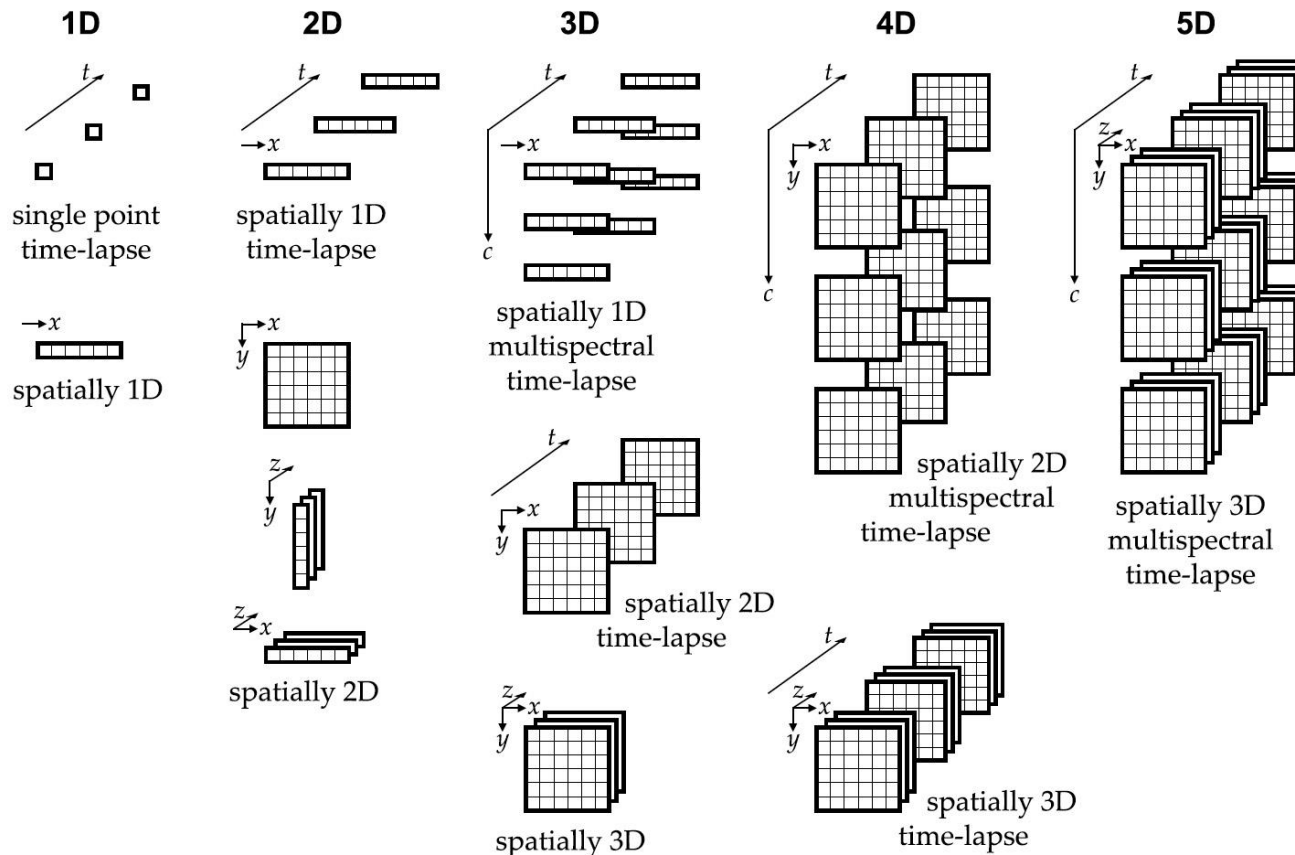


# COMP9517: Computer Vision

## Motion

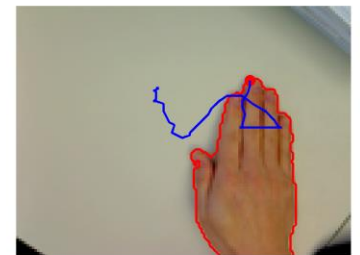
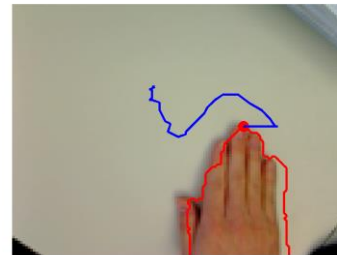
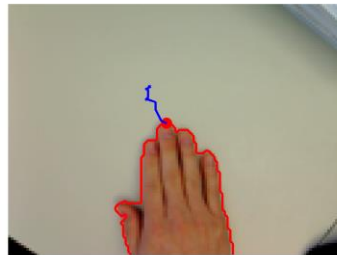
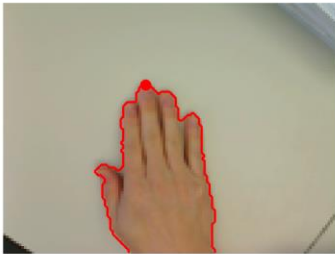
# Introduction

- Adding the time dimension to the image formation



# Introduction

- A changing scene may be observed via a sequence of images



# Introduction

- Changes in an image sequence provide features for
  - detecting objects that are moving
  - computing trajectories of moving objects
  - performing motion analysis of moving objects
  - recognising objects based on their behaviours
  - computing the motion of the viewer in the world
  - detecting and recognising activities in a scene

# Applications

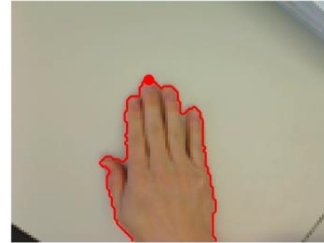
- **Motion-based recognition**
  - human identification based on gait, automatic object detection
- **Automated surveillance**
  - monitoring a scene to detect suspicious activities or unlikely events
- **Video indexing**
  - automatic annotation and retrieval of videos in multimedia databases
- **Human-computer interaction**
  - gesture recognition, eye gaze tracking for data input to computers
- **Traffic monitoring**
  - real-time gathering of traffic statistics to direct traffic flow
- **Vehicle navigation**
  - video-based path planning and obstacle avoidance capabilities

# Scenarios

- Still camera

Constant background with

- single moving object
- multiple moving objects



- Moving camera

Relatively constant scene with

- coherent scene motion
- single moving object
- multiple moving objects



# Topics

- **Change detection**

Using *image subtraction* to detect changes in scenes

- **Sparse motion estimation**

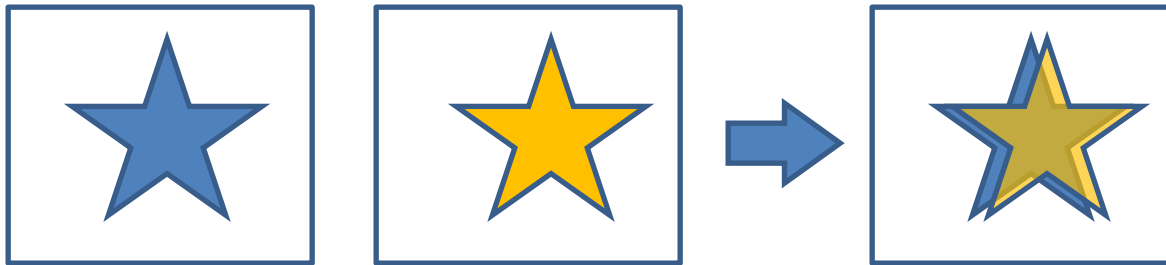
Using *template matching* to estimate local displacements

- **Dense motion estimation**

Using *optical flow* to compute a dense motion vector field

# Change Detection

- Detecting an object moving across a constant background
- The forward and rear edges of the object advance only a few pixels per frame



- By subtracting the image  $I_t$  from the previous image  $I_{t-1}$  the edges should be evident as the only pixels significantly different from zero



# Image Subtraction

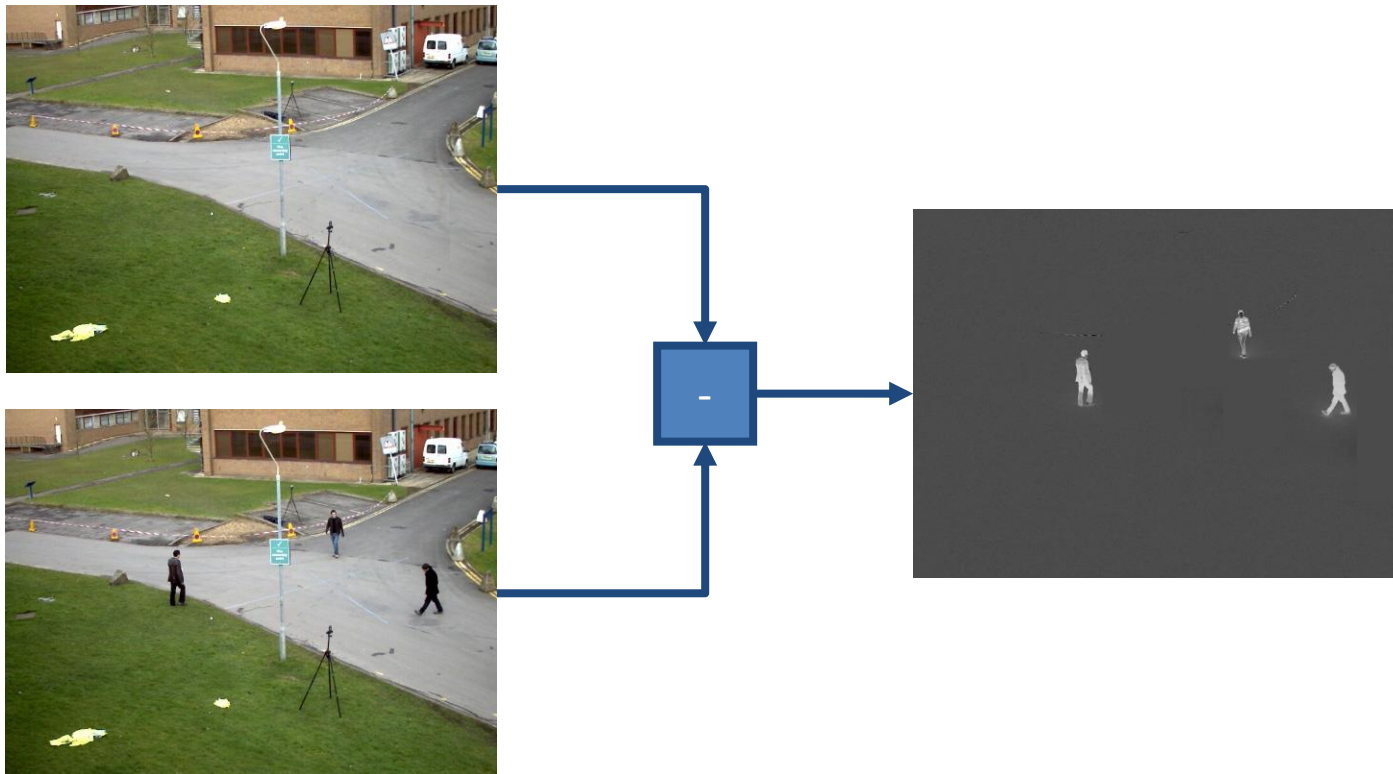
Step: Derive a background image from a set of video frames at the beginning of the video sequence



PETS 2009  
Benchmark

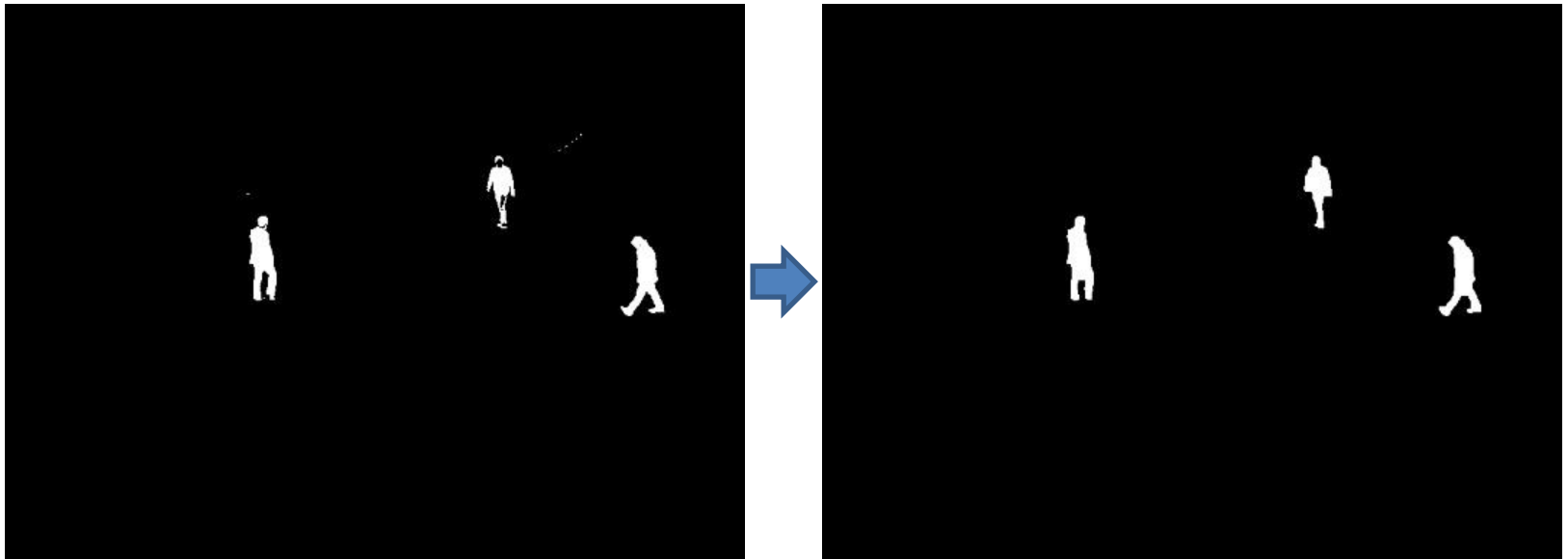
# Image Subtraction

Step: Subtract the background image from each subsequent frame to create a difference image



# Image Subtraction

Step: Threshold and enhance the difference image to fuse neighbouring regions and remove noise

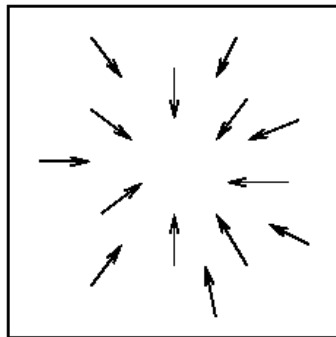


# Change Detection

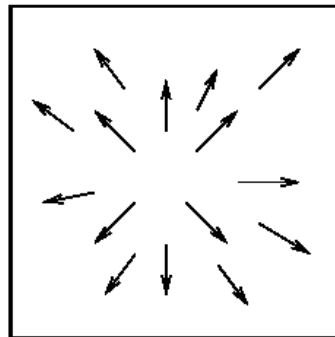
- Image subtraction algorithm
  - Input: images  $I_t$  and  $I_{t-\Delta t}$  (or a model image)
  - Input: an intensity threshold  $\tau$
  - Output: a binary image  $I_{out}$
  - Output: a set of bounding boxes  $B$
- 1. For all pixels  $[r, c]$  in the input images,  
set  $I_{out}[r, c] = 1$  if  $(|I_t[r, c] - I_{t-\Delta t}[r, c]| > \tau)$   
set  $I_{out}[r, c] = 0$  otherwise
- 2. Perform connected components extraction on  $I_{out}$
- 3. Remove small regions in  $I_{out}$  assuming they are noise
- 4. Perform a closing of  $I_{out}$  using a small disk to fuse neighbouring regions
- 5. Compute the bounding boxes of all remaining regions of changed pixels
- 6. Return  $I_{out}[r, c]$  and the bounding boxes  $B$  of regions of changed pixels

# Motion Vector

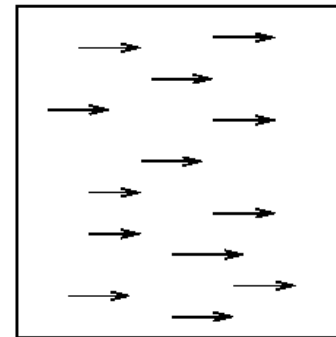
- A motion field is a 2D array of 2D vectors representing the motion of 3D scene points
- A motion vector in the image represents the displacement of the image of a moving 3D point
  - Tail at time  $t$  and head at time  $t+\Delta t$
  - Instantaneous velocity estimate at time  $t$



Zoom out



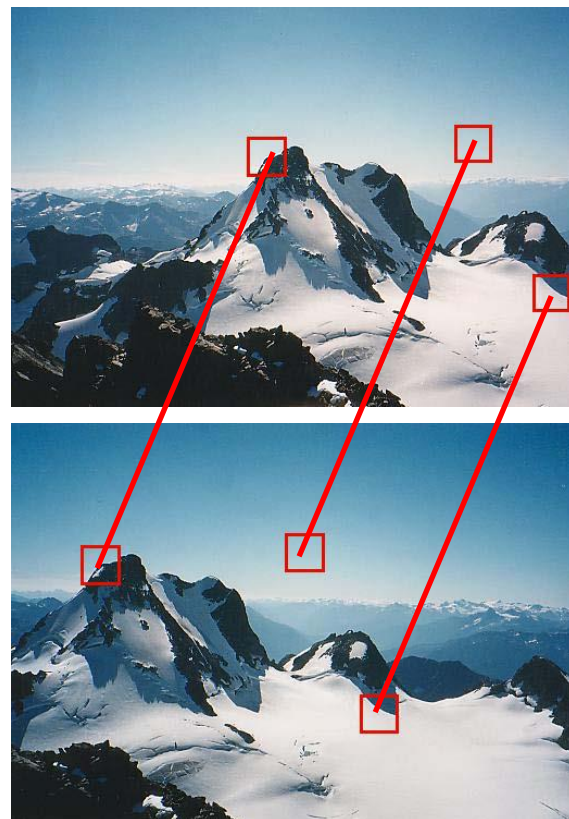
Zoom in



Pan Left

# Sparse Motion Estimation

- A sparse motion field can be computed by identifying pairs of points that correspond in two images taken at times  $t$  and  $t+\Delta t$
- Assumption: intensities of interesting points and their neighbours remain nearly constant over time
- Two steps:
  - Detect interesting points at  $t$
  - Search for corresponding points at  $t+\Delta t$



# Sparse Motion Estimation

- Detect interesting points
  - Image filters
    - Canny edge detector
    - Kirsch edge operator
    - Harris corner detector
    - SUSAN corner detector
    - Frei-Chen ripple operator
    - ...
  - Alternatively, compute Interest operator
    - Computes intensity variance in the vertical, horizontal and diagonal directions
    - Interest point if the minimum of these four variances exceeds a threshold

# Detect Interesting Points

```
Procedure detect_interesting_points(I,V,w,t) {
```

```
    for (r = 0 to MaxRow - 1)
```

```
        for (c = 0 to MaxCol - 1)
```

```
            if (I[r,c] is a border pixel) break;
```

```
            else if (interest_operator(I,r,c,w) >= t)
```

```
                add (r,c) to set V;
```

```
}
```

```
Procedure interest_operator (I,r,c,w) {
```

```
    v1 = variance of intensity of horizontal pixels I[r,c-w]...I[r,c+w];
```

```
    v2 = variance of intensity of vertical pixels I[r-w,c]...I[r+w,c];
```

```
    v3 = variance of intensity of diagonal pixels I[r-w,c-w]...I[r+w,c+w];
```

```
    v4 = variance of intensity of diagonal pixels I[r-w,c+w]...I[r+w,c-w];
```

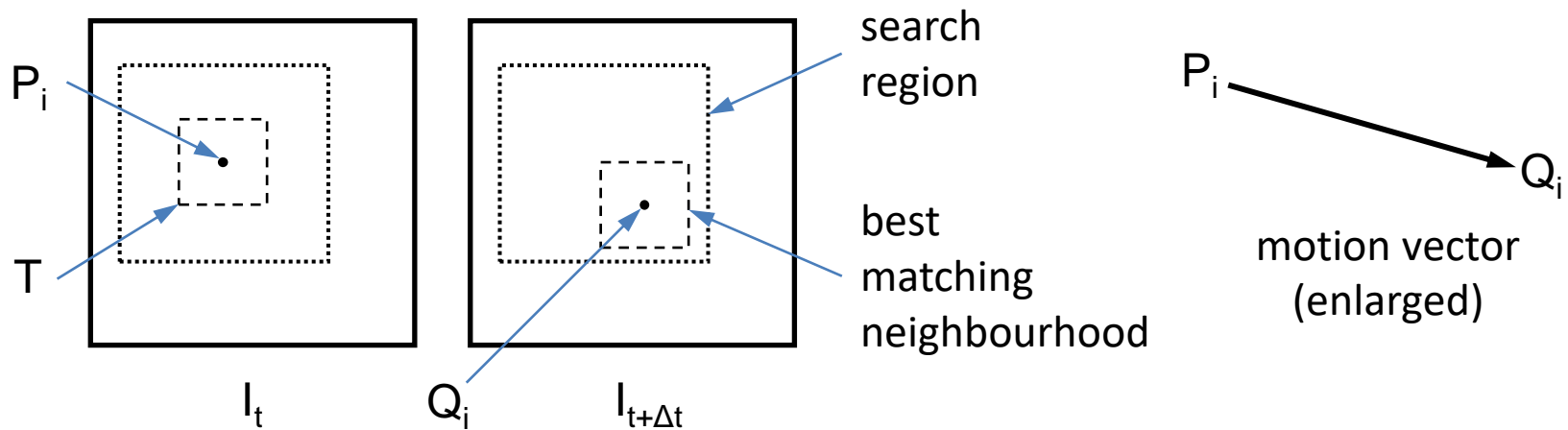
```
    return min(v1, v2, v3, v4);
```

```
}
```



# Sparse Motion Estimation

- Search for corresponding points
  - Given an interesting point  $P_i$  from  $I_t$ , take its neighbourhood in  $I_t$  and find the best matching neighbourhood in  $I_{t+\Delta t}$  under the assumption that the amount of movement is limited



This approach is also known as template matching

# Similarity Measures

- Cross-correlation (to be maximised)

$$\text{CC}(\Delta x, \Delta y) = \sum_{(x,y) \in T} I_t(x, y) \cdot I_{t+\Delta t}(x + \Delta x, y + \Delta y)$$

- Sum of absolute differences (to be minimised)

$$\text{SAD}(\Delta x, \Delta y) = \sum_{(x,y) \in T} |I_t(x, y) - I_{t+\Delta t}(x + \Delta x, y + \Delta y)|$$

- Sum of squared differences (to be minimised)

$$\text{SSD}(\Delta x, \Delta y) = \sum_{(x,y) \in T} [I_t(x, y) - I_{t+\Delta t}(x + \Delta x, y + \Delta y)]^2$$

# Similarity Measures

- Mutual information (to be maximised)

$$MI(A, B) = \sum_a \sum_b P_{AB}(a, b) \log_2 \left( \frac{P_{AB}(a, b)}{P_A(a)P_B(b)} \right)$$

Subimages to compare:

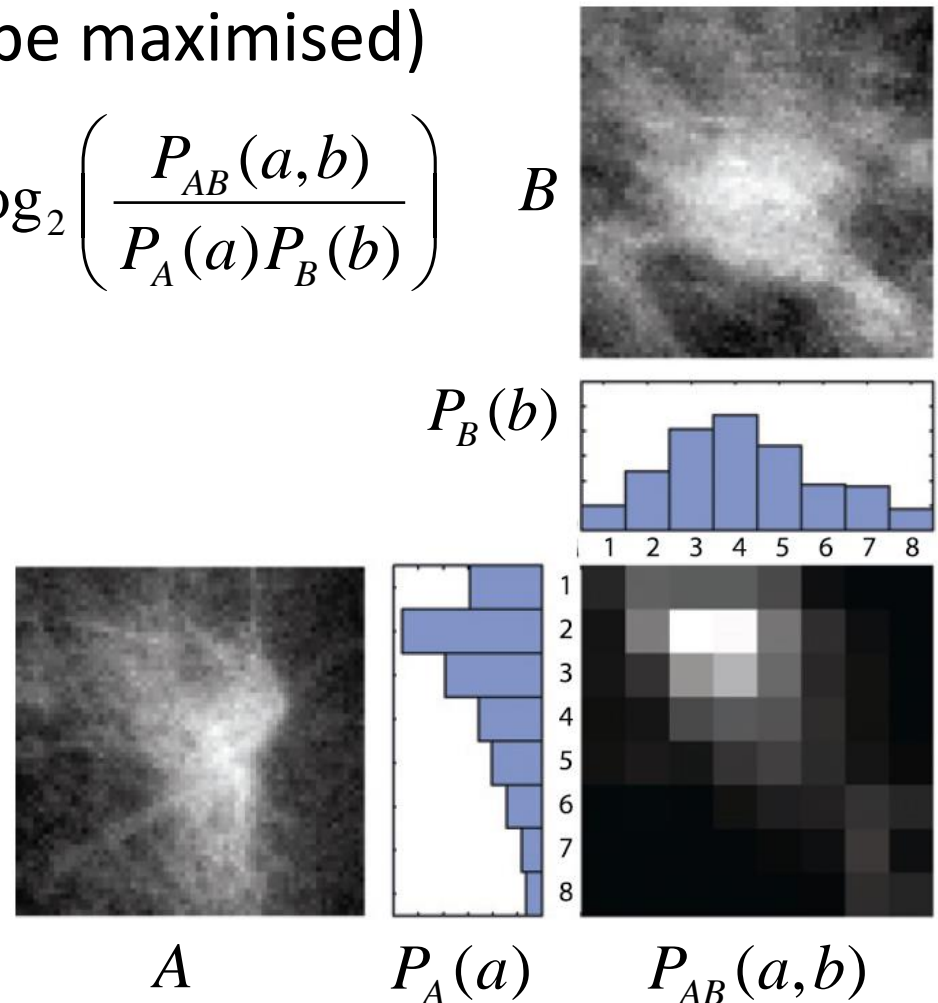
$$A \subset I_t \quad B \subset I_{t+\Delta t}$$

Intensity probabilities:

$$P_A(a) \quad P_B(b)$$

Joint intensity probability:

$$P_{AB}(a, b)$$



# Dense Motion Estimation

- Used to estimate image flow at all points of an image, not just at interesting points
- Assumptions:
  - The object reflectivity and illumination do not change during the considered time interval
  - The distance of the object from the camera and the light sources do not vary significantly over this interval
  - Each small neighbourhood  $N_t(x,y)$  at time  $t$  is observed in some shifted position  $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$  at time  $t+\Delta t$
- These assumptions may not hold tight in reality, but provide useful computation and approximation

# Spatiotemporal Gradient

- Taylor series expansion of function

$$f(x + \Delta x) = f(x) + \frac{\partial f}{\partial x} \Delta x + \text{h.o.t} \Rightarrow$$

$$f(x + \Delta x) \approx f(x) + \frac{\partial f}{\partial x} \Delta x$$

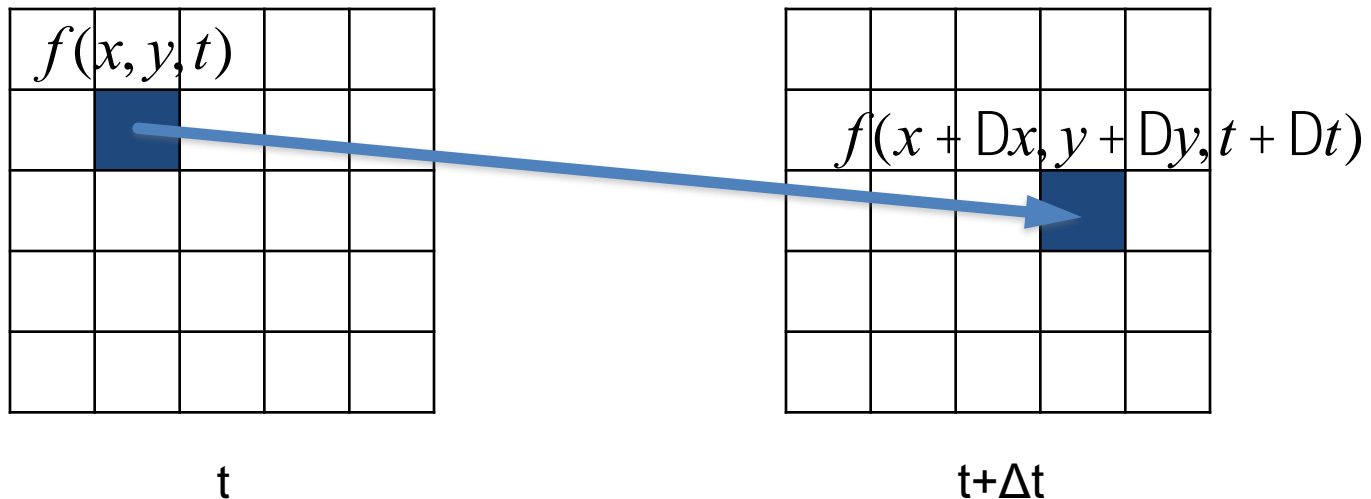
- Multivariable Taylor series approximation

$$f(x + \Delta x, y + \Delta y, t + \Delta t) \approx f(x, y, t) + \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial t} \Delta t \quad (1)$$

# Optical Flow Equation

Assuming intensity neighbourhood  $N_t(x, y)$  at time  $t$  moves over vector  $V=(\Delta x, \Delta y)$  to an identical intensity neighbourhood  $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$  at time  $t+\Delta t$  leads to the optical flow equation:

$$f(x + \Delta x, y + \Delta y, t + \Delta t) = f(x, y, t) \quad (2)$$



# Optical Flow Computation

Combining (1) and (2) yields the following constraint:

$$\frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial t} \Delta t = 0 \Rightarrow$$

$$\frac{\partial f}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y} \frac{\Delta y}{\Delta t} + \frac{\partial f}{\partial t} \frac{\Delta t}{\Delta t} = 0 \Rightarrow$$

$$\frac{\partial f}{\partial x} v_x + \frac{\partial f}{\partial y} v_y + \frac{\partial f}{\partial t} = 0 \Rightarrow$$

$$\nabla f \cdot v = -f_t$$

where  $v = (v_x, v_y)$  is the velocity or *optical flow* of  $f(x, y, t)$   
and  $\nabla f = (f_x, f_y) = (\partial f / \partial x, \partial f / \partial y)$  is the gradient

# Optical Flow Computation

- The optical flow equation provides a constraint that can be applied at every pixel position
- However, the equation does not have unique solution- there may be many neighbourhoods in second image that match neighbourhood in first image
- further constraints are required

For example, by using the optical flow equation for a group of adjacent pixels and assuming that all of them have the same velocity, the optical flow computation task amounts to solving a linear system of equations using the least-squares method

Many other solutions have been proposed (see references)



# Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

Assume the optical flow equation holds for all pixels  $p_i$  in a certain neighbourhood and use the following notation:

$$\mathbf{v} = (v_x, v_y) \quad f_x = \frac{\partial f}{\partial x} \quad f_y = \frac{\partial f}{\partial y} \quad f_t = \frac{\partial f}{\partial t}$$

Then we have the following set of equations:

$$f_x(p_1)v_x + f_y(p_1)v_y = -f_t(p_1)$$

$$f_x(p_2)v_x + f_y(p_2)v_y = -f_t(p_2)$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$f_x(p_N)v_x + f_y(p_N)v_y = -f_t(p_N)$$

# Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

The set of equations can be rewritten as  $Av = b$  where

$$A = \begin{bmatrix} f_x(p_1) & f_y(p_1) \\ f_x(p_2) & f_y(p_2) \\ \vdots & \vdots \\ f_x(p_N) & f_y(p_N) \end{bmatrix} \quad v = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \quad b = \begin{bmatrix} -f_t(p_1) \\ -f_t(p_2) \\ \vdots \\ -f_t(p_N) \end{bmatrix}$$

This can be solved using the least-squares approach:

$$A^T A v = A^T b \quad \Rightarrow \quad v = (A^T A)^{-1} A^T b$$

# Optical Flow Example



<https://www.youtube.com/watch?v=GIUDAZLfYhY>

# References and Acknowledgements

- Chapter 8 of Szeliski 2010
- Chapter 9 of Shapiro and Stockman 2001
- Images drawn from the above references