Computatioanl Project 1

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1. ODE

For my ordinary differential equation I simulated the trajectories of an orbit due to a classical central potential in 2-dimensions. In this case I chose to simulate circular orbits as a simple example case that is easily comparable to the analytical solution. We have a central force of strength,

$$\vec{F} = -\frac{m}{r^3}\vec{r} \tag{1}$$

with our velocities and positions found from the ordinary differential equation as,

$$\vec{v} = \frac{\dot{\vec{F}}}{m} \tag{2}$$

and,

$$\vec{r} = \frac{\ddot{\vec{F}}}{m}. \tag{3}$$

We have kept $m = 1$ for simplicity. We solve this using three distinct methods; Euler's method, Runge-Kutta 2nd order method, and Runge-Kutta 4th order method. Here we show trajectories with different timesteps for all three methods.

Here we give errors in the energies:

Finally, we compre with the Rk4 function from scipy.integrate. Scipy gives infalling orbits and greater errror compared to our RK4 function.

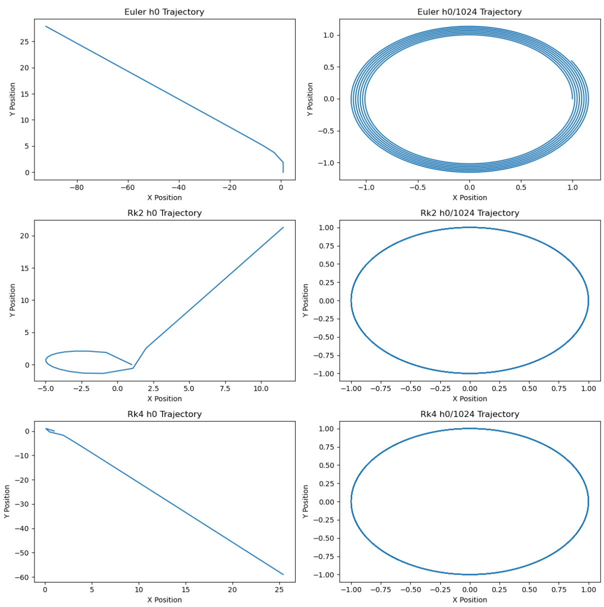

Fig. 1: trajcetories from euler, rk2, rk4

2. Definite Integral:

Here we perform a definite integral of a 3-dimensional time dependant force to get velocity and position over time. This can be applied to any time dependant force, here we give a function of sins and cos. We use a left handed riemann sum, right handed riemann

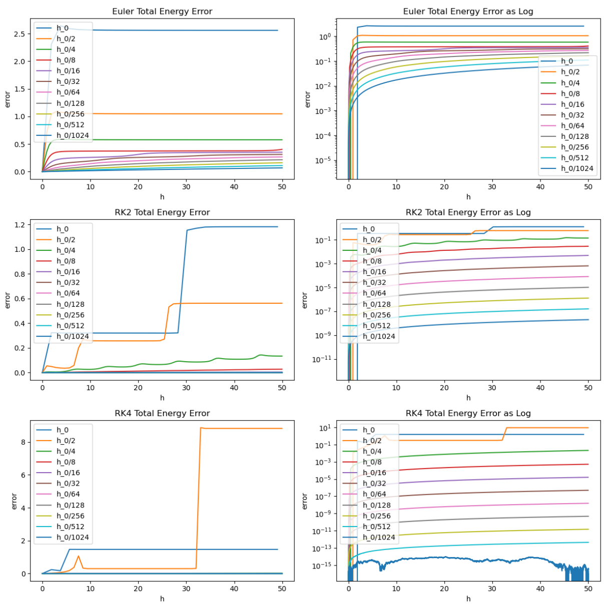

Fig. 2: Energy errors for euler, rk2, rk4, fro various timesteps

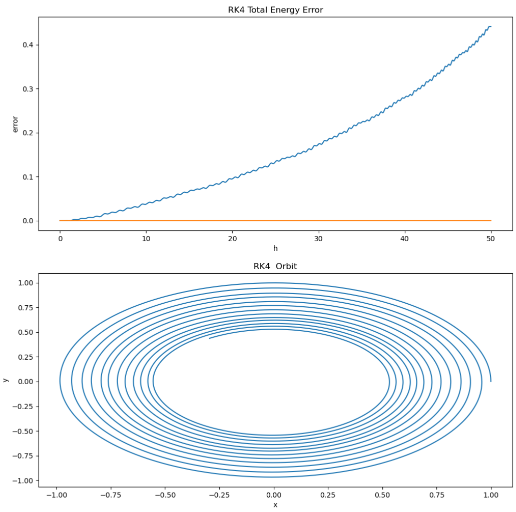

Fig. 3: Scipy solve ivp solution (blue) error and trajectory. Errors are compare to our Rk4 method (orange)

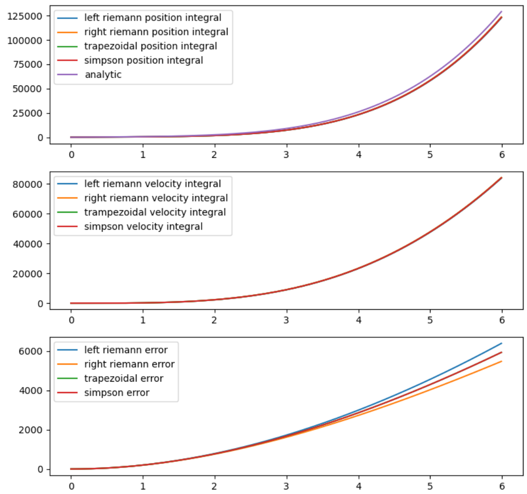

Fig. 4: displacement, velocity, and error propogation over time for our 4 methods

sum, trapezoidal sum, and simpsons rule sum as our 4 integration methods. Here we compare displacements, velocities, and error growth for each of our four methods.