Problem1

1. Assume any element (a, c) ∈ (R1;R2);R3,

Then, there must be (a, x) ∈ R1, (x, y) ∈ R2, (y, c) ∈ R3.

∴(x, c) ∈ R2;R3

(a, c) ∈ R1;(R2;R3)

Therefore, (R1;R2);R3 = R1;(R2;R3)

1. Assume any element (a, b) ∈ R1,

R1 ⊆ S×S

Thus, {a,b} ∈ S,

I = {(x,x) : x ∈S}

Therefore {(a, a), (b, b)} ∈ I

∴ (a, b) ∈ I;R1 and (a, b) ∈ R1;I

So, I;R1 = R1;I = R1

1. Assume R1 = {(a, b)}, R2 = {(b, c)}

R1;R2 = (a, c)

(R1;R2)← = (c, a)

R1← = {(b, a)}, R2←= {(c, b)}

R1← ; R2← = Ø

Thus, (R1;R2)← ≠ R1← ;R2←.

1. Assume (R1∪R2);R3 = (a, b)

Then, (a, x) ∈ (R1∪R2) and (x, b) ∈ R3

∵（a, x）∈ (R1∪R2)

∴There are three conditions

① (a, x) ∈ R1 and (a, x) ∉ R2

Then, (R1;R3) = (a, b), (R2;R3) = Ø

(R1;R3)∪(R2;R3) = (a, b) = (R1∪R2);R3

② (a, x) ∈ R2 and (a, x) ∉ R1

Then, (R2;R3) = (a, b), (R1;R3) = Ø

(R1;R3)∪(R2;R3) = (a, b) = (R1∪R2);R3

③ (a, x) ∈ R1 and (a, x) ∈ R2

Then, (R1;R3) = (a, b) and (R2;R3) = (a, b)

(R1;R3)∪(R2;R3) = (a, b) = (R1∪R2);R3

Therefore, (R1∪R2);R3 = (R1;R3)∪(R2;R3)

1. Assume R1 = {(b, a), (b, b)}, R2 = {(a, b)}, R3 = {(b, b)}

Thus, left hand: R2∩R3 = Ø，

R1;( R2∩R3) = Ø

Right hand: (R1;R2) =(b, b), (R1;R3) = (b, b)

Then (R1;R2)∩(R1;R3) = (b, b) ≠ Ø

So, R1;(R2∩R3) ≠ (R1;R2)∩(R1;R3)

Problem 2

1. Assume Rj = Ri+K,

∵ j >= i

∴ k >= 0

We assume Rj = Ri,

That is Ri+k = Ri,

Base case: k = 0, Ri+k = Ri+0 = Ri

Inductive case: Assume Ri+k = Ri hold,

Ri+k+1 = Ri+k ∪ (R;Ri+k)

∵ Ri+k = Ri and Ri+1 := Ri∪(R;Ri)

∴ Ri+k+1 = Ri∪(R;Ri) = Ri+1 = Ri

Thus, if there is an i such that Ri = Ri+1, then Rj = Ri for all j≥i.

1. ①. If k >= i, just as (a), Rk = Ri,

Then, Rk ⊆ Ri.

②. If 0 <= k < i,

Let i = k + x

Thus, Rk ⊆ Ri can be written as Rk ⊆ Rk+x

Base case: x = 0, Rk = Rk+0 ⊆ Rk

x=1, Rk+1 = Rk ∪ (R;Rk)

thus, Rk ⊆ Rk+1

Inductive case: we assume Rk ⊆ Rk+x hold,

Rk+x+1 = Rk+x ∪ (R;Rk+x)

Rk+x ⊆ Rk+x+1

∵ Rk ⊆ Rk+x

∴ Rk ⊆ Rk+x+1

Thus, if there is an i such that Ri = Ri+1, then Ri ⊆ Ri for all k ≥0.

1. Base case: n = 0,

R0;Rm = I;Rm =Rm, hold. Problem1 (b)

Inductive case: Assume Rn;Rm = Rn+m,

Rn+1;Rm = Rn ∪(R;Rn);Rm

= Rn;Rm∪(R;Rn);Rm problem1 (d)

= Rn;Rm∪R;(Rn;Rm) problem1 (a)

= Rn+m ∪(R;Rn+m) Rn;Rm = Rn+m

= Rn+m+1

Therefore, hold.

1. Firstly, prove R0 ∈ Ri

Base case: i = 1, R1 = R0 ∪(R;R0)

R0 ∈ Ri hold,

Inductive case: Assume R0 ∈ Ri,

Ri+1 = Ri∪(R; Ri)

∵R0 ∈ Ri，

∴R0 ∈ Ri+1，hold

Therefore, R0 ∈ Ri

∵(a, b) ∈ Rk+1

If k = 0,

Then, (a, b) ∈ R1 = R0∪(R;R0)

= I ∪(R;I)

=I ∪ R

∵I = {(x, x) : x ∈S}

∴(a, b) ∈ R and (b, b) ∈ I = R0

∵R0 ∈ Ri-1 (i >= 1)

∴(a, b) ∈ R;Ri-1

Therefore, (a, b) ∈ Ri = Ri-1∪(R; Ri-1)

Thus, RK = Rk+1.

(e)

(f). Let Z = (R∪R←),

Then, (R∪R←)k = Zk

①. When k = 0, Z0 = I = {(x,x) : x ∈S}

∵Z0 ∈Zk

∴Zk is reflective

②.

Problem3

1. Binary tree:

①. Empty tree

②. A node with left\_tree and right\_tree.

1. count(T):

①. T = Ø, count(t) = 0,

②. count(left\_tree) + count(right\_tree) +1

1. leaves(T):

① no successors, return 1

② leaves(left\_tree) + leaves(right\_tree)

1. internal(t):

①. No successor: 0

②. 1 successor: internal(successor)

③. 2 successors: internal(left\_tree) + internal(right\_tree)

1. Base case: when node = 1, leaves(1) = 1, internal(1) = 0,

Then, leaves(T) = 1 + internal(T), hold.

Inductive case: Assume leaves(T) = 1 + internal(T),

①. If leaves(T+1) = leaves(T),

Then, internal(T+1) = internal(T)

Thus, internal(T+1) = 1 + internal(T+1), hold

②. If leaves(T+1) = leaves(T) + 1,

Then, internal(T+1) = internal(T) + 1

Thus, internal(T+1) = 1 + internal(T+1), hold.

Problem4

1. Define hi Alpha as HA, lo Alpha as LA

Define hi Bravo as HB, lo Bravo as LB

Define hi Charlie as HC, lo Charlie as LC

Define hi Delta as HD, lo Delta as LD

(i).(HA∪LA)∩(HB∪LB)∩(HC∪LC)∩(HD∪LD)

(ii).((¬HA∩LA)∪(HA∩¬LA))∩((¬HB∩LB)∪(HB∩¬LB))∩((¬HC∩LC)∪(HC∩¬LC))∩((¬HD∩LD)∪(HD∩¬LD))

(iii).((HA∩LB)∪(HB∩LA))∩((HB∩LC)∪(HC∩LB))∩((HC∩LD)∪(HD∩LC))

1. (i).HA∩LB∩HC∩LD is true

(ii).Alpha and Charlie use hi, Bravo and Delta use lo.