Problem 1

(a). Assume {p,q,r} in PROG,

Firstly, we can easily prove that p≡p,

Thus, F is reflexive

Secondly, assume p≡q, then p ↔ q

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | p → q | q → p | p ↔ q | q ↔ p |
| T | T | T | T | T | T |
| T | F | F | T | T | T |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

As the above truth table, from p ↔ q, we can get q ↔ p,

Thus, F is symmetric

Finally, assume p≡q and q≡r, then p ↔ q and q ↔ r

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p→q | q→p | q→r | r→q | p→r | r→p | p↔q | q↔r | p↔r |
| T | T | T | T | T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | F | T | T | F | F |
| T | F | T | F | T | T | F | T | T | F | F | T |
| T | F | F | F | T | T | T | F | T | F | T | F |
| F | T | T | T | F | T | T | T | F | F | T | F |
| F | T | F | T | F | F | T | T | T | F | F | T |
| F | F | T | T | T | T | F | T | F | T | F | F |
| F | F | F | T | T | T | T | T | T | T | T | T |

As the above truth table, from p ↔ q and q ↔ r, we can get p ↔ r,

Thus, F is transitive.

As a result we can get that the logical equivalence relation, ≡, is an equivalence relation on F .

(b).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p→q | p→r | p∧q | p∧r |
| T | F | F | F | F | F | F |

As the above truth table, {p→q, p→r, p∧q, p∧r} in [⊥].

(c). (i). φ≡φ’, then φ↔φ’, so φ→φ’ φ’→φ

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| φ | φ‘ | φ→φ’ | φ’→φ | ¬φ→¬φ’ | ¬φ’→¬φ | φ↔φ’ | ¬φ↔¬φ’ |
| T | T | T | T | T | T | T | T |
| T | F | F | T | T | F | F | F |
| F | T | T | F | F | T | F | F |
| F | F | T | T | T | T | T | T |

As the above truth table shows, whenever φ↔φ’, we have ¬φ↔¬φ’, then ¬φ≡¬φ’.

(ii). φ∧ψ≡[φ∧ψ]’

Problem 5

1. p1(n+1) = (p2(n)+p4(n))/3

p2(n+1) = (p1(n)+p3(n))/2 + p4(n)/2

p3(n+1) = (p2(n)+p4(n))/3

p4(n+1) = (p1(n)+p3(n))/2 + p2(n)/2