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Transportation Research Part B 38 (2004) 927–952

TRANSPORTATION RESEARCH PART B

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A multi-objective train scheduling model and solution

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Received 24 January 2003; received in revised form 13 June 2003; accepted 6 February 2004

Abstract

This paper develops a multi-objective optimization model for the passenger train-scheduling problem on a railroad network which includes single and multiple tracks, as well as multiple platforms with different train capacities. In this study, lowering the fuel consumption cost is the measure of satisfaction of the railway company and shortening the total passenger-time is being regarded as the passenger satisfaction criterion. The solution of the problem consists of two steps. First the Pareto frontier is determined using the ε-constraint method, and second, based on the obtained Pareto frontier detailed multi-objective optimization is performed using the distance-based method with three types of distances. Numerical examples are given to illustrate the model and solution methodology.

Keywords: Multi-objective programming; Train scheduling; Pareto frontier; ε-Constraint method; Distance-based methods

1. Introduction

Public rail transport planning is a highly complex task. It is based on the interaction of many elements which must be managed simultaneously. Due to the tremendous size of the public rail traffic system, the planning process is usually divided into several steps (see also Assad, 1980; Lindner, 2000; Bussieck et al., 1997). A diagram of this hierarchical decomposition is given in Fig. 1.

In the first step, the *passenger demand* has to be assessed and analyzed. As a result, the amount of travelers wishing to go from certain origins to certain destinations is determined. As a

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0191-2615/$ - see front matter
doi:10.1016/j.trb.2004.02.004

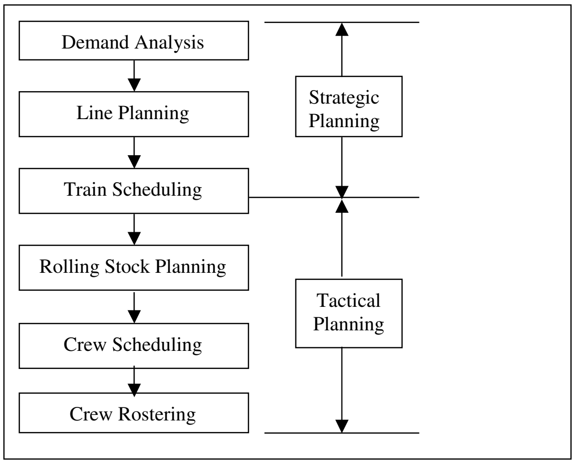


Fig. 1. The hierarchical planning process in public rail transport.

subsequent task, *line planning* is performed which includes decisions about which routes or lines to be operated and how frequently. Then, in the *train schedule planning* step, all arrival and departure times of the lines are determined. Timetables are formulated resulting in schedules of trips with corresponding start and end locations and times schedules for each train. Next engines and coaches have to be assembled to trains, which are assigned to the lines. This is called *rolling stock planning*. A similar task is the *crew management*, in which the distribution of personnel is determined in order to guarantee that each train is equipped with the necessary staff. Crew management components are crew scheduling and crew rostering. In crew scheduling crews are assigned to trains, while the crew rostering process is used for constructing rosters from the crew duties. All these steps are strongly related to each other. Computing an optimal solution in one step may restrict the set of feasible decisions in the next step, therefore in order to reach a global solution the calculation in each step should consider all forthcoming steps and objectives in order to avoid contradictions and a complete recalculation.

The entire planning process can also be divided into *strategic*, *tactical* and *operational* planning levels (see Table 1).

Savas (1978) provided three major criteria for evaluating the provision of public goods and services namely, efficiency, effectiveness and equity. We will focus on efficiency and effectiveness in this study. *Efficiency* is defined as the ratio of the level of a service compared to the cost of the resources required to provide the service. It is clear that for a fixed level of a service, the main variable in determining the efficiency is the total cost incurred by providing the service. Therefore, efficiency can be measured by its cost. It is generally accepted that the capital cost is significantly

Table 1
Planning levels

Planning stages	Time horizon	Objective
Strategic level	5–15 years	Resource acquisition
Tactical level	1–5 years	Resource allocation
Operational level	24 h–1 year	Daily planning

larger than the incremental cost over a year. The *effectiveness* of a service is a measure showing how well the demand for the service is satisfied. In this study efficiency and effectiveness are regarded as the criteria expressing the operator (railway company) and users' (passenger community) viewpoint, respectively.

2. Literature review

Train scheduling is one of the most challenging and difficult problems in railway planning which has been drawing the attention of researchers for decades. The physical railroad network is shared by a large number of trains; it is indeed necessary to synchronize the use of the available resources. Also, the simultaneous scheduling of freight and passenger trains has an important impact on the quality and level of service provided to the public.

Train scheduling had been done manually for more than a century through a trial and error process by humans using a preliminary train diagram. Although there are some countries where such approach is still being done. In this process, rail network is first decomposed into a set of rail corridors. Each corridor typically consists of a set of lines connecting a sequence of stations together. Then planners schedule a train at a time for each corridor separately according to scheduling requirements, and then checking for and eliminating any conflicts arising. Resolving conflict may involve rescheduling other trains. The primary objective of this approach is to produce a feasible timetable.

The strong competition rail carriers are faced with the privatization of many national railroads, deregulation, the ever-increasing speed of computers, and the increasing role of railway in country's economy, all motivate the effort to develop and use more efficient scheduling techniques.

These techniques can be divided into three major groups: simulation, mathematical programming, and expert systems; however, in practical cases their combinations are mostly used. Iida (1988), Komaya and Fukuda (1989), Komaya (1992) are good examples of using expert systems. Simulation approach is used in Frank (1965), Peat et al. (1975), Rudd and Storry (1976), Petersen and Taylor (1982), among others.

Mathematical programming methodology was first applied to this problem by Amit and Goldfarb (1971). Cordeau et al. (1998) presented a survey of relevant optimization models, although the mathematical programming approach is not limited to optimization models. There are studies using heuristic models, such as Sahin (1999). Optimization models are used in Szpigel (1973), Sauder and Westerman (1983), Petersen et al. (1986), Jovanovic (1989), Jovanovic and Harker (1991), Kraay et al. (1991), Kraay and Harker (1995), Carey (1994), Carey and Lockwood (1995), Higgins et al. (1996) to mention a few.

However, the related literature has experienced a slow growth and, until recently, most contributions were dealing with only simplified models or small instances failing to incorporate the characteristics of real-life applications. Surveys of Assad (1980, 1981) and Haghani (1987) suggest that optimization models for rail transportation were not widely employed in practice, instead simulation models were mostly used. In fact, the large size and the complexity of the problem have hindered the development of optimization models for train scheduling.

To our knowledge, all optimization models of train scheduling have a single-objective formulation. The single planning objective is usually constructed from the perspective of either the user such as schedule deviation, or the combination of user and operator objectives such as train delays and operating cost. For example, Jovanovic and Harker (1991) gave a fixed velocity MIP formulation with maximizing reliability objective function. Kraay et al. (1991) gave a variable velocity nonlinear MIP to minimize train delays and fuel costs. Kraay and Harker (1995) presented a fixed velocity nonlinear MIP model with minimizing schedule deviation. Carey (1994) and Carey and Lockwood (1995) presented a fixed velocity linear MIP formulation in order to minimize schedule deviation. Higgins et al. (1996) gave a variable velocity nonlinear MIP formulation to minimize train delays and operating costs.

However, the nature of train scheduling problem is inherently multi-objective. This is primarily due to the multiplicity of conflicting interests of different stakeholders and social concerns. Some recent studies have shown the advantages of dealing with the multi-objective nature of the transportation-planning problems. Multi-objective approach generally produces better planning alternatives, mainly because most relevant factors can be considered as planning objectives and can be evaluated in noncommensurable units.

Good examples of such studies are the work on transportation networks by Agarwal (1973), Current et al. (1987), on traffic management by Colllins (1973), on air service planning by Flynn and Ratick (1988), on bus operations planning by Tzeng and Shiau (1988), on airline flight planning by Teodorovic and Krcmar-Nozic (1989), on freight train planning by Fu and Wright (1994), on urban school bus planning by Bowerman et al. (1995), on transit network design by Israeli and Ceder (1996), on transportation investment planning by Teng and Tzeng (1996), on aircrew rostering by Teodorovic and Lucic (1998), on transportation and assignment problems by Charnes et al. (1969), Lee and Moore (1973), and El-Wahed (2000), on passenger train services planning by Chang et al. (2000), on airline network design by Hsu and Wen (2000), and on vehicle scheduling problems by Park (2000).

Here we mention two special studies, which are related to train planning. Fu and Wright (1994) presented a train-planning model for British rail freight services through the Channel Tunnel. Train planning in that context involves the specification of the origin and destination of every wagon in a train, together with details of "marshalling" activities. The word marshalling is taken to include all operations that change the composition of a train. This model belongs to the rolling stock planning stage of the hierarchical planning process. Chang et al. (2000) gave a multi-objective model for passenger train services planning. They determined the optimal allocation of passenger train services on an inter-city high-speed rail line without branches with specifying subset of stations at which the train must stop. This model belongs to the line-planning category of the hierarchical planning process.

In this paper we develop a multi-objective passenger train scheduling methodology on a railroad network including single and multiple tracks as well as multiple platforms with different train capacities. Tracks can be unidirectional or bi-directional and trains can have different origins and destinations. The model fit at the best in a tactical level of the planning stages.

3. The mathematical model

Although the nature of train scheduling problem is inherently multi-objective, all optimization models in the literature so far have a single-objective formulation. The single planning objective is usually constructed from the perspective of either the user such as schedule deviation (see for example Kraay and Harker, 1995; Carey, 1994), or the combination of user and operator objectives such as train delays and operating cost (see for example Higgins et al., 1996; Kraay et al., 1991). This paper develops a multi-objective optimization model for the passenger train-scheduling problem in which lowering the fuel consumption cost is the measure of satisfaction of the railway company (regarded as a criterion of efficiency) and shortening the total passenger-time is being regarded as the passenger satisfaction criterion (regarded as a criterion of effectiveness). Lowering the total passenger-time needs to increase the velocity of trains, on the other hand increasing the velocity, as it will be seen, leads to more cost of fuel consumption whereas the railway company desires to lower this cost. So a trade-off between these conflicting objectives has to be made in the decision making process. Shortening the total passenger-time as an objective function is a new idea that has not been seen in the literature so far.

We assume that the infrastructure consists of nodes (stations, junctions, bridges, crossings, etc.) and tracks that connect these nodes. The information on each train includes an origin and a destination station as well as all intermediate stations being served by the train. We also suppose that the infrastructure is decomposed according to the following rules:

(1) The set of the nodes (stations, junctions, etc.) is decomposed into pairs of connected nodes. In the case of a station the two nodes are connected with parallel links, the number of which equals the number of platforms.
(2) Each platform with more than one train capacity is divided into a number of sub-links. The number of these sub-links is equal to the capacity of the platform. We assume that the capacity of each platform is predetermined based on certain considerations such as the length of the platform, length of trains, and other safety and technical concerns.
(3) Fig. 2a represents a map of a station with two platforms. For the sake of simplicity, we do not consider separate segments AB and CD. It should be noted that this reduction does not restrict generality, since the length of these segments is usually much smaller than that of any trains.

    Similar reduction is performed in the case of more complicated platforms (see Fig. 2b).
(4) The nodes should be arranged in a way that the trains just move from east to west, or vice versa.
(5) At nodes of a decomposed rail network, with more than one incoming and outgoing links, we can introduce a dummy link in a way that at least one of the numbers of the incoming and

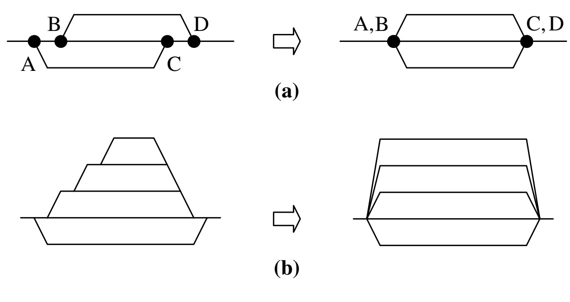


Fig. 2. (a) Reduction of a platform and (b) reduction of a complicated platform.

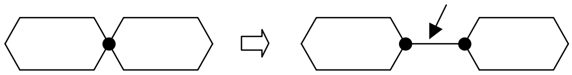


Fig. 3. Adding a dummy link.

outgoing links of any node becomes equal to one (see Fig. 3). Train running time on these dummy links will be set equal to zero (see Carey, 1994 for more information). Note that this rule can be ignored and is not obligated in this model.

3.1. Parameters

In the mathematical model the following notation will be used.

E	set of all trains that are dispatched from west to east during the planning horizon
W	set of all trains that are dispatched from east to west during the planning horizon
$M(k)$	the number of sub-journeys of train k ($k \in E \cup W$)
$k(m)$	mth sub-journey of train k ($k \in E \cup W$, $m = 1, 2, \ldots, M(k)$)
N	set of all nodes on the decomposed network
N_k	set of all possible nodes on the path of train k ($k \in E \cup W$, $N_k \subseteq N$)
N'_k	set of all possible nodes on the path of train k that ought to be covered ($k \in E \cup W$, $N'_k \subseteq N_k$)
N_k^{O}	journey origin node of train k ($k \in E \cup W$, $N_k^{\mathrm{O}} \in N_k$)
N_k^{D}	journey destination node of train k ($k \in E \cup W$, $N_k^{\mathrm{D}} \in N_k$)
$N_{k(m)}^{\mathrm{O}}$	origin node of sub-journey $k(m)$ ($N_{k(m)}^{\mathrm{O}} \in N_k$)
$N_{k(m)}^{\mathrm{D}}$	destination node of sub-journey $k(m)$ ($N_{k(m)}^{\mathrm{D}} \in N_k$)
L	set of all links on decomposed network
L_k	set of all possible links on the path of train k ($k \in E \cup W, L_k \subseteq L$)
$L_k^{\mathrm{I}(n)}$	set of all possible incoming links to node n for train k ($n \in N_k, k \in E \cup W, L_k^{\mathrm{I}(n)} \subseteq L_k$)
$L_k^{\mathrm{O}(n)}$	set of all possible outgoing links from node n for train k ($n \in N_k, k \in E \cup W, L_k^{\mathrm{O}(n)} \subseteq L_k$)
L'_k	set of all links where train k might stop to allow passengers to get off/on the train according to strategic train scheduling ($k \in E \cup W$, $L'_k \subseteq L_k$)
P_k	set of all possible links (here platforms) on the path of train k with more than one train capacity ($k \in E \cup W$, $P_k \subseteq L_k$)
Q_l	train capacity of platform $l \in P_k$
$l(q)$	qth sub-link of link $l \in P_k$ ($q = 1, 2, \ldots, Q_l$)
$P_{k(m)}$	number of passengers at the first node of $k(m)$
$\alpha_{k(m)}$	number of passengers leaving train k at the initial station of $k(m)$
$\beta_{k(m)}$	number of passengers boarding train k at the initial station of $k(m)$
$t_{\alpha_{k(m)}}$	required stopping time for allowing passengers to leave the train at the end of sub-journey $k(m-1)$ (which is also the initial station of $k(m)$)
$t_{\beta_{k(m)}}$	required stopping time for allowing passengers to board the train at the initial station of $k(m)$
d_l	length of link l ($l \in L_k$)
M_k	the mass of train k ($k \in E \cup W$)

R_{kl} the resistance force on train $k \in E \cup W$ while traversing link $l \in L_k$
F_{kl} the force that is required to keep a constant speed of train $k \in E \cup W$ on link $l \in L_k$
P_{kl} required power for generating F_{kl}
c the cost of each unit of fuel
r_k rate of fuel consumption per unit power output for train $k \in E \cup W$
R_0^k, R_1^k, R_2^k resistance coefficients for train $k \in E \cup W$
$\overline{V}_{kl}$ upper velocity limit for train k on link l ($k \in E \cup W$, $l \in L_k$)
$\underline{t}_{kl}$ minimum trip or dwell time on link l for train k ($k \in E \cup W$, $l \in L_k$)
s minimum time interval (headway) between two events
M large arbitrary constant
T time horizon

3.2. Decision variables

Binary variables:

$$X_{kl} = \begin{cases} 1; & \text{if train } k \text{ traverses through link } l \quad (k \in E \cup W, l \in L_k), \\ 0; & \text{otherwise} \end{cases}$$

$$A_{ijl} = \begin{cases} 1; & \text{if train } i \text{ traverses through link } l \text{ after train } j \quad (i \in E, j \in W, l \in L_i \cap L_j), \\ 0; & \text{otherwise} \end{cases}$$

$$B_{ikl} = \begin{cases} 1; & \text{if train } i \text{ traverses through link } l \text{ after train } k \quad (i, k \in E, l \in L_i \cap L_k), \\ 0; & \text{otherwise} \end{cases}$$

$$C_{jkl} = \begin{cases} 1; & \text{if train } j \text{ traverses through link } l \text{ after train } k \quad (j, k \in W, l \in L_j \cap L_k), \\ 0; & \text{otherwise} \end{cases}$$

Continuous variables:

$a_{k(m)}$ time at which train k starts its mth sub-journey ($k \in E \cup W$, $m = 1, 2, \ldots, M(k)$)
$d_{k(m)}$ time at which train k ends its mth sub-journey ($k \in E \cup W$, $m = 1, 2, \ldots, M(k)$)
a_{kl} time at which train k enters link l ($k \in E \cup W$, $l \in L_k$)
d_{kl} time at which train k departs link l ($k \in E \cup W$, $l \in L_k$)
V_{kl} average velocity of train k on link l ($k \in E \cup W$, $l \in L_k$)

3.3. Formulation of the objective functions

3.3.1. Fuel consumption cost objective function

The first objective function reveals mostly the railways companies' viewpoint; however, saving in energy consumption and especially in fossil fuels has environmental and economic consequences as well.

The amount of fuel needed to move a passenger or freight train is proportional to the resistance. For computing resistance there are several methods known from railroad engineering concepts and field studies. Train resistance usually is measured in pounds per ton of train weight and is a function of many factors including (but not limited to): (1) rolling resistance, (2) flange resistance, (3) journal (axle) resistance, (4) track resistance, (5) air resistance, (6) curve resistance, and (7) grade resistance. The first train resistance models were developed by Schmidt and Tuthill (1910–1940s) (Schmidt, 1910; Schmidt and Dunn, 1916; Tuthill, 1948). These findings ultimately lead to the Davis equation for estimating train resistance, which is still in use today. Davis (1926) and the American Railway Engineering Association (1970) derived comprehensive train resistance equations and adjustment factors that incorporate most of the effects described above. These resistance equations, which are detailed in Hay (1982), have been incorporated into many train performance simulators and analytical models.

In order to find an approximate formula to be used in this work; let us use the well-known Davis formula for the resistance R_{kl} of train $k \in E \cup W$ with mass M_k and velocity V_{kl}:

$$\frac{R_{kl}}{M_k} = R_0^k + R_1^k V_{kl} + R_2^k V_{kl}^2,$$

where

R_0^k	resistance coefficients due to grade, which is constant with velocity
R_1^k	resistance coefficients primarily due to rail friction
R_2^k	resistance coefficients primarily due to air friction

For example, for an Advanced Passenger Train in England the values of R_0, R_1, and R_2 are 16.6, 36.6×10^{-2}, 26.10×10^{-3}, respectively. The train resistance of old diesel trains is significantly larger than the train resistance of new high speed trains especially at high speeds (see Jorgensen and Sorenson, 1997).

The force required to keep a constant speed F_{kl} is equal to the resistance force R_{kl}. By the definition of Power, we have

$$P_{kl} = F_{kl} \cdot V_{kl}.$$

Then, the total fuel required as the train traverses link $l \in L_k$ of length d_l with rate r_k of fuel consumption per unit power output is

$$R_{kl} \cdot V_{kl} \cdot \left(\frac{d_l}{V_{kl}}\right) \cdot r_k = R_{kl} \cdot d_l \cdot r_k = M_k\left(R_0^k + R_1^k V_{kl} + R_2^k V_{kl}^2\right) \cdot d_l \cdot r_k.$$

Let c denote the cost of a unit of fuel. The total fuel consumption cost of train k during its trip is

$$\sum_{l \in L_k} M_k\left(R_0^k + R_1^k V_{kl} + R_2^k V_{kl}^2\right) \cdot d_l \cdot r_k \cdot c \cdot X_{kl}.$$

Thus the total fuel consumption cost for all trains can be given as

$$\sum_{k \in E \cup W} \sum_{l \in L_k} M_k\left(R_0^k + R_1^k V_{kl} + R_2^k V_{kl}^2\right) \cdot d_l \cdot r_k \cdot c \cdot X_{kl}.$$

In order to reduce the number of variables, we notice that $V_{kl} = \frac{d_l}{t_{kl}} = \frac{d_l}{d_{kl} - a_{kl}}$.

3.3.2. Trip time objective function

The second objective function is devoted to travel time. This function reflects mainly the user's concerns; however, the railway companies are also interested in saving time due to the more efficient usage of rolling stocks. There are hundreds of studies being undertaken for evaluating savings in travel time (for a review see Wardman, 1998). The passengers want to reach the destination as soon as possible to carry on their activities. From the point of view of the whole society, reductions in individual travel times can also be looked positively for various reasons. First, they result in potential increase of gross domestic product if such reductions translate into productive activities. Second, they help to increase social welfare, as travel conditions improve. For more details see Mackie et al. (2001).

Each train is scheduled to stop at certain stations to allow passengers to board/leave the train. These planned stops are predetermined by the strategic train scheduling planning. Arrival at each of these pre-determined stations starts a new sub-journey. Starting a new sub-journey is the same as terminating the previous sub-journey. Therefore, the trip of train k is divided into $M(k)$ sub-journeys. Fig. 4 illustrates the relationship between the events which occur to train k ($k \in E \cup W$) as it arrives and departs from these stations, where we use the following notation:

e_1 train k enters station A at time $a_{k(m)}$ with number $P_{k(m)}$ of passengers
e_2 all passengers who want to leave train k do so at time $a_{k(m)} + t_{\alpha_{k(m)}}$

With $\alpha_{k(m)}$ passengers leaving the train, the number of train passengers becomes $P_{k(m)} - \alpha_{k(m)}$. It is supposed that the passengers leave the train with a uniform distribution (see Fig. 5).

e_3 all new passengers board train k at time $a_{k(m)} + t_{\alpha_{k(m)}} + t_{\beta_{k(m)}}$

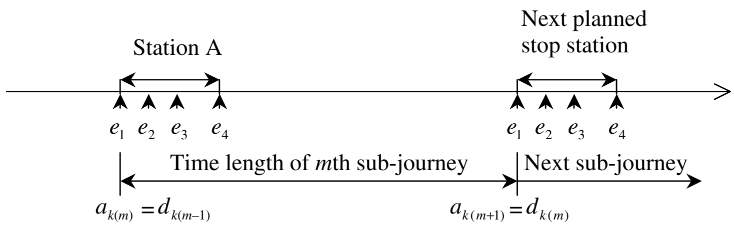


Fig. 4. Relation between events.

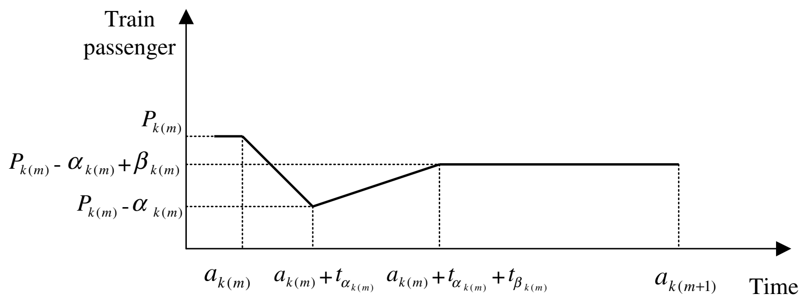


Fig. 5. Train passenger-time diagram.

With the new $\beta_{k(m)}$ passengers, the number of train passengers becomes $P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}$. It is supposed that passengers board the train with uniform distribution (see Fig. 5).

e_4 train k departs from station A and through some intermediate stations it moves toward the next planned stop station

The area under the passenger-time curve gives the total passenger-time during the mth sub-journey of train k. Thus we have

$$\text{Area under the curve} = \left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(a_{k(m+1)} - a_{k(m)} - t_{\alpha_{k(m)}} - t_{\beta_{k(m)}}\right) + \left(P_{k(m)} - \alpha_{k(m)}\right)\left(t_{\alpha_{k(m)}} + t_{\beta_{k(m)}}\right) + \frac{1}{2}\alpha_{k(m)} \cdot t_{\alpha_{k(m)}} + \frac{1}{2}\beta_{k(m)} . t_{\beta_{k(m)}}.$$

This equation can be re-arranged as follows:

$$\begin{aligned}\text{Area under the curve} = {} & \left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(a_{k(m+1)} - a_{k(m)}\right) \\ & + \Big[-\left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(t_{\alpha_{k(m)}} + t_{\beta_{k(m)}}\right) \\ & + \left(P_{k(m)} - \alpha_{k(m)}\right)\left(t_{\alpha_{k(m)}} + t_{\beta_{k(m)}}\right) + \frac{1}{2}\alpha_{k(m)} \cdot t_{\alpha_{k(m)}} + \frac{1}{2}\beta_{k(m)} \cdot t_{\beta_{k(m)}} \Big].\end{aligned}$$

Since the second part of the right hand side of this equation is a constant,

$$\text{Area under the curve} = \left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(a_{k(m+1)} - a_{k(m)}\right) + \text{constant}.$$

Adding this function over all sub-journeys of train k leads to the computation of total passenger-time of train k:

$$\sum_{m=1}^{M(k)} \left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(a_{k(m+1)} - a_{k(m)}\right) + \text{constant}.$$

Therefore, the second objective function is chosen as

$$\sum_{k \in E \cup W} \sum_{m=1}^{M(k)} \left(P_{k(m)} - \alpha_{k(m)} + \beta_{k(m)}\right)\left(a_{k(m+1)} - a_{k(m)}\right).$$

3.4. Formulation of the constraints

Any timetable resulted from the solution of a train scheduling problem should satisfy three main types of constraints: *continuity constraints*, *trip time* and *dwell time constraints*, and *safety constraints*. The rest of this section is devoted to the formulation of these constraints. Much of the formulation has been already appeared in other papers in the literature. For the sake of convenience we present them here in detail.

3.4.1. Train movement continuity constraints

These constraints ensure the continuity of trains' movement. If train $k \in E \cup W$ enters node $n \in N_k$, it can enter only through one of the incoming links of that node. The same situation holds while departing from this node. It can depart the node only through one of its outgoing links (see Fig. 6). Thus,

$$\sum_{l \in L_k^{\mathrm{I}(n)}} X_{kl} = \sum_{l \in L_k^{\mathrm{O}(n)}} X_{kl} \leqslant 1 \quad \left(\forall n \in N_k - N_k', k \in E \cup W\right),$$

$$\sum_{l \in L_k^{\mathrm{I}(n)}} X_{kl} = \sum_{l \in L_k^{\mathrm{O}(n)}} X_{kl} = 1 \quad \left(\forall n \in N_k', k \in E \cup W\right).$$

3.4.2. Events continuity constraints

As train $k \in E \cup W$ enters node $n \in N_k$ an arrival time is recorded from the corresponding incoming link as well as upon departure a departure time to the corresponding outgoing link is recorded. Dwell time on this node is so small that it can be neglected. These constraints ensure that arrival time to a link is equal to the departure time to the next link on any path of train k:

$$\sum_{l \in L_k^{\mathrm{I}(n)}} d_{kl} X_{kl} = \sum_{l \in L_k^{\mathrm{O}(n)}} a_{kl} X_{kl} \quad (\forall n \in N_k, k \in E \cup W),$$

and similar condition is posed for arriving to and departing from any sub-journey:

$$a_{k(m)} = \sum_{l \in L_k^{\mathrm{O}(N^{\mathrm{O}}_{k(m)})}} a_{kl} X_{kl} \quad (\forall k \in E \cup W, m = 1, 2, \ldots, M(k)),$$

$$d_{k(m)} = \sum_{l \in L_k^{I(N^{\mathrm{D}}_{k(m)})}} d_{kl} X_{kl} \quad (\forall k \in E \cup W, m = 1, 2, \ldots, M(k)).$$

3.4.3. Trip times on links and dwell times at platform constraints

Train technology, railroad conditions and other situation of the line and even climate and geographical circumstances impose an upper bound for train velocity on each segment of its path. These constraints ensure that the trip time on each segment is not less than the predetermined minimum amount of time. Similar considerations are posed in platforms for departing and boarding the trains, and maybe for performing planned technical inspections and services on the train. For traversing link $l \in L_k$, it is assumed that train $k \in E \cup W$ needs a minimum time $\underline{t}_{kl}$, so

$$d_{kl} - a_{kl} \geqslant \underline{t}_{kl} \quad (\forall k \in E \cup W, l \in L_k, l \notin P_k).$$

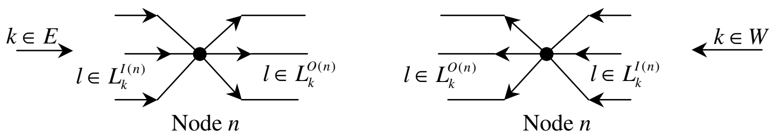


Fig. 6. A node with multiple incoming and outgoing links.

Similar condition is also required for all platforms with one train capacity on the route of any train. However for platforms with more than one train capacity this condition has to be modified. Platform $l \in P_k$ with train capacity Q_l is broken up into the same number of sub-links (see Fig. 7) and we require that

$$d_{kl(Q_l)} - a_{kl(1)} \geqslant \underline{t}_{kl} \quad (\forall k \in E \cup W, l \in P_k).$$

3.4.4. One train at a time on links constraints

These constraints ensure that no collision occurs. Collision may occur between trains of the same or opposite directions. Therefore, these constraints can be divided into three groups:

(1) In the case of eastbound trains we have

$$d_{kl} + s \leqslant a_{il} + M[(1 - X_{kl}) + (1 - X_{il}) + (1 - B_{ikl})] \quad (\forall i, k \in E, l \in L_i \cap L_k),$$
$$d_{il} + s \leqslant a_{kl} + M[(1 - X_{kl}) + (1 - X_{il}) + (1 - B_{kil})] \quad (\forall i, k \in E, l \in L_i \cap L_k),$$
$$B_{ikl} + B_{kil} = X_{kl}X_{il} \quad (\forall i, k \in E, l \in L_i \cap L_k).$$

The third equation requires that, if two trains $i, k \in E$ traverse through link $l \in L_i \cap L_k$ (i.e., $X_{il} = X_{kl} = 1$), then either train i should traverse over that link after train k (i.e., $B_{ikl} = 1$) or train k should traverse over that link after train i (i.e., $B_{kil} = 1$). If train i traverses link l after train k, according to the first inequality we have $d_{kl} + s \leqslant a_{il}$ showing that train i arrives at link l at least s time periods after train k departs that link (see Fig. 8a). Same condition is required in the other case. According to the second inequality in the first case we have $d_{il} + s \leqslant a_{kl}$ requiring that train i must arrive at link l at least s time periods after train i departs the link (see Fig. 8b).

The third nonlinear equation can be replaced by the following linear constraints:

$$B_{ikl} + B_{kil} = Y_{ikl} \quad (\forall i, k \in E, l \in L_i \cap L_k),$$

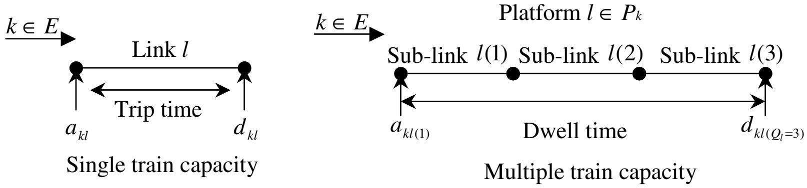


Fig. 7. Platform with one or more train capacity.

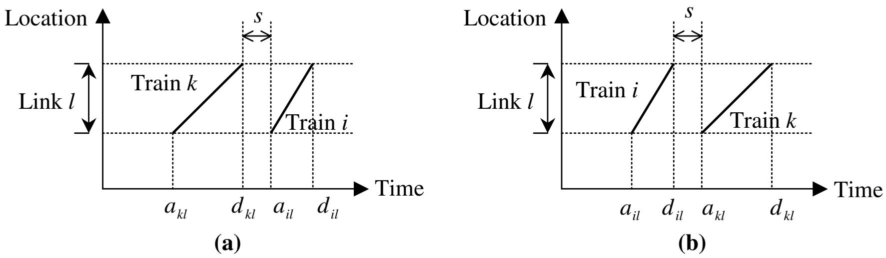


Fig. 8. Location-time diagram for two eastbound trains.

$$X_{kl} + X_{il} - 2Y_{ikl} \leqslant 1 \quad (\forall i,k \in E, l \in L_i \cap L_k),$$
$$X_{kl} + X_{il} - 2Y_{ikl} \geqslant 0 \quad (\forall i,k \in E, l \in L_i \cap L_k),$$
$$Y_{ikl} = 0,1 \quad (\forall i,k \in E, l \in L_i \cap L_k).$$

(2) Similarly to the case of eastbound trains, in order to ensure that no colliding occurs between two westbound trains, we must have (see also Fig. 9a and b):

$$d_{kl} + s \leqslant a_{jl} + M[(1 - X_{kl}) + (1 - X_{jl}) + (1 - C_{jkl})] \quad \left(\forall j,k \in W, l \in L_j \cap L_k\right),$$
$$d_{jl} + s \leqslant a_{kl} + M[(1 - X_{kl}) + (1 - X_{jl}) + (1 - C_{kjl})] \quad \left(\forall j,k \in W, l \in L_j \cap L_k\right),$$
$$C_{jkl} + C_{kjl} = X_{kl}X_{jl} \quad \left(\forall j,k \in W, l \in L_j \cap L_k\right).$$

(3) And finally, to ensure that no collision occurs between two trains of opposite directions, the following constraints should be satisfied (see also Fig. 10a and b):

$$d_{jl} + s \leqslant a_{il} + M[(1 - X_{il}) + (1 - X_{jl}) + (1 - A_{ijl})] \quad \left(\forall i \in E, j \in W, l \in L_i \cap L_j\right),$$
$$d_{il} + s \leqslant a_{jl} + M[(1 - X_{il}) + (1 - X_{jl}) + (1 - A_{jil})] \quad \left(\forall i \in E, j \in W, l \in L_i \cap L_j\right),$$
$$A_{ijl} + A_{jil} = X_{il}X_{jl} \quad \left(\forall i \in E, j \in W, l \in L_i \cap L_j\right).$$

3.4.5. Headway constraints at some nodes

Headway constraints are used in the model to ensure that there is a minimum time interval between two successive events. As it was discussed earlier, all physical nodes at any railroad network such as junctions, intersections, and stations are transformed into links. Headway

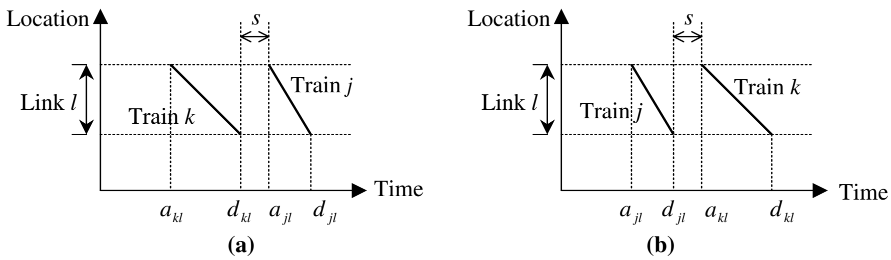


Fig. 9. Location-time diagram for two westbound trains.

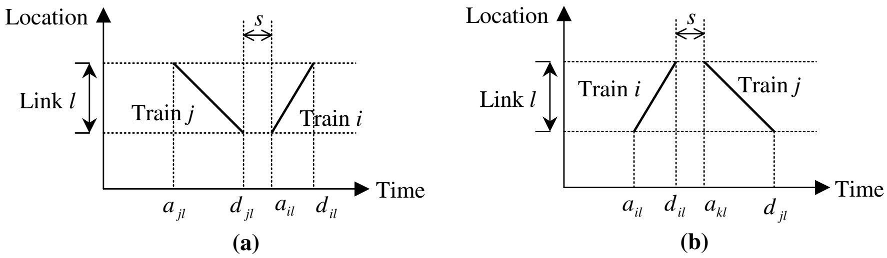


Fig. 10. Location-time diagram for two trains of opposite directions.

constraints at these nodes require that only one train can be present at a time on any link. However, at some nodes of decomposed rail network with more than one incoming and outgoing links (see Figs. 11–13), the headway constraints ensure that no collision of trains of the same or opposite directions occurs:

$$|a_{il} - a_{kl'}| \geqslant s - M\Big[(1 - X_{il}) + (1 - X_{kl'}) + (1 - X_{ip}) + (1 - X_{kp'})\Big]$$
$$\Big(\forall i, k \in E, l \in L_i^{\mathrm{O}(n)}, l' \in L_k^{\mathrm{O}(n)}, p \in L_i^{\mathrm{I}(n)}, p' \in L_k^{\mathrm{I}(n)}, l \neq l', p \neq p', n \in N_i \cap N_k\Big),$$

$$\Big|a_{jp} - a_{kp'}\Big| \geqslant s - M\Big[(1 - X_{jl}) + (1 - X_{kl'}) + (1 - X_{jp}) + (1 - X_{kp'})\Big]$$
$$\Big(\forall j, k \in W, l \in L_j^{\mathrm{I}(n)}, l' \in L_k^{\mathrm{I}(n)}, p \in L_j^{\mathrm{O}(n)}, p' \in L_k^{\mathrm{O}(n)}, l \neq l', p \neq p', n \in N_j \cap N_k\Big),$$

$$\Big|a_{il'} - a_{jp'}\Big| \geqslant s - M\Big[(1 - X_{ip}) + (1 - X_{jl}) + (1 - X_{il'}) + (1 - X_{jp'})\Big]$$
$$\Big(\forall i \in E, j \in W, l \in L_j^{\mathrm{I}(n)}, l' \in L_i^{\mathrm{O}(n)}, p \in L_i^{\mathrm{I}(n)}, p' \in L_j^{\mathrm{O}(n)}, l \neq l', p \neq p', n \in N_j \cap N_k\Big).$$

Since a_k^n denotes the arrival time of train $k \in E \cup W$ to node $n \in N_k$ and d_k^n is the departure time of train $k \in E \cup W$ from node $n \in N_k$, the events continuity constraints can be rewritten in the following way:

$$\sum_{l \in L_k^{\mathrm{I}(n)}} d_{kl} X_{kl} = a_k^n \quad (\forall n \in N_k, k \in E \cup W),$$

$$d_k^n = \sum_{l \in L_k^{\mathrm{O}(n)}} a_{kl} X_{kl} \quad (\forall n \in N_k, k \in E \cup W),$$

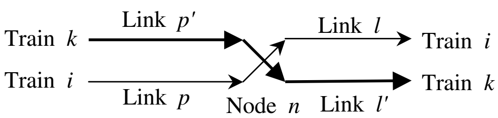


Fig. 11. Headway at node n for two eastbound trains.

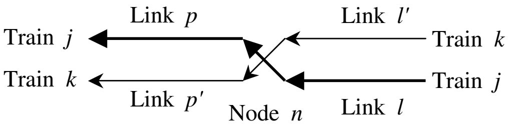


Fig. 12. Headway at node n for two westbound trains.

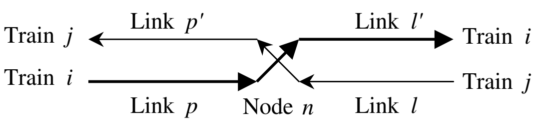


Fig. 13. Headway at node n for two trains of opposite directions.

$$a_k^n = d_k^n \quad (\forall n \in N_k, k \in E \cup W).$$

In addition, any headway constraint can be also rewritten as the following inequality:

$$\left|a_k^n - a_{k'}^n\right| \geqslant s \quad (\forall n \in N_k).$$

3.4.6. Lower and upper bound constraints

These constraints ensure that trains do not depart from their origin nodes earlier than the planned departure times and they do not arrive later to their destinations than some given time horizons. We suppose that train $k \in E \cup W$, according to its strategic train scheduling plan, can not depart from its origin node earlier than its planned time (d_k^*) and also must arrive to its destination before the end of a given time horizon (T), so

$$a_{k(1)} \geqslant d_k^*, \quad \forall k \in E \cup W,$$
$$d_{k(M(k))} \leqslant T, \quad \forall k \in E \cup W.$$

4. Solution of the multi-objective optimization model

A mathematical description of the multi-objective train scheduling problem is as follows:

$$\begin{aligned} &\text{Min} \quad F(\underline{x}) = \begin{bmatrix} \varphi_1(\underline{x}) \\ \varphi_2(\underline{x}) \end{bmatrix} \\ &\text{s.t.} \quad \underline{x} \in X, \end{aligned}$$

where $\underline{x}$ is an n-dimensional decision vector containing all binary and continuous variables, $F(\underline{x})$ is a 2-dimensional objective vector, $\varphi_1(\underline{x})$ and $\varphi_2(\underline{x})$ denote the first and second nonlinear objective functions, and X denotes the feasible set defined by the equality and inequality constraints specified before and by the explicit variable bounds.

In the case of conflicting objectives the optimality concept is replaced by Pareto optimality. A point $\underline{x}^* \in X$ is said to be *Pareto optimal* if and only if there is no $\underline{x} \in X$ such that $\varphi_i(\underline{x}) \leqslant \varphi_i(\underline{x}^*)$ for $i = 1, 2$, with at least one strict inequality. Typically, there are infinitely many Pareto optimal solutions, which form the *Pareto frontier* of the problem. Decision-makers usually select a particular Pareto solution based on additional preference information about the objectives.

The solution of the problem therefore consists of two steps. First the Pareto frontier is determined, and second, based on the obtained Pareto frontier detailed multi-objective optimization is performed.

4.1. Construction of the Pareto frontier

Although there is a large variety of methods in literature of multiple objective decision making to generate the Pareto frontier (such as variants of the *parametric method*, and the *adaptive search method*), in this study we use the *ε-constraint method*, since this method is very simple to apply and it is applicable to nonconvex feasible region problems as well. The *ε-constraint method* can be formulated as follows (see for example Hwang and Masud, 1979, or Szidarovszky et al., 1986):

$$\begin{aligned} \text{Min} \quad & \varphi_2(\underline{x}) \\ \text{s.t.} \quad & \varphi_1(\underline{x}) \leqslant \varepsilon, \\ & \underline{x} \in X, \end{aligned}$$

where ε is a parameter. With varying the value of ε systematically, the optimal solutions of this problem give the points of the Pareto frontier. By repeatedly relaxing the upper bound on φ_1, and re-optimizing φ_2 each time, the points (φ_1, φ_2) provide the Pareto frontier. This procedure is illustrated in Fig. 14.

4.2. Multiobjective analysis

For any multi-objective analysis we first need to normalize the objective functions. The value of the fuel consumption cost is given in monetary units whereas the trip time is measured on the passenger-time scale. Therefore these objectives cannot be compared directly. Normalization transforms them into a common satisfaction scale as follows:

$$\bar{\varphi}_1 = \frac{\varphi_1 - \varphi_{1\,\min}}{\varphi_{1\,\max} - \varphi_{1\,\min}} \in [0, 1],$$
$$\bar{\varphi}_2 = \frac{\varphi_2 - \varphi_{2\,\min}}{\varphi_{2\,\max} - \varphi_{2\,\min}} \in [0, 1],$$

where, $\bar{\varphi}_j$, $\varphi_{j\max}$, $\varphi_{j\min}$ are the normalized, the maximum and the minimum value of the jth objective, respectively. For computing $\varphi_{j\max}$ and $\varphi_{j\min}$, we solve single-objective optimization problems by maximizing and minimizing the corresponding jth objective function. Based on this normalization procedure a new Pareto frontier is formed in the $[0, 1]$ scale.

Distance-based methods find the feasible solution with the minimal distance from an "ideally best" point or with the maximum distance from an "ideally worst" point. In practical applications three distance types are usually selected. They are known as the l_1, l_2, and l_∞ distances. In our case the ideal point is $(0, 0)$ and the worst possible objective vector is $(1, 1)$. Therefore we use six methods to find the solution:

(1) $\text{Min}\, c_1\bar{\varphi}_1 + c_2\bar{\varphi}_2$

(2) $\text{Min}\, \sqrt{c_1\bar{\varphi}_1^2 + c_2\bar{\varphi}_2^2}$

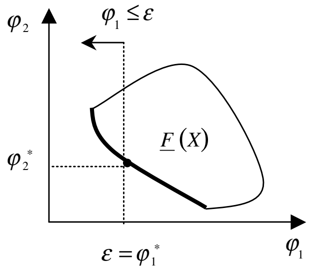


Fig. 14. Illustration of the ε-constraint method.

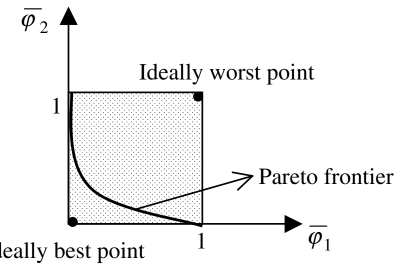


Fig. 15. Illustration of distance based methods.

(3) $\mathrm{Min}\,\mathrm{Max}\{c_1\bar{\varphi}_1, c_2\bar{\varphi}_2\}$
(4) $\mathrm{Max}\, c_1(1-\bar{\varphi}_1)+c_2(1-\bar{\varphi}_2)$
(5) $\mathrm{Max}\sqrt{c_1(1-\bar{\varphi}_1)^2+c_2(1-\bar{\varphi}_2)^2}$
(6) $\mathrm{Max}\,\mathrm{Max}\{c_1(1-\bar{\varphi}_1), c_2(1-\bar{\varphi}_2)\}$.

Here c_1, c_2 denote the importance weights of the two objectives. These weights are usually provided by the managers or decision makers of the train company. Another way is to transform travel times into monetary units. For more information on obtaining decision maker's preference weights see Szidarovszky et al. (1986) and for guidance on estimating the monetary value of travel times see Wardman (1998).

The first three methods look for a solution being as close as possible to the ideally best point and the last three methods seek a solution, which is as far as possible from the worst point (see also Fig. 15).

5. Numerical example

In this section, we first present a simple numerical example to illustrate the model and the solution methods. The example includes three trains without intermediate stop, two of which dispatch from west to east and the third one from east to west. The rail network used in the example is illustrated in Fig. 16. Table 2 shows the data.

In Fig. 17 we illustrate the Pareto frontier of the problem. The first part of Fig. 17 illustrates the Pareto frontier in its original scale and the normalized Pareto frontier is shown in the second part of the figure. Since the ideally best point and the worst points are ($\varphi_{1\,\mathrm{min}} = 11600$, $\varphi_{2\,\mathrm{min}} = 115879$) and ($\varphi_{1\,\mathrm{max}} = 36500$, $\varphi_{2\,\mathrm{max}} = 399198$), normalizing was done based on the following equations:

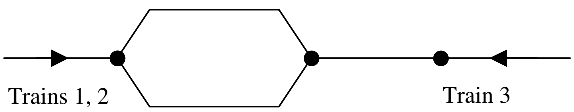


Fig. 16. Railroad network of the numerical example.

Table 2
Data used in numerical examples

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
t_{11}	10	$P_{2(1)}$	100	$t_{\beta_{2(1)}}$	0	T	100	N_3^{O}	3
t_{12}	8	$P_{3(1)}$	200	$t_{\beta_{3(1)}}$	0	R_0^1	1.5	N_1^{D}	3
t_{13}	15	$\alpha_{1(1)}$	0	d_1	40	R_0^2	1.5	N_2^{D}	3
t_{21}	10	$\alpha_{2(1)}$	0	d_2	30	R_0^3	1.5	N_3^{D}	1
t_{22}	8	$\alpha_{3(1)}$	0	d_3	50	R_1^1	1.5	$N_{1(1)}^{\mathrm{O}}$	1
t_{23}	15	$\beta_{1(1)}$	0	M_1	50	R_1^2	1.5	$N_{2(1)}^{\mathrm{O}}$	1
t_{31}	10	$\beta_{2(1)}$	0	M_2	50	R_1^3	1.5	$N_{3(1)}^{\mathrm{O}}$	3
t_{32}	8	$\beta_{3(1)}$	0	M_3	50	R_2^1	1.5	$N_{1(1)}^{\mathrm{D}}$	3
t_{33}	15	$t_{\alpha_{1(1)}}$	0	r_1	1	R_2^2	1.5	$N_{2(1)}^{\mathrm{D}}$	3
s	5	$t_{\alpha_{2(1)}}$	0	r_2	1	R_2^3	1.5	$N_{3(1)}^{\mathrm{D}}$	1
M	10^5	$t_{\alpha_{3(1)}}$	0	r_3	1	N_1^{O}	1		
$P_{1(1)}$	100	$t_{\beta_{1(1)}}$	0	c	1	N_2^{O}	1		

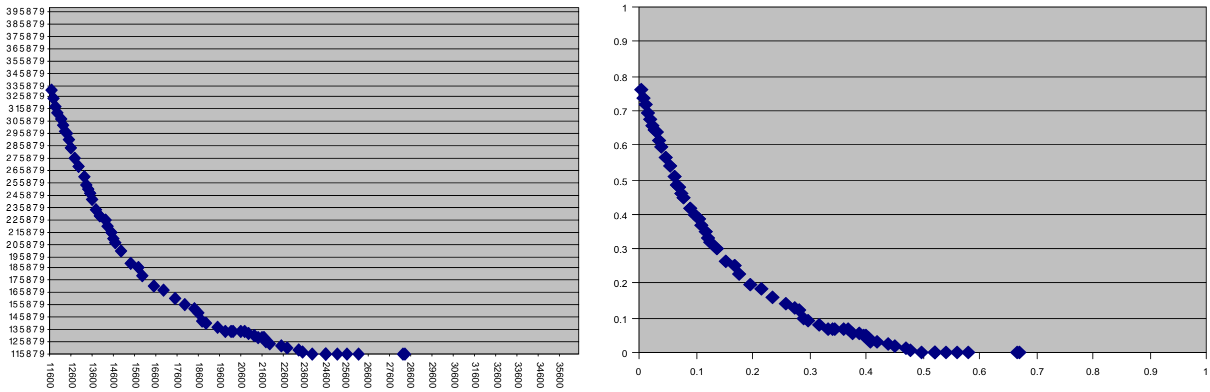


Fig. 17. Original and normalized Pareto frontiers.

$$\bar{\varphi}_1 = \frac{\varphi_1 - 11600}{36500 - 11600} \quad \text{and} \quad \bar{\varphi}_2 = \frac{\varphi_2 - 115879}{399198 - 115879},$$

where φ_1 is the passenger-time function, and φ_2 is the cost function.

Table 3 shows the 70 obtained Pareto optimal solutions. With the systematic variation of the weights all of the six distance based methods were applied based on the obtained Pareto frontier. Details of the solutions are shown in Table 4. Figs. 18 and 19 show the sensitivity of the first and second objective functions to the six applied methods.

As it is clear from Figs. 18 and 19, the first method ($\mathrm{Min}\, c_1\bar{\varphi}_1 + c_2\bar{\varphi}_2$) and the fourth method ($\mathrm{Max}\, c_1(1-\bar{\varphi}_1) + c_2(1-\bar{\varphi}_2)$) give the same result, since these methods are equivalent to each other (and also to the weighting method). The solution by using the sixth method ($\mathrm{Max}\,\mathrm{Max}\{c_1(1-\bar{\varphi}_1), c_2(1-\bar{\varphi}_2)\}$) depends only on the relation between the weights c_1 and c_2. The solution for the case of $c_1 < c_2$ is $(0.547879, 0)$, whereas the solution obtained for the case of

Table 3
Pareto optimal solution

No.	Obj. 1	Obj. 2	Norm obj. 1	Norm obj. 2
1	11700	332021	0.00402	0.762893
2	11800	324566	0.00803	0.73658
3	11900	318506	0.01205	0.71519
4	12000	312851	0.01606	0.69523
5	12100	307490	0.02008	0.676308
6	12200	302404	0.0241	0.658357
7	12300	297576	0.02811	0.641316
8	12400	296484	0.03213	0.637462
9	12500	290485	0.03614	0.616288
10	12600	284372	0.04016	0.594711
11	12800	276250	0.04819	0.566044
12	13000	268560	0.05622	0.538901
13	13200	260251	0.06426	0.509574
14	13300	253544	0.06827	0.485901
15	13400	251197	0.07229	0.477617
16	13500	246992	0.07631	0.462775
17	13600	242984	0.08032	0.448629
18	13800	233564	0.08835	0.41538
19	14000	228647	0.09639	0.398025
20	14200	224745	0.10442	0.384252
21	14350	220727	0.11044	0.37007
22	14538.6	215727	0.11802	0.352423
23	14600	210414	0.12048	0.33367
24	14700	206749	0.1245	0.320734
25	15000	200666	0.13655	0.299263
26	15400	190594	0.15261	0.263713
27	15800	187370	0.16867	0.252334
28	16000	179620	0.17671	0.22498
29	16500	170892	0.19679	0.194173
30	17000	168325	0.21687	0.185113
31	17500	161851	0.23695	0.162262
32	18000	156057	0.25703	0.141812
33	18450	152517	0.2751	0.129317
34	18600	149843	0.28112	0.119879
35	18800	143114	0.28916	0.096128
36	19000	141440	0.29719	0.09022
37	19500	137625	0.31727	0.076754
38	19900	135243	0.33333	0.068347
39	20100	135243	0.34137	0.068347
40	20200	135243	0.34538	0.068347
41	20564.1	135220	0.36	0.068266
42	20800	134532	0.36948	0.065837
43	21000	132374	0.37751	0.058221
44	21250.7	131374	0.38758	0.054691
45	21437.4	130154	0.39508	0.050385
46	21600	129232	0.40161	0.047131
47	21650	128832	0.40361	0.045719

(*continued on next page*)

Table 3 (*continued*)

No.	Obj. 1	Obj. 2	Norm obj. 1	Norm obj. 2
48	21682.3	128632	0.40491	0.045013
49	21800	125427	0.40964	0.033701
50	22000	124393	0.41767	0.030051
51	22500	122056	0.43775	0.021802
52	22800	121105	0.4498	0.018446
53	23300	118926	0.46988	0.010755
54	23500	118279	0.47791	0.008471
55	24000	115879	0.49799	0
56	24600	115879	0.52209	0
57	25100	115879	0.54217	0
58	25600	115879	0.56225	0
59	26100	115879	0.58233	0
60	24600	115879	0.52209	0
61	24575	115879	0.52108	0
62	28300	115879	0.67068	0
63	24600	115879	0.52209	0
64	24600	115879	0.52209	0
65	24600	115879	0.52209	0
66	24600	115879	0.52209	0
67	24600	115879	0.52209	0
68	24600	115879	0.52209	0
69	28200	115879	0.66667	0
70	24600	115879	0.52209	0

$c_1 \geqslant c_2$ is $(0, 0.779952)$. The fifth method $(\text{Max}\sqrt{c_1(1-\bar{\varphi}_1)^2 + c_2(1-\bar{\varphi}_2)^2})$ for all values of $c_1 \in [0, 0.3] \cup [0.7, 1]$ is approximately insensitive. The solution obtained by this method for $c_1 \in [0, 0.3]$ is $(0.54879, 0)$ and for $c_1 \in [0.7, 1]$ is $(0, 0.77995)$. The second method $(\text{Min}\sqrt{c_1\bar{\varphi}_1^2 + c_2\bar{\varphi}_2^2})$ in comparison to the third one $(\text{Min}\,\text{Max}\{c_1\bar{\varphi}_1, c_2\bar{\varphi}_2\})$ for $c_1 \in [0, 0.4)$ has more sensitivity to weight changes, whereas the third method is more sensitive for $c_1 \in [0.4, 1]$.

To test the accuracy of the model and results we worked out 21 numerical examples which are shown in Table 5 in which n_l and n_t denote the number of links in the rail network, and the number of trains dispatching in the planning horizon. Half of the trains in each case study are selected eastbound and the rest westbound. The problems are solved by the Lingo v7.0 optimization software. All results are produced on a Pentium 2 with 300 MHz speed.

A real-world problem may include thousands of links in its rail network as well as hundreds of dispatching trains in a given time horizon. Although these numerical examples can be interpreted as segments of a real-world problem they will illustrate the applicability of the models and solution methodology. It is interesting that computation time is more sensitive to the number of trains than to the number of links in the network.

6. Conclusion

This paper developed a multi-objective optimization model for the passenger train-scheduling problem on a railroad network including single and multiple tracks, and multiple platforms with

Table 4
Multiobjective solutions

	(c_1, c_2)										
	0, 1	0.1, 0.9	0.2, 0.8	0.3, 0.7	0.4, 0.6	0.5, 0.5	0.6, 0.4	0.7, 0.3	0.8, 0.2	0.9, 0.1	1, 0
Method 1											
Objective 1	0.54788	0.547879	0.490761	0.388498	0.31257	0.24949	0.18714	0.1219	0.042594	0	0
Objective 2	0	0	0.012312	0.046223	0.08748	0.139304	0.21557	0.3379	0.583459	0.779936	0.77995
Method 2											
Objective 1	0.54788	0.344439	0.296154	0.264346	0.23937	0.217076	0.19497	0.17118	0.144101	0.107325	0
Objective 2	0	0.067922	0.099101	0.12522	0.1497	0.175086	0.2042	0.24069	0.290022	0.373545	0.77995
Method 3											
Objective 1	0.54788	0.394194	0.322914	0.273466	0.23371	0.198822	0.16614	0.1336	0.09894	0.05822	0
Objective 2	0	0.043799	0.080728	0.1172	0.1558	0.198822	0.24921	0.31174	0.39576	0.52398	0.77995
Method 4											
Objective 1	0.54879	0.54879	0.490761	0.388498	0.31257	0.24949	0.18714	0.1219	0.042594	0	0
Objective 2	0	0	0.012312	0.046223	0.08748	0.139304	0.21557	0.3379	0.583459	0.779942	0.77994
Method 5											
Objective 1	0.54879	0.54879	0.54879	0.53071	0.39196	0.284368	0.17446	0	0	0	0
Objective 2	0	0	0	0.003324	0.04474	0.108185	0.2353	0.77995	0.779952	0.779952	0.77995
Method 6											
Objective 1	0.547879	0.547879	0.547879	0.547879	0.547879	0	0	0	0	0	0
Objective 2	0	0	0	0	0	0.779952	0.77995	0.77995	0.779952	0.779952	0.77995

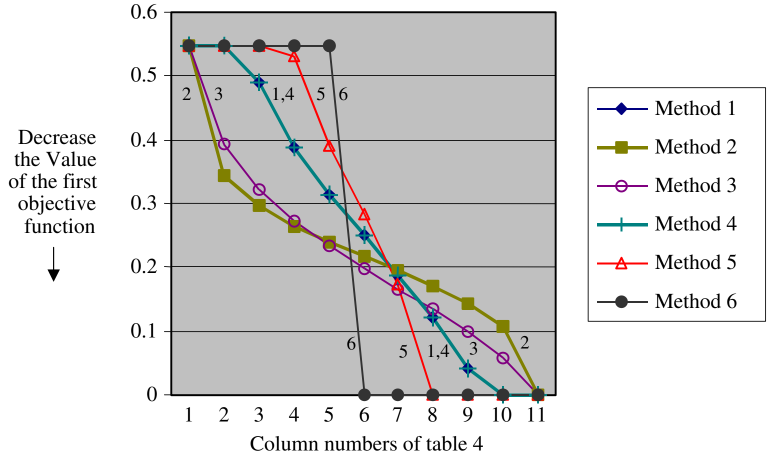


Fig. 18. Sensitivity of φ_1.

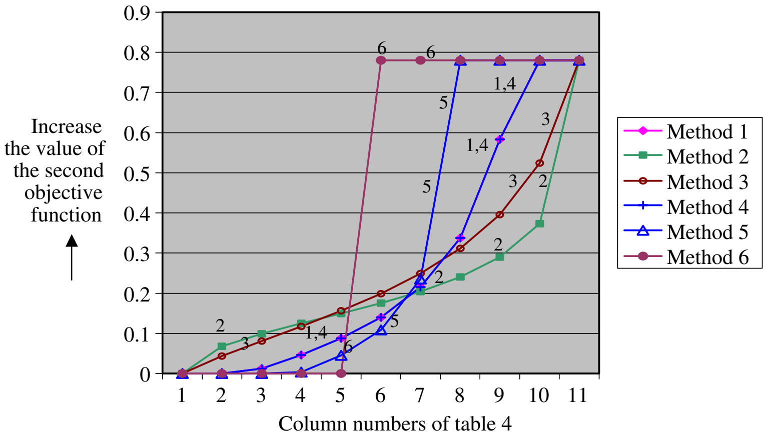
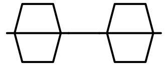
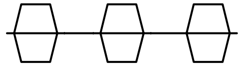
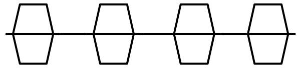
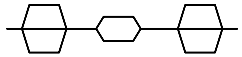
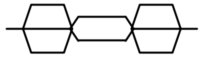
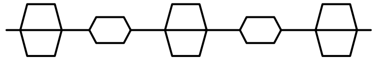
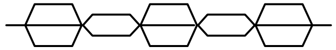
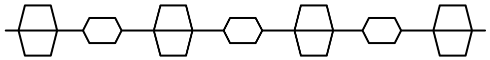
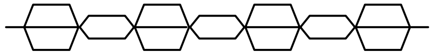
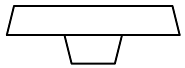
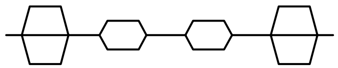
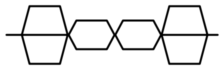


Fig. 19. Sensitivity of φ_2.

different train capacities. In this study, lowering the fuel consumption cost was the measure of satisfaction of the railway company and shortening the total passenger-time was regarded as the passenger satisfaction criterion. The solution of the problem consisted of two steps. First the Pareto frontier was determined by using the ε-constraint method, and second, based on the obtained Pareto frontier detailed multi-objective optimization was performed in which distance-based methods were used with three types of distances. A simple numerical example was given to illustrate the model, solution method, and results. The sensitivity of the solutions was also examined. Results of 21 worked numerical examples were given to show the applicability of the models and solution procedure.

Table 5
Results of worked numerical examples

Case no.	Problem size ($n_l \times n_t$)	Run time	Number of variables	Number of constraints	Figure of rail networks applied in the case studies
1	7×2	00:00:09	83	39	
2	7×4	00:02:28	221	147	
3	7×6	00:05:23	415	325	
4	11×2	00:01:21	129	61	
5	11×4	00:02:35	345	231	
6	11×6	00:04:25	649	511	
7	15×2	00:02:15	175	83	
8	15×4	00:03:29	469	315	
9	15×6	00:43:12	883	697	
10	10×2	00:00:01	117	57	 or 
11	10×4	00:02:00	313	213	
12	17×2	00:00:21	197	97	 or
					
13	17×4	00:09:47	529	363	
14	24×2	00:01:57	277	137	
					
15	24×4	00:15:30	745	513	
16	5×2	00:00:03	61	33	
17	5×4	00:00:31	161	115	
18	5×6	00:09:29	301	247	
19	4×4	00:01:41	187	135	
20	6×6	00:31:12	212	178	
21	13×4	00:15:05	405	279	 or 

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