Appendix A: Tables

Table 1 Cumulative Areas Under the Standard Normal Curve, $P(Z < z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Table 2 Critical Values of the Student t-Distribution*

df	α					
	0.10	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**
1	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088
2	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271
3	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145
4	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732
5	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076
7	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853
8	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247
12	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520
14	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874
15	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852
35	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400
40	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069
45	1.3006	1.6794	2.0141	2.4121	2.6896	3.2815
50	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614
60	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317
70	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108
80	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953
90	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833
100	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737
∞	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902

*Table entries are t-values under the specified degrees of freedom with the area a to its right (e.g., $t_{0.05,3} = 2.3534$ is the t-value with 3 degrees of freedom and an area of 0.05 to its right)

Appendix B: Key Formulas

Numerical Measures

Class Size	$c = \frac{R}{d}$
Class Mark	$CM = \frac{\text{lower limit + upper limit}}{2}$
Relative frequency	$RF = \frac{f}{n}$

Measures of Central Tendency

Population Mean	$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{x_1 + x_2 + \ldots + x_N}{N}$
Sample Mean	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$
Median position	$md\ position = \frac{n+1}{2}$
Range	R = largest value – smallest value
Population Variance	$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$
Sample Variance	$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$
Population Standard Deviation	$\sigma = \sqrt{\sigma^2}$
Sample Standard Deviation	$s = \sqrt{s^2}$

Measures of Variability

Coefficient of Variation	$CV = \frac{\text{standard deviation}}{\text{mean}} \cdot 100\%$
Position of the kth Percentile	$p = \left(\frac{k}{100}\right) n$
z-score	$\frac{x - \bar{x}}{s}$
Interquartile Range	$\text{IQR} = Q_3 - Q_1$
Inner fence	minimum value – 1.5(IQR) maximum value + 1.5(IQR)
Outer fence	minimum value – 3(IQR) maximum value + 3(IQR)

Measures of Skewness and Kurtosis

Coefficient of Skewness	$\text{SK} = \frac{3(\bar{x} - md)}{s}$
Measure of Kurtosis	$\text{Kurtosis} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{(n-1)s^4}$
Pearson Product-Moment Correlation Coefficient	$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]\left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}}$

Counting Methods

Fundamental Principle of Counting	Number of ways = $n_1 n_2 \ldots n_k$
Counting Technique for Alternative Cases	Number of ways = $m_1 + m_2 + m_3 \ldots m_r$
Factorial Notation	$n! = (n)(n-1)(n-2) \ldots . (3)(2)(1)$
Permutation of n Things Take r at a Time	${}_nP_r = \frac{n!}{(n-r)!}, \; r \leq n$
Permutations of n Things Taken n at a Time	${}_nP_n = n!$
Circular Permutations	Number of arrangements = $(n-1)!$
Distinguishable Permutations of n Things where Some Things are Alike	$\frac{n!}{n_1!\, n_2! \cdots n_k!}$
Partitioning Formula	${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}, \; r \leq n$ where $n = n_1 + n_2 + \ldots + n_k$
Combination of n Things Taken r at a Time:	${}_nC_r = , \; r \leq n$

Probability Rules

A Priori or Classical Probability Approach	$P(E) = \frac{n}{N}$
A Posteriori or Relative Frequency Approach	$P(E) = \frac{\text{number of times event } E \text{ occurred}}{\text{number of trials}}$
Addition Rule of Probability	$P(A \text{ or } B) = P(A \cup B)$ $= P(A) + P(B) - P(A \cap B)$
Addition Rule for Mutually Exclusive Events	$P(A \cup B) = P(A) + P(B)$
Probability of Complementary Events	$P(A^c) = 1 - P(A)$

Conditional Probability	$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$
Conditional Probability for Independent Events A and B	$P(B \mid A) = P(B)$ $P(A \mid B) = P(A)$
Multiplication Rule of Probability	$P(A \text{ and } B) = P(A \cap B) = P(A)P(B \mid A)$
Multiplication Rule for Independent Events	$P(A \cap B) = P(A)P(B)$
Theorem of Total Probability or Rule of Elimination	$P(A) = \sum_{i=1}^{k} P(B_i \cap A)$ $= \sum_{i=1}^{k} P(B_i)(PA \mid B_i)$
Bayes's Rule	$P(B_r \mid A) = \dfrac{P(B_r \cap A)}{P(A)}$ $= \dfrac{P(B_r)P(A \mid B_r)}{\sum_{i=1}^{k} P(B_i)(P A \mid B_i)}$

Random Variable Formulas

Probability Mass Function	$p(x) = \begin{cases} P(X = x), & \text{if } x = x_1, x_2, \ldots, x_k \\ 0, & \text{otherwise} \end{cases}$
Mean of a Discrete Random Variable	$\mu_X \text{ or } E(X) = \sum_{i=1}^{k} x_i \cdot p(x_i)$
Variance of a Discrete Random Variable	$\sigma_X^2 \text{ or } \mathrm{Var}(X) = E\left[(X - \mu_X)^2\right]$ $= \sum_{i=1}^{k} (x_i - \mu_X)^2 \cdot p(x_i)$ $\sigma_X^2 \text{ or } \mathrm{Var}(X) = E(X^2) - [E(X)]^2$ $= \sum_{i=1}^{k} x_i^2 \cdot p(x_i) - (\mu_X)^2$
Standard Deviation of a Random Variable	$\sigma_X = +\sqrt{\mathrm{Var}(X)}$

Special Probability Distributions

Discrete Uniform Distribution	$p(x) = \frac{1}{k},\ x = x_1,\ x_2, \ldots, x_k$ $\mu_X = \frac{1}{k}\sum_{i=1}^{k} x_i$ $\sigma_X^2 = \frac{1}{k}\sum_{i=1}^{k} (x_i - \mu_X)^2$
Binomial Distribution	$p(x) = \binom{n}{x} p^x (1-p)^{n-x},$ $x = 0, 1, 2, \ldots, n$ $\mu_X = np$ $\sigma_X^2 = np(1-p)$
Bernoulli Distribution	$p(x) = p^x (1-p)^{1-x}, x = 0, 1$ $\mu_X = p$ $\sigma_X^2 = p(1-p)$
Hypergeometric Distribution	$p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}},$ $x = \max(0, n-N+K), \ldots,$ $\min(n, K);\ n = 1, 2, \ldots, N$ $\mu_X = n\frac{K}{N}$ $\sigma_X^2 = n\frac{K}{N}\left(1 - \frac{K}{N}\right)\left(\frac{N-n}{N-1}\right)$
Negative Binomial Distribution	$p(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k},$ $x = k,\ k+1,\ k+2, \ldots$ $\mu_X = \frac{k}{p}$ $\sigma_X^2 = \frac{k(1-p)}{p^2}$

Geometric Distribution	$p(x) = p(1-p)^{x-1}, x = 1, 2, 3, \ldots$ $\mu_X = \frac{1}{p}$ $\sigma_X^2 = \frac{1-p}{p^2}$
Poisson Distribution	$p(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, x = 0, 1, 2, \ldots,$ where $e \approx 2.71828$ $\mu_X = \lambda$ $\sigma_X^2 = \lambda$
Continuous Uniform (or Rectangular) Distribution	$f(x) = \frac{1}{b-a}, a \le x \le b$ $\mu_X = \frac{a+b}{2}$ $\sigma_X^2 = \frac{(b-a)^2}{12}$
Normal Distribution	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty; -\infty < \mu < \infty,$ $\sigma > 0$
z-score	$z = \frac{x-\mu}{\sigma}$

Sampling Distributions and Estimation

Mean of $\bar{x}$	$\mu_{\bar{x}}$ or $E(\bar{x}) = \sum \bar{x} \cdot p(\bar{x}) = \mu_X$
Variance of $\bar{x}$	$\sigma_{\bar{x}}^2 = \text{Var}(\bar{x})$ $= E\left[(\bar{x} - \mu_{\bar{x}})^2\right] = E\left[(\bar{x} - \mu_{\bar{x}})^2\right]$ $= \sum \bar{x}^2 \cdot p(\bar{x}) - (\mu_{\bar{x}})^2$
Variance of $\bar{x}$: Sampling Without Replacement	$\sigma_{\bar{x}}^2 = \frac{\sigma_X^2}{n} \cdot \frac{N-n}{N-1}$
Variance of $\bar{x}$: Sampling With Replacement	$\sigma_{\bar{x}}^2 = \frac{\sigma_X^2}{n}$

Standard Error of $\bar{x}$	$\sigma_{\bar{x}} = +\sqrt{\text{Var}(\bar{x})} = \frac{\sigma_X}{\sqrt{n}}$
z-score for $\bar{x}$	$Z = \frac{\bar{x} - \mu_X}{\frac{\sigma_X}{\sqrt{n}}}$
Point Estimator of μ	$\bar{x} = \frac{\sum_{i=1}^{n} X_i}{n}$
Confidence Interval for μ: σ known	$[\bar{x} - e,\ \bar{x} + e]$ where $e = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
Confidence Interval for μ: σ Unknown and n Large	$[\bar{x} - e, \bar{x} + e]$ where $e = Z_{\alpha/2}\frac{s}{\sqrt{n}}$
Confidence Interval for μ: σ Unknown and n Small	$[\bar{x} - e,\ \bar{x} + e]$ where $e = t_{\alpha/2(n-1)}\frac{s}{\sqrt{n}}$
Point Estimator for p	$\hat{p} = \frac{x}{n}$
Confidence Interval for p: Sufficiently Large n	$[\hat{p} - e,\ \hat{p} + e]$ where $e = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$
Sample Size for Estimating μ	$n = \left(\frac{Z_{\alpha/2}^2\sigma}{e}\right)^2$
Sample Size for Estimating p	$n = \frac{Z_{\alpha/2}^2\hat{p}\hat{q}}{e^2}$
Point Estimator of $\mu_1 - \mu_2$: Independent Samples	$\bar{x}_1 - \bar{x}_2$
Confidence Interval for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Known	$\left[(\bar{x}_1 - \bar{x}_2) - e, (\bar{x}_1 - \bar{x}_2) + e\right]$ where $e = Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Confidence Interval for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Unknown, and n_1, n_2 Large	$\left[(\bar{x}_1 - \bar{x}_2) - e, (\bar{x}_1 - \bar{x}_2) + e\right]$ where $e = e = Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Confidence Interval for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Unknown but Assumed Equal	$\left[(\bar{x}_1 - \bar{x}_2) - e, (\bar{x}_1 - \bar{x}_2) + e\right]$ where $e = t_{\alpha/2\,(n_1+n_2-2)}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ and $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$
Confidence Interval for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Unknown and Assumed Unequal	$\left[(\bar{x}_1 - \bar{x}_2) - e, (\bar{x}_1 - \bar{x}_2) + e\right]$ where $e = t_{\alpha/2\,(v)}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ and $v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{(n_1 - 1)} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{(n_2 - 1)}}$
Point Estimator of $\mu_1 - \mu_2$: Related Samples	$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$
Confidence Interval for $\mu_1 - \mu_2$: Related Samples	$\left[\bar{d} - e, \bar{d} + e\right]$ where $e = t_{\alpha/2\,(n-1)}\frac{s_d}{\sqrt{n}}$
Point Estimator of $p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$
Confidence Interval for $p_1 - p_2$: Sufficiently Large n_1 and n_2	$\left[(\hat{p}_1 - \hat{p}_2) - e, (\hat{p}_1 - \hat{p}_2) + e\right]$ where $e = Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

Tests of Statistical Hypothesis

Test for μ: σ Known	$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
Test for μ: σ Unknown and $n \geq 30$	$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

Test for μ: σ Unknown and n Small	$t = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ with $v = n - 1$
Test for p: Sufficiently Large n	$Z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}}$
Test for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Known (Independent Samples)	$Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_o}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$
Test for $\mu_1 - \mu_2$: σ_1^2, σ_2^2 Unknown, and n_1, n_2 Large (Independent Samples)	$Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$
Test for $\mu_1 - \mu_2$: σ_1^2 amd σ_2^2 Unknown but Assumed Equal (Independent Samples)	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $v = n_1 + n_2 - 2$ and $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$
Test for $\mu_1 - \mu_2$: σ_1^2 and σ_2^2 Unknown and Assumed Unequal	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ with $v = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{(n_1 - 1)} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{(n_2 - 1)}}$
Test for $\mu_1 - \mu_2$: Related Samples	$t = \dfrac{\bar{d} - d_o}{\dfrac{s_d}{\sqrt{n}}}$ with $v = n - 1$
Test for $p_1 - p_2$: Sufficiently Large n_1 and n_2	$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

Appendix C: Rubrics

Rubric: Report/Presentation/Video clips on Data Organization/Official Statistics Learning Module in Descriptive Statistics

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Significance	Project is not significant; Demonstrates no evidence that the topic was selected, researched and designed	Project is somewhat significant; Demonstrates limited evidence that the topic was selected, researched and designed	Project is significant; Demonstrates some evidence that the topic was selected, researched and designed	Project is significant; Demonstrates evidence that the topic was selected, researched and designed well	**/4**
Content	Demonstrates minimal understanding of statistical concepts and skills with so many errors	Demonstrates some understanding of statistical concepts and skills with minimal errors	Demonstrates understanding of statistical concepts and skills with one or two errors	Demonstrates in-depth understanding of statistical concepts and skills with no error	**/4**
Appropriateness of descriptive Statistics	Inaccurate presentation of data using inappropriate tables/ graphs/numerical measures	Accurate presentation of data using inappropriate tables/graphs/ numerical measures	Accurate presentation of data using appropriate tables/ graphs/numerical measures without proper labels/titles	Accurate presentation of data using appropriate tables/ graphs/ numerical measures with proper labels/titles	**/4**
Extensiveness of descriptive Statistics	Presentation of data uses minimal number of graphs, tables, and numerical measures discussed	Presentation of data uses fewer than 3 kinds of graphs, one-way table, two-way table, and not all numerical measures discussed.	Presentation of data uses at least 3 kinds of graphs, one-way table, two-way table, and all numerical measures discussed with incorrect or missing interpretations.	Presentation of data uses at least 3 kinds of graphs, one-way table, two-way table, and all numerical measures discussed with correct interpretations.	**/4**

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Organization	Presented concepts/ skills which were poorly organized and lacked supporting evidence	Presented concepts/skills which were minimally organized with minimal supporting ideas	Presented concepts/ skills which were logically organized with some supporting ideas	Presented concepts/ skills which were logically organizedwith complete supporting ideas	**/4**
Integration	Demonstrates no integration with other fields or disciplines	Demonstrates limited integration with other fields or discipline	Demonstrates integration with other fields or discipline	Demonstrates integration with other fields or discipline and has long-term impact	**/4**
Use of Resources/Visual Aids	Demonstrates no use of quality resources and materials	Demonstrates limited use of quality resources and materials	Demonstrates some uses of quality resources and materials	Demonstrates appropriate uses of quality resources and materials	**/4**
Overall Presentation and creativity	Overall presentation is neither creative nor artistic with no innovative ideas	Overall presentation shows limited effort in its creativity and artistic value with limited innovative ideas	Overall presentation shows some effort in its creativity and artistic value with some innovative ideas	Overall presentation is creative and artistic with innovative ideas	**/4**
TOTAL					**/32**

Rubric: Games of Chance Exhibit

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Accuracy of Probabilities Calculated	Probabilities are calculated incorrectly.	Probabilities are calculated with some errors.	Probabilities are calculated correctly but not explained well.	Probabilities are calculated and explained correctly with no error	**/4**
Use of Resources/ Visual Aids	Demonstrates no use of quality resources and materials	Demonstrates limited use of quality resources and materials	Demonstrates some uses of quality resources and materials	Demonstrates appropriate uses of quality resources and materials	**/4**
Overall Presentation and creativity	Overall presentation is neither creative nor artistic with no innovative ideas	Overall presentation shows limited effort in its creativity and artistic value with limited innovative ideas	Overall presentation shows some effort in its creativity and artistic value with some innovative ideas	Overall presentation is creative and artistic with innovative ideas	**/4**
Mechanics and Organization	Presented no background and unorganized mechanics of the game	Presented background of the game and mechanics which are not logically organized	Presented background of the game and mechanics which are logically organized	Presented background of the game and mechanics which are logically organized and understood by every participant	**/4**
TOTAL					**/16**

Rubric: Movie /Article/Book Review/Game Critique

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Organization	Manuscript is disorganized and the flow of information.	Manuscript shows organization but has several portions that are not relevant.	Manuscript shows organization but lacks coherence.	Manuscript is well-organized and structured.	**/4**
Statistical Accuracy	Statistical explanations or facts presented/cited are all misconceptions.	Statistical explanations or facts presented/cited show a significant number of inaccuracies.	Statistical explanations or facts presented/cited show some inaccuracies.	Statistical explanations or facts presented are 100% accurate.	**/4**
Presentation of Explanations	Explanations presented do not in any way address the problem/topic.	Explanations presented only partially addressed the problem/topic.	Explanations are clear, valid and convincing but have several flaws.	Explanations are clear, valid and convincing.	**/4**
TOTAL					**/12**

Rubric: Portfolio for Statistical Analysis

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Significance	Project is not significant; Demonstrates no evidence that the topic was selected, researched and designed	Project is somewhat significant; Demonstrates limited evidence that the topic was selected, researched and designed	Project is significant; Demonstrates some evidence that the topic was selected, researched and designed	Project is significant; Demonstrates evidence that the topic was selected, researched and designed well	/4
Content	Demonstrates minimal understanding of statistical concepts and skills with so many errors	Demonstrates some understanding of statistical concepts and skills with minimal errors	Demonstrates understanding of statistical concepts and skills with one or two errors	Demonstrates in-depth understanding of statistical concepts and skills with no error	/4
Accuracy of Statistical Analysis	Incorrect use of inferential statistical analysis	Some statistical analyses are not correct.	All statistical analyses are correct with missing or lacking interpretations.	All statistical analyses are accurate with correct interpretations.	/4
Extensiveness of Statistical Analysis	No estimation procedure/ statistical test included in the analysis.	Only one estimation procedure/statistical test is included in the analysis.	At least 2 estimation procedures are presented or at least 2 statistical tests are included in the analyses.	At least 2 estimation procedures are presented and at least 2 statistical tests are included in the analyses.	/4
Organization	Presented concepts/ skills which were poorly organized and lacked supporting evidence	Presented concepts/skills which were minimally organized with minimal supporting ideas	Presented concepts/skills which were logically organized with some supporting ideas	Presented concepts/ skills which were logically organized with complete supporting ideas	/4

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent/4*	*Score*
Integration	Demonstrates no integration with other fields or disciplines	Demonstrates limited integration with other fields or discipline	Demonstrates integration with other fields or discipline	Demonstrates integration with other fields or discipline and has long-term impact	/4
Use of Resources/Visual Aids	Demonstrates no use of quality resources and educational materials	Demonstrates limited use of quality resources and educational materials	Demonstrates some uses of quality resources and educational materials	Demonstrates appropriate uses of quality resources and materials	/4
Overall Presentation and creativity	Overall presentation is neither creative nor artistic with no innovative ideas	Overall presentation shows limited effort in its creativity and artistic value with limited innovative ideas	Overall presentation shows some effort in its creativity and artistic value with some innovative ideas	Overall presentation is creative and artistic with innovative ideas	/4
TOTAL					/32

Rubric: Group Member Assessment

Criteria	*Needs Improvement/1*	*Satisfactory/2*	*Good/3*	*Excellent / 4*	*Score*
Contribution	Group members contributions were insignificant or nonexistent .	Group member contributed little toward the project	Group member contributed significantly, but other members clearly contributed more	Group member completed an equal share of work and strived to maintain that equity throughout the project	**/4**
Dependability	Group member was undependable forcing other members to take up the slack	Group member contributions were regularly late and often missed scheduled group work	Group member contributions were mostly punctual and almost always appeared for group work	Group member provided contributions with 100% punctuality and always appeared for group work	**/4**
Efficiency	Work performed was completely ineffective and useless in the final product	Work performed was inappropriate and mostly useless toward the final product	Participation was inefficient and thus contributions were less than expected	Work performed was very useful and contributed significantly to the final product	**/4**
Attitude	Group member often complained and generally demoralized the group	Group member sometimes complained and was somewhat of a burden	Group member didn't complain but offered little enthusiasm	Group member was very positive and pleasant to work with	**/4**
Quality of Work	Group member's work was routinely of poor quality and lacked a sense of pride	Group member's work product was frequently unpolished and/ or incomplete	Group member's work product was sometimes unpolished and/or incomplete	Group member's work product was of extremely high quality	**/4**
Cooperation	You would never work with this person again under any circumstances	You would prefer to never work in a group setting with this person again	You would work with this person again but would choose someone else if given the opportunity	This person was an excellent partner, and you would eagerly work with them again	**/4**
TOTAL					**/24**