

## Application of sets

This problem is a simple application of the set data structure. The first part is from the ICPC world finals 2019. The actual description of the problem can be found at <https://icpc.kattis.com/problems/azulejos>. A more formal description is given below.

Let  $S_1$  and  $S_2$  be two sequences of length  $n$ , the elements of which are ordered pairs of integers, not necessarily distinct. Let  $S_1 = (a_1, b_1), \dots, (a_n, b_n)$  and  $S_2 = (c_1, d_1), \dots, (c_n, d_n)$ . The problem is to find permutations  $p$  and  $q$  of  $\{1, \dots, n\}$ , if they exist, satisfying the following properties.

1.  $a_{p(i)} \leq a_{p(i+1)}$  and  $c_{q(i)} \leq c_{q(i+1)}$  for  $1 \leq i < n$ .
2.  $d_{q(i)} \leq b_{p(i)}$  for  $1 \leq i \leq n$ .

In other words, the elements in  $S_1$  and  $S_2$  must be ordered in non-decreasing order of their first coordinate, such that the  $i$ th element in the ordering of  $S_2$  has second coordinate less than or equal to the second coordinate of the  $i$ th element in the ordering of  $S_1$ . Such orderings may not always be possible.

The basic algorithm is as follows. Let  $a$  be the minimum value of the  $a_i$  and  $c$  the minimum of the  $c_i$ . Let  $A$  be the subset of pairs in  $S_1$  whose first coordinate is equal to  $a$ , and  $C$  the subset of pairs in  $S_2$  with first coordinate  $c$ . The elements in  $A$  must come first in the ordering of  $S_1$  and the elements in  $C$  must come first in the ordering of  $S_2$ . Suppose  $|A| \leq |C|$ . Then for every element of  $A$  the corresponding element in the ordering of  $S_2$  must be an element in  $C$ . Choose an element in  $C$  such that its second coordinate is the largest possible but does not exceed the second coordinate of the element in  $A$ . Delete these elements from  $A$  and  $C$ . If for some element in  $A$ , no such element in  $C$  is found, then an ordering is not possible. If  $A$  becomes empty, some elements may be left in  $C$ . Now consider the second smallest value of the first coordinate in  $S_1$ , let  $A$  be the set of those elements and repeat. If  $|C| \leq |A|$ , do the same for elements in  $C$ , except that the element of  $A$  chosen is the one with smallest second coordinate that is greater than or equal to the second coordinate of the element in  $C$ . The `lower_bound` and/or `upper_bound` methods of the set class can be used to find these elements.

The second part of the problem, which was not in the ICPC, is to determine whether the solution is unique, that is, there exists exactly one pair of permutations  $p, q$  that satisfies the given property.

**Input/Output:** The first line of input specifies  $n$ , which will be at most  $5 \times 10^5$ . The next 4 lines contain  $n$  numbers each separated by a space, with each number at most  $10^9$ . The first line gives the values of  $a_i$ , the next  $b_i$ ,  $c_i$ ,  $d_i$  in that order. If there exist permutations  $p, q$  satisfying the required properties, print  $p$  on the first line and  $q$  on the second. If there are many solutions, any one is okay. Note that indices are considered from 1 to  $n$ . If there are no such permutations, print “impossible”, without quotes. For the second part, if there exist such permutations, print “unique” if the answer is unique, otherwise print “not unique”. Nothing needs to be done if it is impossible. Time limit in the contest was 10sec but it should probably take less. The test cases from the contest,

for the first part, are put up on teams. I may use different ones, especially for the second part.

**Submission:** Submit a single file named RollNo\_9.cpp

**Note:** It may be necessary to compare ordered pairs of numbers based on the first coordinate, or the second coordinate at different times. (It may in fact be easier to consider triples, where the third coordinate is the index). By default, sets and functions like sort use the < operator for comparison. If a different comparison function is required, it can be specified when defining the set or calling the sort function. This is done using function objects, for which the function call operator () is defined. A sample declaration would be

```
struct compare{
    bool operator()(T const &t1, T const &t2) const
    {
        // comparison for elements of type T
    }
};

sort(v.begin(), v.end(), compare());

// v is a vector of type T, a dummy object of type compare is passed
// to the sort function. If f is an object of type compare, f(t1,t2)
// returns a boolean value, comparing t1 with t2.

set<T,compare> S;

// defines a set with elements of type T using compare
// as the comparison operation.

Sample Input 1 Sample Output 1
4                3 2 4 1
3 2 1 2          4 2 1 3
2 3 4 3          not unique
2 1 2 1
2 2 1 3

Sample Input 2 Sample Output 2
2                impossible
1 2
2 3
2 8
2 1
```