Optimization for Exciting Trajectory

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OPTIMIZATION PROBLEM:

$$q_d(t) \rightarrow q_d(\underline{p}, t) \rightarrow Y(q_d, \dot{q}_d, \ddot{q}_d)$$

$$\underline{p}^* = \arg\max_{\underline{p}} \left\{ \sigma_{min} \left\{ \underbrace{\int_0^T Y^T Y \, d\tau}_{G(\underline{p}, T)} \right\} \right\}$$

OPTION 1

$$\underline{p}^* = \arg\max_{\underline{p}} \left\{ \det G(\underline{p}, T) \right\}$$

ADVANTAGES: – simple

DISADVANTAGES: – no guarantees on maximizing σ_{min}

OPTION 2

$$\underline{p}^* = \arg\max_{\underline{p}} \left\{ \lambda_{min} \left\{ G(\underline{p}, T) \right\} \right\}$$

ADVANTAGES: – maximizes σ_{min}

DISADVANTAGES: – problems on convergence of optimization

OPTION 3

$$\underline{p}^* = \arg\max_{\underline{p}, z} \; z$$

$$\text{s.t.} \quad \det\left(G(\underline{p}, T) - z I \right) > 0$$

– ADVANTAGES: – attempts to maximize σ_i

- better optimization
- DISADVANTAGES : - still no guarantee that σ_{min} maximized

OPTION 4

$$\underline{p}^* = \arg\max_{\underline{p}} \underline{\sigma}^T W \underline{\sigma}$$

where $\underline{\sigma} = \left(\sigma_i \left(\underline{q}(\underline{p}, T) \right) \right)$

W = weight matrix

- ADVANTAGES : - with high weight on σ_{min} we hope to maximize it
- DISADVANTAGES : - ?

IMPORTANT OBSERVATIONS

1) There are bounds on $\underline{p}$ related to the limitations on q_d. This depends on the used robot.

2) In general, OPTIONS 1, 2 and 3 try to maximize λ_{min}, hence

this considers also making $\lambda_i > 0$.

This is actually not required!

3) Is it guaranteed that there exists a $q_d(t)$ that maximizes σ_{min} above a threshold?

Could it be that two columns of Y are always dependent for all $q_d(t)$?

4) Probably also the time window T can be added to the optimization variable $\underline{x} = [\underline{p}^T, T]^T$. But This makes the optimization much more complex.

5) We are optimizing offline.

Is there a way to optimize online modifying $q_d(t)$ in real-time?

6) Once an optimal $q_d(t)$ is found learning depends on control gains Λ and R.

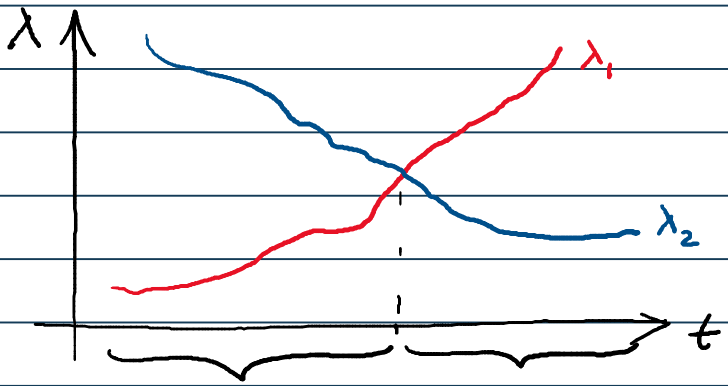
$$\hookrightarrow \quad \dot{q}_r = \dot{q}_d + \Lambda e; \quad e = q_d - q$$

$$\dot{\hat{\pi}} = -R^{-1} Y^T s; \quad s = \dot{q}_r - \dot{q}$$

$$\tau = M\ddot{q}_r + C\dot{q}_r + G + K_d s + e$$

PROBLEM WITH OPTION 2

Given $A(t)$, for simplicity assume $A(t) \in \mathbb{R}^2$, the following can happen:



$\lambda_{min} = \lambda_1$ $\qquad$ $\lambda_{min} = \lambda_2$

It can happen that during optimization

this type of exchange happens and the opt. algorith doesn't know in which direction to proceed.

FINAL OPTION

$$\underline{p}^* = \arg \min_{\underline{p}} \{ cond(G(\underline{p}, I)) \}$$

!

where $cond(A) = \frac{\sigma_{max}}{\sigma_{min}}$

If condition number of G goes to 1 then rank is full.