b ∈ Z)
Therefore, k = amd + bnd = (am + bn)d and k ∈ Z
∵ m, n ∈ Z
∴ k = pd(p = am+bn and p ∈ Z)
∵ d|n and n ∈ Z
∴ n = qd(q∈ Z)
Thus, k **in** {n : n ∈ Z and d|n}.
So the former assumption is wrong.
Therefore, we can get Sx,y ⊆ {n : n ∈ Z and d|n}.

(d). Assume {n : n∈ Z and z|n} ⊆ $S_{x,y}$ is invalid
Thus, there must be an element k in {n : n∈ Z and z|n} not in $S_{x,y}$
k = n / z(k ∈ Z), n = k*z = k*(min(mx+ny))
∵ m,n ∈ Z
∴ k*m, k*n ∈ Z
Therefore, n in $S_{x,y}$, so assumption is wrong
As a result, {n : n∈ Z and z|n} ⊆ $S_{x,y}$

(e). ∵ $d = \gcd(x,y)$

∴ $x = a*d$ and $y = b*d$ ($a,b \in N_{>0}$) and $d >= 1$

∵ z is the smallest positive number in $S_{x,y}$

∴ $z = \min(mx+ny) = \min(m*a*d+n*b*d) = (ma+nb)d >= 1$

∵ $d >= 1$ and $(ma+nb)d >= 1$

∴ $(ma+nb) >= 1$

∵ $m,n,a,b \in Z$

∴ $(ma+nb) \in Z$

∵ z is a positive number and $d >= 1$

∴ $(ma+nb) >= 1$

∴ $z = (ma+nb)d >= d$

(f). Assume $z <= d$ is invalid, so that is $z > d$ for all case.

∵ z is the smallest positive number

∴ $z >= 1$. So when $x = 0$ and $y = 1$ and $m = 0$ and $n = 1$

$z = 1$

∵ $d = \gcd(x,y) = \gcd(0,1) = 1$

Then $d = z$

So $z > d$ is not valid.

Therefore, $z <= d$.

Problem 3

(a) $(A*B)*(A*B)$

$= (A^c \cup B^c)*(A^c \cup B^c)$	$A*B := A^c \cup B^c$
$= (A^c \cup B^c)^c \cup (A^c \cup B^c)^c$	$A*B := A^c \cup B^c$
$= ((A^c)^c \cap (B^c)^c) \cup ((A^c)^c \cap (B^c)^c)$	De morgan
$= (A \cap B) \cup (A \cap B)$	Double Complementation
$= A \cap B$	Idempotence

(b) $A*A$

$= (A^c \cup A^c)$	$A*B := A^c \cup B^c$
$= A^c$	Idempotence

(c) $(A*(A*A))*(A*(A*A))$

$= (A \cap (A*A))$	(a)
$= A \cap A^c$	(b)
$= \varnothing$	Complementation

(d) $(A*(B*B))*(A*(B*B))$

$= (A*B^c)*(A*B^c)$	(b)
$= A \cap B^c$	(a)
$= A \setminus B$	

Problem 4

(a) $w = aa$, $v = bb$

(b) ∵ $R^{\leftarrow}(\{aba\})$, $(w,v) \in R$

Case1: $w = aba$, $v = \lambda$.

∵ $v = wz$

∴ $v \neq \lambda$

Invalid.

Case2: $w = \lambda$, $v = aba$

∵ $v = wz = \lambda z = aba$

∴ $z = aba$

Valid.

Therefore, $w = \lambda$, $v = aba$, $z = aba$.

$R^{\leftarrow}(\{aba\}) = (\lambda, aba)$

(c) Firstly,$(w,v) = (w,zw)$

Assume $z = \lambda \in \Sigma^*$, for all $(w,v) \in R$: $(w,v) = (w,w) \in R$

Thus, R is Reflexive

Secondly, if (w,v) and $(v,w) \in R$

$(v,w) = (wz,w)$

$wz = w$

so, $z = \lambda$,

therefore $w = zw = v$

thus, R is Antisymmetric

Finally, if (x,y) and $(y,p) \in R$

∴ $y = zx$ and $p = zy = z*(zy) = (z*z)y$

∵ $z \in \Sigma^*$(I am not sure whether I can get the result)

∴ $z*z \in \Sigma^*$

$z*z \in z$

Thus, $(x,p) \in R$

Therefore, R is Transitive

As a result, R is Reflexive, Antisymmetric and Transitive, so we can say R is a partial order.

Problem 5

1. If $y = 0$, ∵ $\gcd(x,y) = 1$

∴ $x|y = 1$ so $x = 1$

Thus, $x|z$

2. If $y \neq 0$ and $z = 0$,

∵ $x|yz$,

∴ $x \neq 0$

Therefore, $x|z$

3. If y != 0 and z != 0

∵gcd(x,y) = 1

∴(a). If x = 1,then x|z

(b). If x != 1 and y = 1,

x|yz = x|1*z = x|z

(c). If x != 1 and y != 1

∵x|yz

∴yz = kx(k ∈ Z)

k = (y*z)/x

thus, x|y or x|z

∵gcd(x,y) = 1 and x != 1 and y != 1

∴x|y is not valid.

Therefore, we get x|z.

As a result, for all conditions, we always have x|z.