## The Vigenere cipher

Eric Martin, CSE, UNSW

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Consider a message all written in capital letters. The  $Caesar\ code$  fixes a natural number n between 1 and 25 and shifts all letters n positions to the right, wrapping around when needed, leaving all other characters such as spaces unchanged. For instance, the message ALEA JACTA EST becomes

```
• with n=1: BMFB KBDUB FTU
• with n=2: CNGC LCEVC GUV
• ...
• with n=10: KVOK TKMDK OCD
• ...
• with n=25: ZKDZ IZBSZ DRS
```

The *Vigenere cipher* offers a generalisation as follows. Choose a word written all in uppercase letters as the key, for instance, RAW. Since RAW consists of 3 letters, write the letters that make up the message over 3 columns:

ALE AJA CTA EST

As R, A and W are the 18th, 1st and 23rd letters of the alphabet, respectively, all letters in the first column are shifted 18 positions to the right, all letters in the second column are shifted 1 position to the right, and all letters in the third column are shifted 23 positions to the right, wrapping around when needed:

SMB SKX UUX WTQ

This results in the encrypted message: SMBS KXUUX WTQ

Let us not distinguish between (capital) letters and other characters, but rather let the Vigenere cipher encrypt all characters in a message with a key that itself can be any sequence of characters. The string module defines a string consisting of all printable ASCII characters; it could be a good candidate for which characters to allow both in message and key:

ASCII characters can be represented as \x followed by their ASCII code in the form of two hexadecimal digits:

'\x0b' and '\x0c' are the vertical tab and form feed characters, respectively; Jupyter ignores them. For code run from the command line, '\x0b' and '\x0c' behave the same: the next character follows the previous one but on the next line (form feed is meant to force a printer to move to the next sheet of paper). Both '\x0b' and '\x0c' actually have escaped characters as alternatives, namely, '\v' and '\f', respectively:

```
In [4]: '\v', '\f'
Out[4]: ('\x0b', '\x0c')
```

It is sensible to ignore the last 3 characters in printable and accept all others as the possible constituents of messages and keys:

Any character in characters can be used in keys and determine by how much to shift characters in messages; the values of the shifts are given by the positions in characters of the characters that make up the key. It would be inefficient to look for the position of a given character in characters every time it is needed: for instance, if a key was 7 characters long and started with 'G', it would be inefficient to scan characters left to right from the beginning, to find 'G' at position 42 and infer that the first, eight, fiftieth... characters in the message to encrypt should be shifted 42 positions to the right, wrapping around if needed. We define a dictionary with the characters in characters as keys, and their positions in characters as values, so the latter can be obtained from the former in constant time:

```
In [6]: shifts = {characters[i]: i for i in range(len(characters))}
        print(shifts)
{'0': 0, '1': 1, '2': 2, '3': 3, '4': 4, '5': 5, '6': 6, '7': 7, '8': 8,...
        ...'9': 9, 'a': 10, 'b': 11, 'c': 12, 'd': 13, 'e': 14, 'f': 15,...
        ...'g': 16, 'h': 17, 'i': 18, 'j': 19, 'k': 20, 'l': 21, 'm': 22,...
        ...'n': 23, 'o': 24, 'p': 25, 'q': 26, 'r': 27, 's': 28, 't': 29,...
        ...'u': 30, 'v': 31, 'w': 32, 'x': 33, 'y': 34, 'z': 35, 'A': 36,...
        ...'B': 37, 'C': 38, 'D': 39, 'E': 40, 'F': 41, 'G': 42, 'H': 43,...
        ...'I': 44, 'J': 45, 'K': 46, 'L': 47, 'M': 48, 'N': 49, '0': 50,...
         ...'P': 51, 'Q': 52, 'R': 53, 'S': 54, 'T': 55, 'U': 56, 'V': 57,...
         ...'W': 58, 'X': 59, 'Y': 60, 'Z': 61, '!': 62, '"': 63, '#': 64,...
         ...'$': 65, '%': 66, '&': 67, "'": 68, '(': 69, ')': 70, '*': 71,...
         ...'+': 72, ',': 73, '-': 74, '.': 75, '/': 76, ':': 77, ';': 78,...
        ...'<': 79, '=': 80, '>': 81, '?': 82, '@': 83, '[': 84, '\\': 85,...
         ...']': 86, '^': 87, '_': 88, '`': 89, '{': 90, '|': 91, '}': 92,...
                                      ...'~': 93, ' ': 94, '\t': 95, '\n': 96}
```

Let us give an example of a message and its encryption with the key 'Take it Easy!' (we will see below how the encryption has been computed, now it is given and supposed to have been properly computed). 'Take it Easy!' is a key of length 13, so we align the key, the message displayed over 13 columns, and the encrypted message displayed over 13 columns. As the message length is not a multiple of 13, the last lines of message and encrypted message are less than 13 characters long; formatted strings can given a field width to display a string, padding with spaces to the right for shorter strings, ignoring the field width for longer strings:

Alice was | beginning to | get | very | tired

Next to key and message, we display the ASCII codes of the characters in key and message, similarly aligned. We observed that characters has 97 characters.

- The first character in key, namely, 'T', has position 55 in characters. The first character in message, namely 'A', has position 36 in characters; it is therefore encrypted in encrypted\_message as the character that in characters, has position  $(55 + 36) \mod 97 = 91 \mod 97 = 91$ , namely, '|'.
- The fifth character in key, namely a space, has position 94 in characters. The fifth character in message, namely 'e', has position 14 in characters; it is therefore encrypted in encrypted\_message as the character that in characters, has position  $(94 + 14) \mod 97 = 108 \mod 97 = 11$ , namely, 'b'.

```
"twice she had peeped into the book her sister was reading, "
                    "but it had no pictures or conversations in it, 'and what "
                    "is the use of a book,' thought Alice, 'without pictures or "
                    "conversations?'"
                   )
         encrypted message = ('|vCqbfZ7\'7DM;,xHwkyqq#7IM|QFyFvfWf&oFv])7MwqLL'
                              'kU7D\'X+oLbpAVqSBpW\\QDBs|tDkY@pI\\\'7It|zDsWxI'
                              'v\\<DBwkygg#7FWGQyHgbfRoBDYQ-(7MvbfK7R7RM/=oxbf'
                              'FWlBDJMX%yIy|zHoBCK!|(BhK7KqoSkFQ\\*@hprLqf(7JI'
                              '.QxIbmAFq)BG!X<BhqlFYb&CC"=<xMbfFqf(@p5+;nhKesW'
                              '|WCp''<(70GbfRcBkpJ]<u\sim?|LKl)qJ''X|vCqb|q$+sVP]\\'
                              'DhDfuWr&oUv]?7wCkNHo\'kVQ];C5?'
                             )
         key = 'Take it Easy!'
         print(f'{key}' + ' ' + ' '.join(f'{shifts[c]:2}' for c in key))
         print()
         print('\n'.join(f'{message[i * 13: (i + 1) * 13]:13}' + ' ' +
                         ' '.join(f'{shifts[c]:2}'
                                   for c in message[i * 13: (i + 1) * 13]
                                 ) for i in range(len(message) // 13 + 1)
                        )
              )
         print()
         print('\n'.join(f'{encrypted_message[i * 13: (i + 1) * 13]:13}' + ' ' +
                         ' '.join(f'{shifts[c]:2}'
                                    for c in encrypted_message[i * 13: (i + 1) * 13]
                                 ) for i in range(len(encrypted_message) // 13 + 1)
                        )
              )
Take it Easy! 55 10 20 14 94 18 29 94 40 10 28 34 62
Alice was beg 36 21 18 12 14 94 32 10 28 94 11 14 16
inning to get 18 23 23 18 23 16 94 29 24 94 16 14 29
very tired o 94 31 14 27 34 94 29 18 27 14 13 94 24
f sitting by
              15 94 28 18 29 29 18 23 16 94 11 34 94
her sister on 17 14 27 94 28 18 28 29 14 27 94 24 23
the bank, an 94 29 17 14 94 11 10 23 20 73 94 10 23
d of having n 13 94 24 15 94 17 10 31 18 23 16 94 23
othing to do: 24 29 17 18 23 16 94 29 24 94 13 24 77
once or twic 94 24 23 12 14 94 24 27 94 29 32 18 12
e she had pee 14 94 28 17 14 94 17 10 13 94 25 14 14
ped into the
              25 14 13 94 18 23 29 24 94 29 17 14 94
book her sist 11 24 24 20 94 17 14 27 94 28 18 28 29
er was readin 14 27 94 32 10 28 94 27 14 10 13 18 23
```

```
g, but it had 16 73 94 11 30 29 94 18 29 94 17 10 13
               94 23 24 94 25 18 12 29 30 27 14 28 94
 no pictures
              24 27 94 12 24 23 31 14 27 28 10 29 18
or conversati
ons in it, 'a 24 23 28 94 18 23 94 18 29 73 94 68 10
nd what is th
              23 13 94 32 17 10 29 94 18 28 94 29 17
e use of a bo 14 94 30 28 14 94 24 15 94 10 94 11 24
ok,' thought
               24 20 73 68 94 29 17 24 30 16 17 29 94
Alice, 'witho 36 21 18 12 14 73 94 68 32 18 29 17 24
ut pictures o 30 29 94 25 18 12 29 30 27 14 28 94 24
r conversatio 27 94 12 24 23 31 14 27 28 10 29 18 24
ns?'
               23 28 82 68
vCqbfZ7'7DM;
              91 31 38 26 11 15 61
                                     7 68
                                          7 39 48 78
              73 33 43 32 20 34 26 26 64
                                           7 44 48 91
,xHwkygg#7IM|
QFyFvfWf&oFv]
               52 41 34 41 31 15 58 15 67 24 41 31 86
)7MwqLLkU7D'X 70
                  7 48 32 26 47 47 20 56
                                          7 39 68 59
+oLbpAVqSBpW\
              72 24 47 11 25 36 57 26 54 37 25 58 85
QDBs|tDkY@pI\
              52 39 37 28 91 29 39 20 60 83 25 44 85
'7It|zDsWxIv\
                  7 44 29 91 35 39 28 58 33 44 31 85
<DBwkyqq#7FWG
              79 39 37 32 20 34 26 26 64 7 41 58 42
QyHqbfRoBDYQ-
               52 34 43 26 11 15 53 24 37 39 60 52 74
(7MvbfK7R7RM/
                  7 48 31 11 15 46
                                     7 53
                                          7 53 48 76
=oxbfFWlBDJMX 80 24 33 11 15 41 58 21 37 39 45 48 59
%yIy|zHoBCK!|
              66 34 44 34 91 35 43 24 37 38 46 62 91
(BhK7KqoSkFQ\
              69 37 17 46
                           7 46 26 24 54 20 41 52 85
*@hprLqf(7JI.
               71 83 17 25 27 47 26 15 69
                                          7 45 44 75
0xIbmAFq)BG!X
              52 33 44 11 22 36 41 26 70 37 42 62 59
<BhqlFYb&CC"=
              79 37 17 26 21 41 60 11 67 38 38 63 80
<xMbfFqf(@p5+
              79 33 48 11 15 41 26 15 69 83 25
;nhKesW|WCp"< 78 23 17 46 14 28 58 91 58 38 25 63 79
(70GbfRcBkpJ]
                  7 50 42 11 15 53 12 37 20 25 45 86
<u~?|LKl)qJ"X 79 30 93 82 91 47 46 21 70 26 45 63 59
              91 31 38 26 11 91 26 65 72 28 57 51 86
[vCqb[q$+sVP]
\DhDfuWr&oUv]
              85 39 17 39 15 30 58 27 67 24 56 31 86
?7wCkNHo'kVQ]
                  7 32 38 20 49 43 24 68 20 57 52 86
               78 38 5 82
;C5?
```

Let us write a function, encrypt(), to encrypt a message, and a function, decrypt(), to decrypt an encrypted message, both with a key K provided as first argument, the message being provided as second argument. Both functions perform almost identically:

- To encrypt a message M, one goes over each character in M, determine its index i modulo the length of K, and shifts it to the right in characters by n with n the position in characters of the character in K of index i, wrapping around if needed.
- To decrypt an encrypted message E, one goes over each character in E, determine its index i modulo the length of K, and shifts it to the left in characters by n with n equal to the position in characters of the character in K of index i, wrapping around if needed.

It is therefore natural to define an auxiliary function, encrypt\_or\_decrypt() called by both encrypt() and decrypt(), being directed by either whether shifting should be "to the right" or "to the left", which corresponds to adding or subtracting positions:

```
In [9]: def encrypt(key, text):
            return encrypt_or_decrypt(key, text, 1)
        def decrypt(key, text):
            return encrypt_or_decrypt(key, text, -1)
        def encrypt or decrypt(key, text, mode):
            return ''.join(characters[(shifts[text[i]] +
                                       shifts[key[i % len(key)]] * mode
                                      ) % len(shifts)
                                     ] for i in range(len(text))
                          )
        encrypt(key, message)
        print()
        decrypt(key, encrypted_message)
        print()
        encrypt(key, decrypt(key, encrypted_message))
        print()
        decrypt(key, encrypt(key, message))
Out[9]: '|vCqbfZ7\'7DM;,xHwkyqq#7IM|QFyFvfWf&oFv])7MwqLLkU7D\'X+oLbpAVqSBpW\\Q...
   ...DBs|tDkY@pI\\'7It|zDsWxIv\\<DBwkyqq#7FWGQyHqbfRoBDYQ-(7MvbfK7R7RM/=oxbf...
   ...FWlBDJMX%yIy|zHoBCK!|(BhK7KqoSkFQ\\*@hprLqf(7JI.QxIbmAFq)BG!X<BhqlFYb&CC...
   ..."=<xMbfFqf(@p5+;nhKesW|WCp"<(70GbfRcBkpJ]<u~?|LKl)qJ"X|vCqb|q$+sVP]\\DhD...
                                                  ...fuWr&oUv]?7wCkNHo\'kVQ];C5?'
Out[9]: "Alice was beginning to get very tired of sitting by her sister on the...
   ...bank, and of having nothing to do: once or twice she had peeped into the...
    ...book her sister was reading, but it had no pictures or conversations in...
    ...it, 'and what is the use of a book,' thought Alice, 'without pictures or...
                                                                ...conversations?'"
Out[9]: '|vCqbfZ7\'7DM;,xHwkyqq#7IM|QFyFvfWf&oFv])7MwqLLkU7D\'X+oLbpAVqSBpW\\Q...
   ...DBs|tDkY@pI\\'7It|zDsWxIv\\<DBwkyqq#7FWGQyHqbfRoBDYQ-(7MvbfK7R7RM/=oxbf...
   ...FWlBDJMX%yIy|zHoBCK!|(BhK7KqoSkFQ\\*@hprLqf(7JI.QxIbmAFq)BG!X<BhqlFYb&CC...
   \dots"=<xMbfFqf(@p5+;nhKesW|WCp"<(70GbfRcBkpJ]<u^?|LKl)qJ"X|vCqb|q$+sVP]\\DhD...
                                                   ...fuWr&oUv]?7wCkNHo\'kVQ];C5?'
```

```
Out[9]: "Alice was beginning to get very tired of sitting by her sister on the...
...bank, and of having nothing to do: once or twice she had peeped into the...
...book her sister was reading, but it had no pictures or conversations in...
...it, 'and what is the use of a book,' thought Alice, 'without pictures or...
...conversations?'"
```

Let us now tackle the task of breaking the code of an encrypted message E, that is, discovering the key K that was used to encrypt a message M into E, and decrypting E into M with K. A program can only tentatively suggest candidates for K that the user would then have to validate. Also, the program might not always succeed, especially when the encrypted message is short. Let us fix an upper bound on the length of K, the first of a number of parameters we will use in our heuristics:

```
In [10]: max_key_length = 16
    With that value, the number of possible keys is huge:
In [11]: sum(len(characters) ** i for i in range(1, 17))
Out[11]: 62065212901958868055012327674640
```

If the message is long enough, it is likely to contain many n-grams (i.e., n consecutive characters) that occur many times, and less likely but still likely enough, n-grams with many vertically aligned occurrences. For instance, "the", "ing", "able" could be n-grams (with n equal to 3, 3 and 4, respectively) some of whose occurrences in a message could be aligned:

```
....the...
.able....
.ing
.....ing
.....ing
.....the...
....the...
.able....
....the...
```

All vertically aligned occurrences of a given n-gram are identically encrypted. Hence one can look for n-grams in the encrypted message that occur many times, and assume that they have a significant chance to correspond to n-grams in the original message that happen to be vertically aligned. Suppose for instance that %)O occurs many times in the encrypted message. Suppose that the second occurrence of %)O is 40 positions to the right of the first occurrence of %)O. Conjecturing that both occurrences of %)O are aligned, and assuming that the key is at most 16 characters long, there are 4 options (writing  $\sim$  for a slot for a character):

• The key is 4 characters long and the second occurrence of %)O is 10 lines below the first occurrence of %)O, e.g.:

~ % ) O

~ ~ ~ ~

~ ~ ~ ~

~ ~ ~ ~

~ ~ ~ ~

~ ~ ~ ~

~ ~ ~ ~

~ ~ ~ ~

~ % ) O

• The key is 5 characters long and the second occurrence of %)O is 8 lines below the first occurrence of %)O, e.g.:

~ % ) O ~

~ ~ ~ ~ ~

~ ~ ~ ~ ~

~ ~ ~ ~ ~ ~

~ ~ ~ ~ ~

~ ~ ~ ~ ~

~ ~ ~ ~ ~

~ % ) O ~

• The key is 8 characters long and the second occurrence of %)O is 5 lines below the first occurrence of %)O, e.g.:

~ ~ % ) O ~ ~ ~

~~%)O~~~

• The key is 10 characters long and the second occurrence of %)O is 4 lines below the first occurrence of %)O, e.g.:

~ ~ ~ % ) O ~ ~ ~ ~

~ ~ ~ ~ ~ ~ ~ ~ ~ ~

~ ~ ~ % ) O ~ ~ ~ ~

In other words, the conjecture that two successive occurrences of an n-gram are aligned leads to the conclusion that the length of the key is a divisor at least equal to n and at most equal to the value of  $\max_{k \in \mathcal{N}} \text{length}$  of the distance between (the start of) both occurrences. The following function returns a generator expression for the relevant divisors of its first argument, namely, those of its factors that are at least equal to its second argument and at most equal to the value of  $\max_{k \in \mathcal{N}} \text{length}$ :

factors\_from\_successive\_n\_grams() returns a generator expression; each factor will be retrieved on demand, e.g., by calls to next(), or thanks to a for statement. We would like to generate on demand not just the relevant factors of the distance between two successive occurrences of a given n-gram, but the relevant factors of the distances of all pairs of successive occurrences of all n-grams (for the chosen values of n). To this aim, we introduce another mechanism, namely, generator functions. A generator function is a function with yield statements in its body. A yield statement can, but does not have to, be followed by an expression. In case an expression follows, the value of that expression is provided when the yield statement is executed; otherwise, None is provided. In any case, execution of the generator function then stops, waiting to be resumed, possibly, on demand:

```
In [13]: def f(n):
             print(f'Before yielding {n}')
             vield n
             print(f'Before yielding {n + 1}')
             vield n + 1
             print('Before yielding None')
             yield
             print(f'Before yielding {n + 2}')
             yield n + 2
             print(f'After yielding {n + 2}')
         F = f(10)
         F
         next(F)
         next(F)
         next(F)
         next(F)
         next(F)
Out[13]: <generator object f at 0x108745308>
Before yielding 10
Out[13]: 10
Before yielding 11
Out[13]: 11
```

```
Before yielding None
Before yielding 12
Out[13]: 12
After yielding 12
                                                   Traceback (most recent call last)
        StopIteration
        <ipython-input-13-e01b2fdaadee> in <module>()
         17 next(F)
         18 next(F)
    ---> 19 next(F)
        StopIteration:
   A generator function can also contain return statements:
In [14]: def f(n):
             print(f'Before yielding {n}')
             yield n
             print(f'Before yielding {n + 1}')
             yield n + 1
             print('Before returning')
             return
             # Won't be printed out
             print('After returning')
             # Won't be yielded
             yield n + 2
         F = f(10)
         F
         next(F)
         next(F)
         next(F)
         next(F)
         next(F)
Out[14]: <generator object f at 0x1086892b0>
Before yielding 10
```

```
Out[14]: 10
Before yielding 11
Out[14]: 11
Before returning
                                                     Traceback (most recent call last)
        StopIteration
        <ipython-input-14-83fd57077d03> in <module>()
         16 next(F)
         17 next(F)
    ---> 18 next(F)
         19 next(F)
         20 next(F)
        StopIteration:
   A generator function can yield values yielded by another generator function or any other iterable:
In [15]: def f():
             yield 1
             yield 2
         def g():
             yield 0
             for x in f():
                  yield x
             for x in range(3, 6):
                 yield x
             for x in (x for x in [6, 7, 8]):
                 yield x
             for x in [9, 10]:
                  yield x
         G = g()
```

In such a context, yield from statements offer a more concise alternative:

for x in G:

0 1 2 3 4 5 6 7 8 9 10

print(x, end = ' ')

```
In [16]: def f():
        yield 1
        yield 2

def g():
        yield 0
        yield from f()
        yield from range(3, 6)
        yield from (x for x in [6, 7, 8])
        yield from [9, 10]

list(g())

Out[16]: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Let us get back to the problem of discovering successive occurrences of n-grams. As part of our heuristics, let us set the sizes of n-grams to look for to the default values of 3, 4 and 5:

```
In [17]: n_gram_range = range(3, 6)
```

To look for substrings within a string, we can use either find() or index() str's methods. They are exactly the same, except that find() returns -1 whereas index() raises a ValueError exception when they fail to find their argument as a substring. Both methods take an optional second argument and a second optional third argument to limit the search within a narrower range than the full string to which the method is applied:

When we find a pair of occurrences of a 5-gram, we are guaranteed to find as well 2 pairs of occurrences of 4-grams, and 3 pairs of occurrences of 3-grams, but there does not seem to be a good reason to prevent the redundancy (in particular because between two consecutive occurrences of an n-gram that extends a substring s, there can be other occurrences of s in between. The following code fragment takes as example the first 91 characters of message (so the first 7 lines of message displayed over 13 columns as above) and collects in a list the relevant factors of the distances between all successive occurrences of all 3, 4 and 5-grams in this text. We trace execution and display, in particular, the contexts in which two successive occurrences of a 3, 4 or 5-gram have been detected:

```
print(f'Looking for consecutive occurrences of {n}-grams:')
             for i in range(len(text) -2 * n + 1):
                 n_{gram} = text[i: i + n]
                 j = text.find(n_gram, i + n)
                 if j != -1:
                     print(f' Found "{n\_gram}" at indexes {i} and {j}\n'
                           f' {text[i: j + n]}\n'
                           f' Distance: {j - i}. Adding as factors:'
                           f' {tuple(factors from successive n grams(j - i, n))}\n'
                     relevant_factors.extend(
                                        factors_from_successive_n_grams(j - i, n)
         print(relevant_factors)
Looking for consecutive occurrences of 3-grams:
  Found "ing" at indexes 16 and 45
  ing to get very tired of sitting
  Distance: 29. Adding as factors: ()
  Found "ng " at indexes 17 and 46
  ng to get very tired of sitting
 Distance: 29. Adding as factors: ()
  Found "d o" at indexes 36 and 78
  d of sitting by her sister on the bank, and o
  Distance: 42. Adding as factors: (3, 6, 7, 14)
  Found " of" at indexes 37 and 79
  of sitting by her sister on the bank, and of
  Distance: 42. Adding as factors: (3, 6, 7, 14)
  Found "of " at indexes 38 and 80
  of sitting by her sister on the bank, and of
  Distance: 42. Adding as factors: (3, 6, 7, 14)
  Found " si" at indexes 40 and 55
  sitting by her si
  Distance: 15. Adding as factors: (3, 5, 15)
  Found "ing" at indexes 45 and 86
  ing by her sister on the bank, and of having
  Distance: 41. Adding as factors: ()
  Found "ng " at indexes 46 and 87
  ng by her sister on the bank, and of having
  Distance: 41. Adding as factors: ()
```

```
Found "er " at indexes 53 and 60
  er sister
  Distance: 7. Adding as factors: (7,)
Looking for consecutive occurrences of 4-grams:
  Found "ing " at indexes 16 and 45
  ing to get very tired of sitting
  Distance: 29. Adding as factors: ()
  Found "d of" at indexes 36 and 78
  d of sitting by her sister on the bank, and of
  Distance: 42. Adding as factors: (6, 7, 14)
  Found " of " at indexes 37 and 79
  of sitting by her sister on the bank, and of
  Distance: 42. Adding as factors: (6, 7, 14)
  Found "ing " at indexes 45 and 86
  ing by her sister on the bank, and of having
  Distance: 41. Adding as factors: ()
Looking for consecutive occurrences of 5-grams:
  Found "d of " at indexes 36 and 78
  d of sitting by her sister on the bank, and of
  Distance: 42. Adding as factors: (6, 7, 14)
[3, 6, 7, 14, 3, 6, 7, 14, 3, 6, 7, 14, 3, 5, 15, 7, 6, 7, 14, 6, 7, 14, 6, 7, 14]
```

It turns out that in these first seven lines, we do not get a single factor of 13: no 3-, 4- or 5-gram has two aligned successive occurrences. The search for successive occurrences of n-grams is anyway meant to be done in the encrypted message, and results are different; we used the original message for a friendlier illustration of the technique.

The following generator function adapts the previous code to, for an arbitrary text passed as argument, not collect in a list the relevant factors of the distances between all successive occurrences of all n-grams in the text (for the chosen values of n), but have a mechanism to return them on demand:

```
...4, 5, 10, 13, 6, 13, 6, 13, 6, 13, 6, 13, 5, 10, 13, 6, 13, 6, 13, 6, 13]
```

Arguably, the more frequent a factor is, the more likely it is to be equal to the length of the key that has been used to encrypt the message. We therefore want to rank, from most frequent to least frequent, all factors that have been collected for all consecutive pairs of n-grams. The first step is to count each factor. Thanks to the Counter class from the collections module, it is straightforward to get from a collection a (type of object that embeds a) dictionary whose keys and values are the members of the collection and the number of times they occur in the collection, respectively:

```
In [21]: Counter([4, 5, 10, 13, 4, 5, 10, 13, 4, 5, 10, 13, 4, 8, 13])
Out[21]: Counter({4: 4, 5: 3, 10: 3, 13: 4, 8: 1})
```

Factors should be ranked from the most frequent ones to the least frequent ones, but what to do when two factors have equal counts? E.g., what if both 8 and 16 occur 6,811 times? (This is the case when encrypting the text in carroll.txt with the key '0123456789ABCDEF'.) It seems probable that if the key length was 8 rather than 16, then we would have more n-grams that reveal vertical alignments over 8 columns than n-grams that reveal vertical alignments over 16 columns (actually, not having a single extra factor of 8 strongly suggests that the key length is 16 indeed). So we decide to rank the divisors from most frequent ones to least frequent ones, and for a given count, from largest ones to smallest ones. So the key of the sorting method needs to take into account first the values of the dictionary embedded in the counter, and then the keys. We can retrieve from a dictionary (be it embedded in a counter) keys only, values only, or both keys and values, in the form of list-like objects:

The third way of sorting the items of a dictionary offers the solution to ranking properly our collected factors:

```
key = lambda x: (x[1], x[0]), reverse = True
)

Out[23]: [(13, 4), (10, 3), (8, 1), (5, 3), (4, 4)]

Out[23]: [13, 10, 8, 5, 4]

Out[23]: [(4, 4), (13, 4), (5, 3), (10, 3), (8, 1)]

Out[23]: [4, 13, 5, 10, 8]

Out[23]: [(13, 4), (4, 4), (10, 3), (5, 3), (8, 1)]

Out[23]: [13, 4, 10, 5, 8]
```

Rather than using a lambda expression, we can also use itemgetter() from the operator module:

```
In [24]: L = ['A', 'B', 'C']
    itemgetter(0)(L)
    itemgetter(0, 2)(L)
    itemgetter(2, 0, 1)(L)

D = {4: 4, 5: 3, 10: 3, 13: 4, 8: 1}
    sorted(D.items(), key = itemgetter(1, 0), reverse = True)
    [x[0] for x in sorted(D.items(), key = itemgetter(1, 0), reverse = True)]

Out[24]: 'A'

Out[24]: ('A', 'C')

Out[24]: ('C', 'A', 'B')

Out[24]: [(13, 4), (4, 4), (10, 3), (5, 3), (8, 1)]

Out[24]: [13, 4, 10, 5, 8]
```

Having with all\_collected\_factors() a mechanism to generate some key lengths from most promising to least promising, we still would like to consider all possible key lengths between 1 and the value of max\_key\_length, starting with 1 (in case the message has actually been encrypted with the simple Caesar code), then those given by all\_collected\_factors(), and finally all others, from shortest to longest. This is the purpose of the following generator function:

```
)
             yield from key_lengths
             yield from (i for i in range(2, max_key_length + 1)
                                if i not in key lengths
                         )
         list(key_lengths_from_most_to_least_promising(encrypted_message))
Out[25]: [1, 13, 6, 10, 5, 3, 4, 8, 2, 7, 9, 11, 12, 14, 15, 16]
   For all n < 13, the n-th character of key encrypts the n-th column of message, yielding the n-th
column of encrypted message, which can be decrypted back to the n-th column of message:
In [26]: for n in range(13):
             decrypt(key[n], encrypted_message[n: : 13])
Out[26]: 'Ai fh do epbeg ooneoAurn'
Out[26]: 'Inv et to eor, nrnd klt s'
Out[26]: 'inesrhohnsdo o s u,i c?'
Out[26]: "ciri efich kwb c ws'cpo'"
Out[26]: 'enyts neei aupoihe ein'
Out[26]: ' g tibhg nhstinna t,cv'
Out[26]: 'w tisaa ohte cv toh te'
Out[26]: "atintnvtraorritei fo'ur"
Out[26]: 'sorgekio d eturti uwrs'
Out[26]: ' e r,n t tsa rs,sagiea'
Out[26]: 'bgdb gdwphidhea
                              htst'
Out[26]: "ee yoa oieesiast'tbth i"
Out[26]: 'gto nnn:ce tnd iaho ooo'
   Compare with the result obtained when decrypting the n-th column of encrypted_message not with
the right character (the n-th character in key), but with an arbitrary character:
In [27]: for n in range(13):
             decrypt('m+2I|b~!AxZ=I', encrypted_message[n: : 13])
```

Out[27]: '(10q;F+hgAj@pN::z[W@tNNh'

```
Out[27]: '9WDYusb-\t*YP{ZWz]t~y%3*-'
Out[27]: "g'w4RqM+7f(Z)}(f4nD0,;\nF"
Out[27]: '4VD\\hhx&^\tLP23Ao#Qv]Z35l'
Out[27]: "]Jt<v=\tT+.PbY5Lj'k0\tK/["
Out[27]: '{Xd3GiD(/<:Q2pZD y4Pt|g'
Out[27]: 'DPU3"sHZhd Y<4%W<#G0Zma'
Out[27]: "?Pd,w9wZ\\*VF:{P9'01p3__"
Out[27]: 'K`$cYN!21k,SaL\t$p#q-avz'
Out[27]: '?wmYH+BG3*.T,?!ADI9u"\\['
Out[27]: 'h(D}veM/ok>" n&A;veN}ko'
Out[27]: 'q,to#xz~gf[<8m^ZW(y&]}j'
Out[27]: 'Uj[f|-`:CH\tbFR[;s\\."oOR'</pre>
```

The columns derived from the correct key contain more letters. But is that all? Is the distribution of letters in a given column good, in some sense? A column consisting of nothing but q's, z's and j's would not appear as natural.

"etaoinshrdlcumwfgypbvkjxqz" is all lowercase letters ordered in decreasing frequency of use in English: "e" is most common, then comes "t", then "a"...

A good distribution of letters in a given column should be relatively consistent with the ordering of letters given by the previous sequence. The notion of etaoin score will turn this idea into a precise measure.

Consider the first column, "Ai fh do epbeg ooneoAurn". Let us convert all letters to lowercase and get a count of the number of occurrences of the letters that occur in the resulting string:

Let us order the letters by decreasing frequency in column. When two letters  $\alpha$  and  $\beta$  share the same count, let us be "pessimistic" about column's quality and rank  $\alpha$  before  $\beta$  if  $\alpha$  occurs after  $\beta$  in "etaoinshrdlcumwfgypbvkjxqz". So "o" comes first with 4 occurrences, then comes "e" with 3 occurrences. Both "a" and "n" have 2 occurrences, but "a" is more frequent than "n" in English (they occur at index 2 and 5 in 'etaoinshrdlcumwfgypbvkjxqz', respectively), so we rank "n" before "a". All other letters occur only once in column, with "p" least frequent in English, so ranked next, and "i" most frequent in English, so ranked last:

The previous code can be improved by defining from 'etaoinshrdlcumwfgypbvkjxqz' a dictionary thanks to which the index in that string of a lowercase letter can be more effectively retrieved, similarly to the way the dictionary shifts has been defined from the string characters:

The etaoin score of ranked\_letters\_in\_column is actually a function of both ranked\_letters\_in\_column and a nonzero natural number l at most equal to the length of this string: for such a number l, it is equal to the number of elements common to the initial segments of 'oenabpgfudrhi' and 'etaoinshrdlcumwfgypbvkjxqz' of length l. Let us display for l at most equal to 10, l itself, then those initial segments of length l, then the letters both segments have in common, then the number of those letters, so the corresponding etaoin score:

```
In [31]: for i in range(1, 11):
             s_1 = ranked_letters_in_column[: i]
             s 2 = 'etaoinshrdlcumwfqypbvkjxqz'[: i]
             s = ''.join(set(s_1) & set(s_2))
             print(f'{i:2} {s_1:10} {s_2:10} {s:9} {len(s)}')
 1 o
                                    0
 2 oe
                                    1
              et
                          е
 3 oen
              eta
                                    1
 4 oena
              etao
                          oea
                                    3
 5 oenab
                                    3
              etaoi
                          oae
 6 oenabp
                                    4
              etaoin
                          noae
```

```
7 oenabpg etaoins noae 4
8 oenabpgf etaoinsh noae 4
9 oenabpgfu etaoinshr noae 4
10 oenabpgfud etaoinshrd nedoa 5
```

Converting strings to sets and taking their intersections is not the most efficient approach here. We can instead look whether each the l first letters in ranked\_letters\_in\_column is one of the first l characters in 'etaoinshrdlcumwfgypbvkjxqz'. To get the etaoin score of ranked\_letters\_in\_column for l=10, we need to see how many times the if statement below is executed:

```
In [32]: for i in range(10):
             s = ranked letters in column[i]
             print(f'{s}', end = ' ')
             if etaoin[s] < 10:</pre>
                  print('occurs', end = ' ')
             else:
                  print('does not occur', end = ' ')
             print(f"in {'etaoinshrdlcumwfgypbvkjxqz'[: 10]}")
o occurs in etaoinshrd
e occurs in etaoinshrd
n occurs in etaoinshrd
a occurs in etaoinshrd
b does not occur in etaoinshrd
p does not occur in etaoinshrd
g does not occur in etaoinshrd
f does not occur in etaoinshrd
u does not occur in etaoinshrd
d occurs in etaoinshrd
   This is easily done with the sum() function:
In [33]: sum(1 for i in range(10) if etaoin[ranked_letters_in_column[i]] < 10)</pre>
Out[33]: 5
   Putting it all together in a function:
In [34]: def etaoin_score(text, length):
             letter_counts = Counter(c.lower() for c in text if c.isalpha())
             ranked_letters = sorted(letter_counts,
                                      key = lambda x: (letter_counts[x], etaoin[x]),
                                       reverse = True
             return sum(1 for i in range(min(length, len(ranked_letters)))
                               if etaoin[ranked_letters[i]] < length</pre>
                        )
         etaoin_score('Ai fh do epbeg ooneoAurn', 10)
```

## Out[34]: 5

As part of the heuristic, we have to chose a value for the second argument length of etaoin\_score(), which clearly should be neither too small not too large:

```
In [35]: etaoin_length = 6
```

The 8th column of message displayed over 13 columns has be encrypted with "E", the 8th character of "Take it Easy!". Deciphering this column with "E" and "e" shows similar results, with at many positions, the same letter, but lowercase for one, and uppercase for the other. The etaoin score is the same for both:

If we assume that the original message is "usual" text, with mostly lowercase letters, then we should consider "E" to be more promising than "e". More generally, when deciphering a given column of the encrypted message with two letters, if the ataoin scores differ, then the letter with the highest score is arguably more likely to be correct; but if both ataoin scores are the same, then the letter that yields the highest proportion of lowercase letters over all letters is arguably more likely to be correct.

The following code fragment tentatively deciphers the 8th column of encryped\_message with each of the 97 members of characters (1-character subkeys). It computes the 97 etaoin scores as well as the 97 proportions of lowercase letters over all letters plus 1 (to prevent division by 0). It then ranks each of the 97 1-character subkeys in decreasing value of etaoin score, and for a given etaoin score, in decreasing proportion of lower letters. Eventually, it displays the top 10 results, together with the corresponding tentative deciphering of the 8th column:

```
nb_of_lowercase_letters / nb_of_letters
        scores.sort(key = itemgetter(1, 2), reverse = True)
        for scored subkey in scores[: 10]:
            print(scored subkey, decrypt(scored subkey[0],
                                       encrypted_message[8: : 13]
                 )
('E', 3, 0.95) sorgekio d eturti uwrs
('I', 3, 0.9473684210526315) okncagek{9{{apqnpe{qsno}}
('K', 3, 0.9411764705882353) mila8eci_7__8nolnc_oqlm
('S', 3, 0.9230769230769231) ead2064a=
==0fgdf4=gide
('J', 2, 0.9411764705882353) njmb9fdj`8``9opmod`prmn
('0', 2, 0.9285714285714286) ieh64a8e[3[[4jkhj8[kmhi
('W', 2, 0.875) a69
                         ~206/}//~bc9b0/ce9a
('r', 2, 0.45833333333333333) FBEtrxvBagaarGHEGvaHJEF
('n', 2, 0.416666666666667) JFIxvBzFeueevKLIKzeLNIJ
```

Given a tentative key length l, hoped to be the actual length of the hidden key, there are l 1-character subkeys to discover, one for each column of the message displayed over l many columns. One would like to consider, for each column, nothing but its most promising 1-character subkeys. Restricting ourselves to, for each column, its k most promising subkeys, a total of  $k^l$  full keys can then be assembled from the l 1-character subkeys. So k has to be small. Let us still make it another parameter of the overall heuristics and set it to a default of 2:

```
In [38]: nb_of_options_for_subkey = 2
```

To easily generate all possible keys from all choices of 1-character subkeys, the product class from the itertools module is useful:

```
In [39]: # The cartesian product {0, 1} x {'A', 'B'} x {'c', 'd'}
    list(product(range(2), ['A', 'B'], (x for x in 'cd')))
    # The cartesian product ({0, 1} x {'A', 'B'})^2
    list(product(range(2), ['A', 'B'], repeat = 2))

Out[39]: [(0, 'A', 'c'),
        (0, 'A', 'd'),
        (0, 'B', 'c'),
        (0, 'B', 'c'),
        (1, 'A', 'c'),
        (1, 'A', 'd'),
        (1, 'B', 'c'),
        (1, 'B', 'c'),
        (1, 'B', 'c'),
        (1, 'B', 'd')]
```

```
Out[39]: [(0, 'A', 0, 'A'),
          (0, 'A', 0, 'B'),
          (0, 'A', 1, 'A'),
          (0, 'A', 1, 'B'),
          (0, 'B', 0, 'A'),
          (0, 'B', 0, 'B'),
          (0, 'B', 1, 'A'),
          (0, 'B', 1, 'B'),
          (1, 'A', 0, 'A'),
          (1, 'A', 0, 'B'),
          (1, 'A', 1, 'A'),
          (1, 'A', 1, 'B'),
          (1, 'B', 0, 'A'),
          (1, 'B', 0, 'B'),
          (1, 'B', 1, 'A'),
          (1, 'B', 1, 'B')]
```

The following code fragment passes as an argument to list() a generator expression to generate all keys of length 3 from a triple of 2 possible options for each of three 1-character subkeys, with '0' or '1' for the first subkey, 'A' or 'B' for the second subkey, 'c' or 'd' for the third subkey:

Having assembled a complete key from 1-character subkeys, the encrypted message can be tentatively decrypted as a whole. Whereas 1-character subkeys look more or less promising depending on the distribution of letters in a tentatively decrypted column, whole keys look more or less promising depending on whether the tentatively decrypted message contains enough letters amongst all characters, and enough English words amongst all words, here defining a word as a longest sequence of consecutive letters. So any longest sequence of consecutive nonletters is a word separator; that is something not for the split() method of the str class, but for the split() function of the re module, using the syntax of regular expressions:

```
# or digits as a separator
         split('[^a-zA-Z0-9]', '0+$abc(^?DEF=_')
Out[41]: ['0', '', 'abc', '', '', 'DEF', '', '']
Out[41]: ['0', 'abc', 'DEF', '']
Out[41]: ['0+$', '', '', '(^?DEF=_']
Out[41]: ['', '', '', 'abc', '', '', '', '', '', '', '']
Out[41]: ['0+$', '(^?', '= ']
Out[41]: ['0', '', 'abc', '', '', 'DEF', '', '']
   So '[^a-zA-Z]+' is the regular expression we need:
In [42]: print(split('[^a-zA-Z]+', message))
['Alice', 'was', 'beginning', 'to', 'get', 'very', 'tired', 'of', 'sitting',...
       ...'by', 'her', 'sister', 'on', 'the', 'bank', 'and', 'of', 'having',...
              ...'nothing', 'to', 'do', 'once', 'or', 'twice', 'she', 'had',...
      ...'peeped', 'into', 'the', 'book', 'her', 'sister', 'was', 'reading',...
       ...'but', 'it', 'had', 'no', 'pictures', 'or', 'conversations', 'in',...
              ...'it', 'and', 'what', 'is', 'the', 'use', 'of', 'a', 'book',...
        ...'thought', 'Alice', 'without', 'pictures', 'or', 'conversations', '']
```

To complete the heuristics, we make use of two last parameters. First a desired fraction of letters over all letters in the tentatively deciphered message, set to a default of 70%. Second a desired fraction of English words over all words in the tentatively deciphered message, set to a default of 50%:

With dictionary denoting a list of all lowercase English words (first read from a file in practice), the following function returns True or False depending on whether the tentatively decrypted message passed as first argument looks right or not. In case the function returns True, the tentatively decrypted message will be displayed to the user to accept, or to reject, in which case the program will resume its search for the correct key:

We are ready to put everything together.

- A function encrypt\_file() is designed to encrypt the text contained in a file, whose name is provided as second argument, the first argument being the encryption key. By default, the function displays the encrypted message to the screen, but the third argument, set by default to None, can be changed to the name of a file and the encrypted message will then be written to this file instead.
  - To read from a file, the second argument should be the name of an existing file. Otherwise, open() will raise a FileNotFoundError exception, which the codes catches in an except statement.
  - To write to a new file or overwrite the contents of an existing file, the open() function is given 'w' as second argument.
  - encrypt\_file() just calls the encrypt() function to perform the encryption. That function expects the message to encrypt to be given as a single string, with as many embedded '\n' characters in the string as there are lines in the message. To read the whole contents of a file as a single string, we use the read() method of the object returned by the open() function.
- A function decrypt\_file() is designed to decrypt the text contained in a file, whose name is provided as second argument, the first argument being the encryption key. By default, the function displays the decrypted message to the screen, but the third argument, set by default to None, can be changed to the name of a file and the decrypted message will then be written to this file instead, provided that it does not exist.
  - To write to a new file, the open() function is given 'x' as second argument; in case the file exists, then open() will not overwrite it but instead, raise a FileExistsError exception.
  - decrypt\_file() just calls the decrypt() function to perform the decryption, also using read() to get the message to decrypt as a single string.
- A function break\_key\_for\_file() is designed to try and break the key of an encrypted message stored in a file whose name is passed as argument to break\_key\_for\_file(). Here again, read() is used to get the text to decrypt (from a key that first, has to be discovered) as a single string, that is passed an an argument to another function, break\_key().
  - break\_key() opens a file meant to contain a list of English words, one per line. The strip() method is used to discard the new line character that ends each line of this file, the lower() method to convert each word to all lowercase.
  - break\_key() quickly displays all keys it comes up with to tentatively decrypt the encrypted message; carriage return is used to quickly display all keys of a given length on a single line, each such key overwriting the previous one.
  - When looks\_like\_English() returns True, break\_key() displays the first 200 characters of the tentatively deciphered message, and prompts the user to express whether the message has been successfully decrypted, or whether search for another key should be resumed. In case it exhausts all keys it comes up with, none of which is either proposed to or validated by the user, break\_key() admits defeat.

```
else:
                return encrypt(key, file.read())
    except FileNotFoundError:
        print(f'Could not open {filename}, giving up.')
def decrypt file(key, filename, decrypted filename = None):
        with open(filename) as file:
            if decrypted_filename:
                try:
                    with open(decrypted_filename, 'x') as decrypted_file:
                        print(decrypt(key, file.read()),
                              file = decrypted_file
                             )
                except FileExistsError:
                    print(f'{decrypted_filename} exists, giving up.')
            else:
                return decrypt(key, file.read())
    except FileNotFoundError:
        print(f'Could not open {filename}, giving up.')
def break key for file(filename):
   try:
        with open(filename) as file:
            break key(file read())
    except FileNotFoundError:
        print(f'Could not open {filename}, giving up.')
def break_key(text):
   try:
        with open(dictionary_file) as file:
            dictionary = {w.strip().lower() for w in file}
    except FileNotFoundError:
        print(f'Could not open the file {dictionary_file}, giving up.')
    for key_length in key_lengths_from_most_to_least_promising(text):
        print(f'\nNow working with keys of length {key length}')
        subkeys = []
        for n in range(key_length):
            scores = []
            for subkey in characters:
                decrypted_column = decrypt(subkey, text[n: : key_length])
                nb_of_lowercase_letters = 0
                nb of letters = 1
                letters = (c for c in decrypted_column if c.isalpha())
                for c in letters:
                    nb_of_letters += 1
                    if c.islower():
```

```
nb_of_lowercase_letters += 1
                          scores.append((subkey,
                                         etaoin_score(decrypted_column, etaoin_length),
                                         nb_of_lowercase_letters / nb_of_letters
                                        )
                                       )
                      scores.sort(key = itemgetter(1, 2), reverse = True)
                      subkeys.append(x[0] for x in scores[: nb_of_options_for_subkey])
                  for key in (''.join(subkey) for subkey in product(*subkeys)):
                      print('\r', key, end = '')
                      decrypted_text = decrypt(key, text)
                      if looks_like_English(decrypted_text, dictionary):
                          print('\nWhat about this?\n')
                          print(decrypted_text[: 200], '...', sep = '')
                          print('Enter Y[es] if happy, otherwise press any key '
                                "and I'll keep working.")
                          yes_or_no = input('> ')
                          if yes_or_no in {'YES', 'Yes', 'yes', 'Y', 'y'}:
                              print(f'The key is: "{key}"')
                              return
              print('Sorry, I did my best...')
In [46]: encrypt_file('Take it Easy!', 'carroll.txt', 'encrypted_carroll.txt')
         break key for file('encrypted carroll.txt')
Now working with keys of length 1
Now working with keys of length 13
Take it Easy!
What about this?
Alice was beginning to get very tired of sitting by her sister on the bank, and of...
    ...having nothing to do: once or twice she had peeped into the book her sister...
                                      ...was reading, but it had no pictures or co...
Enter Y[es] if happy, otherwise press any key and I'll keep working.
> Yes
The key is: "Take it Easy!"
```