Robust Train Timetabling Problem: Mathematical Model and Branch and Bound Algorithm

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***Abstract*—This paper illustrates the results of an investigation into developing a new robust train-timetabling problem in a single-track railway line. The proposed model is formulated as a robust form of the mixed integer approach. A branch-and-bound (B&B) algorithm, along with a new heuristic beam search (BS) algorithm, is presented to solve the model for large-scale problems in reasonable time. We also propose two different methods to measure the required buffer times under the assumption of unknown and known distribution functions of disturbances. We have generated some random instances, and the efficiency of the B&B and BS algorithms are demonstrated by comparing the results with common software packages as well as a new lower bound method. The results demonstrate that the B&B algorithm can find optimum solutions in a shorter amount of time compared with common software packages such as Lingo. Moreover, the BS algorithm can effectively find a near-optimum solution in a rational amount of time.**

***Index Terms*—Branch-and-bound (B&B) algorithm, disturbance, robustness, train scheduling.**

I. Introduction

There are many different practical cases in optimization problems in which a small alteration of the data can make the optimal solution virtually infeasible. In train timetabling, the word "disturbance" is a common term for this phenomenon and is formally defined as the mistakes, malfunctions, or deviating conditions that occur in the railway system or its environment and influence the railway traffic. In train-timetabling problems and real-time traffic management, two kinds of delay are investigated: 1) primary delays, which are directly caused by disturbances; and 2) secondary delays or knock-on delays, which are caused by delays of earlier trains. According to these definitions, delay propagation is the phenomenon in which a secondary delay occurs. To prevent the propagation of delays among the trains, the time interval among the occupation of block sections by trains, which are called buffer times, might need to be increased. The performance of a schedule, which does not significantly reduce under the condition of disturbance occurrences, is known as robustness, and the schedule is called robust. In this paper, the robustness of a railway network indicates how much the system is affected by disturbances. According to these definitions, the performance of a robust schedule should not be sensitive to disturbances.

A. Previous Works

During the past decade, the train-scheduling problem has become one of the most interesting research topics. Zhou and Zhong [1] introduced a modified branch-and-bound (B&B) algorithm, which contains three methods to reduce the solution space. B&B algorithms are also presented by Walker *et al.* [2], Zhou and Zhong [3], and D'Ariano *et al.* [4].

In addition to exact algorithms, Caprara *et al.* [5] studied a double-track railway line and presented a heuristic algorithm based on the Lagrange relaxation method. Cacchiani *et al.* [6] presented exact and heuristic algorithms to solve the train-timetabling problem based on the solution of the linear programming (LP) relaxation of an integer linear programming (ILP) formulation. Fischetti *et al.* [7] exhibited four different methods to find robust solutions. The proposed procedure contains two steps: 1) generating an optimal timetable and 2) finding a robust solution, given fixed event precedences. This approach is more carefully investigated in [8]. Vromans *et al.* [9] considered the shared use of the same infrastructure by different railway services, origins, and, destinations, different speeds, and different halting patterns, as the main keys of heterogeneity of the timetables, which would result in propagation of delays throughout the network.

Carey [10] presented some heuristic measures of stability of train timetables in two different categories including using and not using probability of disturbances. Vansteenwegen and Oudheusden [11] computed ideal buffer times in connections based on the delay distributions of the arriving trains and the weighting of different types of waiting times. Khan and Zhou [12] proposed a two-stage stochastic recourse model for the double-track train-timetabling problem. Their proposed method is based on adding additional time supplements to the traveling times of trains, as well as the dwell times, to achieve a robust timetable. Liebchen and Stiller [13] mathematically justified the sampling approach for the aperiodic delay resistant timetabling problem. Liebchen *et al.* [14] studied the construction of delay-resistant periodic timetables. Fischetti and Monaci

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[15] investigated a heuristic method to model uncertainty through a modeling framework based on combining the robust optimization with a simplified two-stage stochastic programming approach. D'Angelo *et al.* [16] proposed an algorithm for solving the robust timetabling problem. The algorithm ensures that, if a delay occurs, no more than a specific number of activities are influenced by the propagation of such a delay. Shafia *et al.* [17] proposed a fuzzy approach to reach the required robustness in train-timetabling problem.

To handle the disturbances, some authors proposed to use decision support systems [18]–[20].

Generating robust timetables in the planning stage may not be sufficient to absorb all potential disturbances; as a result, real-time scheduling methods are applied in practice. For example, D'Ariano *et al.* [21] studied real-time scheduling, considering train speed coordination. Furthermore, a new concept is recently defined, which is called recoverable robust timetable [22], [23]. The idea is that, in addition to robustness concept during the planning phase, the recovery features that might be applied at runtime must also be considered in a way that the disturbances can be easily managed.

B. Contribution of the Paper

In this paper, a new formulation for the robust train-timetabling problem is presented. The proposed model is capable of absorbing disturbances, which normally happen during the traveling of trains. As the common software packages are able to solve only small-size instances (i.e., problems of scheduling up to ten trains in ten block sections) in a reasonable amount of time, a B&B algorithm is proposed to efficiently solve the examples. We present two novel computational methods to compute the buffer times. The first one is based on the unknown distribution functions of disturbances, and the second one is based on the stochastic behavior of disturbances when the input data are perturbed. To find some good solutions that are not guaranteed to be optimal, i.e., near-optimal solutions in a limited amount of time, a heuristic beam search (BS) method is presented.

C. Outline

This paper is organized as follows: In Section II, a mathematical model is presented, along with the capacity consumption concept. In Section III, after presenting the robust optimization approach, the robust train-timetabling model is proposed. In Section IV, two novel methods are proposed to compute the required buffer times to reach the desired level of robustness. Section V deals with the explanation of the B&B, along with a heuristic BS algorithm. In Section VI, the validity of the proposed algorithms, as well as their effectiveness, are demonstrated. Finally, the concluding remarks are given at the end to summarize the contribution of this paper.

II. Mathematical Model

The train-timetabling problem is defined as follows: A railway network consists of m block sections with n trains passing the block sections one after another. Given the routes of trains, the block section occupation time of trains, the origins and the destinations, and the importance weight of trains, it is necessary to find the arrival and the departure times of trains to/from block sections to optimize the desired objective function and ensure that all the operational and safety requirements are satisfied.

The applied notations in this paper are illustrated in Table I.

The proposed mathematical model is presented as follows:

A. Mathematical Model

$$\min Z = \sum_{i \in T} w_i \times \left(t_{ie_i} + \sum_{s=1}^{|S_{e_i}|} dep_{ie_i s} \right). \tag{1}$$

The objective function shown in (1) minimizes the sum of the weighted arrival times of trains to their destinations. Note that, based on the following constraints, there exist only one $s \in \{1, \ldots, |S_{e_i}|\}$, where $dep_{ie_i s} \neq 0$, subject to

$$\sum_{s=1}^{|S_j|} d_{ijs} = 1 \quad \forall i \in T, j \in B_i. \tag{2}$$

Equation (2) ensures that just one sequence is assigned to train i in passing block section j

$$\sum_{i \in S_j} d_{ijs} = 1 \quad \forall j \in B_i, s \in \{1, 2, \ldots, |S_j|\}. \tag{3}$$

Equation (3) guarantees that at most one train can pass block section j in sequence s

$$dep_{ijs} \leq M \times d_{ijs} \quad \forall i \in T, j \in B_i, s \in \{1, 2, \ldots, |S_j|\}. \tag{4}$$

Inequality (4) establishes the relation of d_{ijs} and dep_{ijs} variables. In other words, dep_{ijs} is assigned a positive value only if the binary variable d_{ijs} is equal to 1, i.e.,

$$\sum_{s=1}^{|S_{o_i}|} dep_{io_i s} \geq od_i \quad \forall i \in T. \tag{5}$$

Inequality (5) ensures that the departure time of trains from their origins cannot be earlier than one that is predetermined, i.e.,

$$\sum_{s=1}^{|S_j|} dep_{ijs} + t_{ij} \leq \sum_{s=1}^{|S_{j'}|} dep_{ij's} \quad \forall i \in T, j \in B_i - \{e_i\} \tag{6}$$

where j' is the block section, which is met by train i after block section j. Inequality (6) provides the condition that trains must pass block sections one after another based on the predetermined routes, i.e.,

$$\sum_{s=1}^{|S_{j'}|} dep_{ij's} - \left(\sum_{s=1}^{|S_j|} dep_{ijs} + t_{ij} \right) \leq \xi_{ij} \quad \forall i \in T, j \in B_i - \{e_i\}. \tag{7}$$

TABLE I
LIST OF EMPLOYED SYMBOLS

Symbol	Definition
(i,j)	A doublet which represents train i, and block section j, and indicates the transit of train i from block section j, here in after called as transit doublet.
ANL	Active Node List
arr_{ij}^{v}	The arrival time of train i to the end of block section j at node v.
B_i	Set of block sections which must be passed by train i
b_{ij}^{v}	i-th existing buffer time in block section j at node v
b_{sj}	s-th existing buffer time in block section j
BS	Set of block sections
D_{sj}	Delay in departure of s-th train from beginning of block section j
d_{ijs}	A binary variable which equal to 1, if train i passes block section j in sequence s, *and* 0 otherwise
dep_{ijs}	Denotes the departure time of train i from the beginning of block section j, where train i is in sequence s in passing block section j
dep_{ij}^{v}	The departure time of train i from beginning of block section j at node v.
$d\varepsilon_{ij}$	Differential of ε_{ij}
e_i	The last block section which must be met by train i
$E(D_{sj})$	Expected value of D_{sj}
$f(\varepsilon_{ij})$	*pdf* of ε_{ij}
i	Train index
j	Block section index
k	Counter
la_{j}^{v}	The arrival time associated with the last train that passes block section j at node v
LB_v	The lower bound at node v
M	A large positive number
ma_v	The minimum arrival times of trains associated with the doublets belong to Ω_v, at node v
mb_{ij}^{v}	Minimum required buffer time before train with sequence i in passing block section j at node v
mb_{sj}	Minimum required buffer time before travel of s-th train in block section j, which is computed based on the desired level of robustness.
o_i	The first block section which must be met by train i
od_i	The pre-determined departure time of train i from its origin
s	Sequence index defined for each train in passing block sections
S_j	Set of trains which pass block section j
sq_{j}^{v}	The sequence of the last train which is passed block section j at node v
T	Set of trains
t_{ij}	Occupation time of block section j by train i
TWT_v	The total weighted tardiness at node v
UB	The upper bound
w_i	The importance weight of train i
ε_{ij}	A random variable that represents the disturbance occurrence of train i in passing block section j.
$\bar{\varepsilon}_{ij}$	A random variable that represents the delay occurrence of train i in passing block section j, disregarding the potential remaining disturbances during previous block sections.
ζ_i	A threshold which specifies the maximum allowable travelling time of train i
ξ_{ij}	A threshold which specifies the maximum allowable dwell time of train i at the end of block section j
v	Node Index
Ω_v	A set which consists of at last $\lvert T\rvert$ transit doublets each of which shows the trains and the next associated block sections which must be passed, at node v.
Ω'_v	A set which contains those transit doublets which originates a new branch at node v, *and* $\Omega'_v \in \Omega_v$.

Inequality (7) ensures that the occupation time of train i in block section j cannot be more than a threshold. In a busy rail network, it is very time consuming and frustrating for passengers to stop in a station for a long period of time without any acceptable reason, i.e.,

$$\sum_{s=1}^{|S_{e_i}|} dep_{ie_is} + t_{ie_i} - od_{io_i} - \sum_{j\in B_i} t_{ij} \le \zeta_i \quad \forall i \in T. \tag{8}$$

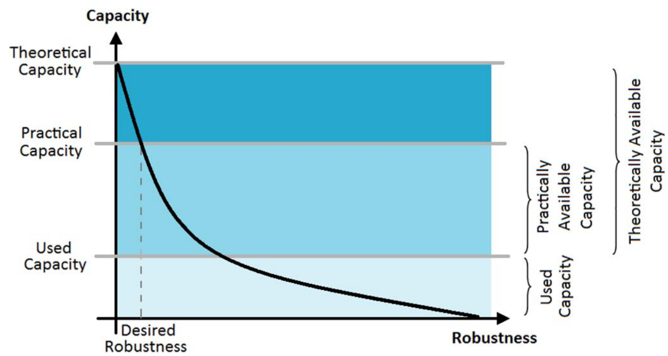

Fig. 1. Different types of capacity.

Inequality (8) guarantees that the total delays of a specific train cannot be more than a threshold. By this inequality, it is intended to generate smooth timetables in which all trains with the same characteristics have similar traveling times, i.e.,

$$\sum_{i\in S_j} dep_{ijs} + \sum_{i\in S_j}(d_{ijs}\times t_{ij}) \leq \sum_{k\in S_j} dep_{kjs+1} \qquad j\in BS, s\in[1,|S_j|-1]. \quad (9)$$

Inequality (9) provides the safety movements in passing block sections. In other words, train k with sequence number $s+1$ can only enter block section j if train i with sequence number s has left the block section

$$dep_{ijs} \geq 0 \quad \forall i\in T, j\in B_i, s\in[1,|S_j|] \quad (10)$$

$$d_{ijs}\in\{0,1\} \quad \forall i\in T, j\in B_i, s\in[1,|S_j|]. \quad (11)$$

Remark 1: The preceding problem can also be formulated in such a form that the distinct index for presenting the sequence of trains in block sections is disregarded. In other words, one can formulate the problem in a way that variable dep_{ijs} would be replaced with dep_{ij}, and therefore, the number of variables would be considerably reduced. However, the main reason for such formulation is illustrated in applying a robust approach to the model introduced in the next section, where the order of trains passing the block sections, as well as the existing time interval among them, is exploited to compute the required buffer times.

B. Capacity Consumption

Theoretical capacity is defined as the number of trains that could run over a railway line, during a specific amount of time, in a strict generated environment. Moreover, practical capacity is considered under more realistic assumptions, where the minimum desired robustness is applied for providing smooth and robust train running (see Fig. 1).

As described in Fig. 1, by increasing the desired robustness level, the practical capacity is reduced. On the other hand, constraints 7 and 8 define the minimum thresholds to maintain the passengers' satisfaction. Therefore, the capacity utilization (i.e., the number of trains going to be scheduled) could be increased to an extent where a certain level of robustness and a certain level of passengers' satisfaction are provided.

In the next section, after a brief explanation of a robust approach, we expand to the proposed model to arrive at a robust train-timetabling mathematical model.

III. Robust Train-Timetabling Model

Before presenting the robust train-timetabling model, suppose that a disturbance has occurred for one train in passing a block section. Depending on the size of disruption and the existing buffer times among departure and arrival times of trains from the beginning and to the end of this block section. The disturbance not only affected this train but also may be propagated to other trains as well. In this section, it is intended to propose the methods of absorbing the effect of disturbances on trains.

Generally, robust optimization ensures the feasibility and the optimality of the solution for the worst-case values of parameters. In this case, we accept a suboptimal solution for the nominal values of the data to be sure that the solution remains feasible when the data changes. Soyster [24], Ben-Tal and Nemirovski [25]–[27], El Ghaoui and Lebret [28], and El Ghaoui *et al.* [29] have applied robust optimization to linear programming problems. Bertsimas and Sim [30] introduced a robust approach where the robust counterpart is of the same size and class as the nominal problem, i.e., if the nominal problem is linear/mixed integer, the final robust model will remain in the same shape. By this approach, it is possible to control the degree of conservatism for every constraint and the feasibility of the robust optimization problem is guaranteed. Considering the advantages of the Bertsimas and Sim method, the authors have used this idea for the mathematical model presented in Section II. For details about the Bertsimas and Sim approach, see [30]. Shafia *et al.* [31] applied the Bertsimas and Sim robust approach to the problem of train routing and makeup. The idea is also extended to the Job shop scheduling problem by Shafia *et al.* [32].

Considering the recent methods to find reliable travel times [33] is not sufficient in a timetabling problem. Therefore, investigating the disturbances that affect the block section occupation times is vital. For this reason, it is required to investigate those constraints that contain block section occupation time t_{ij} as an input, i.e., (6) and (9). In this paper, the Bertsimas and Sim robust optimization approach is applied on (9), and the results can be extended to (6) as well.

Applying the Bertsimas and Sim robust approach leads to the inclusion of some buffer times in each constraint. The amounts of buffer times are dependent on uncertain parameters, which exist in the defined constraint.

To reduce the effects of potential delay propagation among the trains, the relative position of all trains must be considered. In the next section, this fact is proved, considering the stochastic behavior of disturbances. It is concluded that, to apply the robust optimization approach, some constraints should be added to protect trains against possible disturbances of all trains with earlier departure times.

According to the preceding explanation, at the first step, (9) must be extended to the following inequality:

$$\sum_{i\in S_j} dep_{ijs'} + \sum_{i\in S_j}\sum_{s''=s'}^{s-1}(d_{ijs''}\times t_{ij}) \leq \sum_{k\in S_j} dep_{kjs}$$
$$j\in BS, s'\in[1,|S_j|-1], s\in[s'+1,|S_j|] \quad (12)$$

so that, for each block section, a constraint corresponds to each pair of train sequences, i.e., s', and s. It can be found that (12) results in generating $\binom{|S_j|}{2}$ combination of constraints, whereas (9) contains $|S_j|-1$ constraints.

In the next step based on the Bertsimas and Sim approach, the authors propose the robust train-timetabling model proposed by Bertsimas and Sim [30]. As we have already explained, the disturbances normally happen in the block section occupation times of trains. It is assumed that each uncertain coefficient $\widetilde{t}_{ij}$ that represents the true value of the occupation time of train i in passing block section j independently take values according to a symmetric distribution with mean equal to the nominal value t_{ij} and of half length $\hat{t}_{ij}$. In other words, $\widetilde{t}_{ij}$ belongs to the interval $[t_{ij}-\hat{t}_{ij}, t_{ij}+\hat{t}]_{ij}$.

Consider (12), and let $Q_{j,s',s}$ represent the set of trains which pass block section j with a sequence number which belongs to the set $\{s', s'+1,\ldots,s\}$.

In Soyster's approach [24], it is assumed that all uncertain parameters take the worst possible values. This approach leads to achieving the highest protection, but the objective function is determined in the worst-case condition. In real-world applications, it is unlikely to suppose that all uncertain parameters are equal to their worst-case bound; as a result, parameter $\Gamma_{j,s',s}$, $0\leq\Gamma_{j,s',s}\leq|Q_{j,s',s}|$, is used to adjust the conservatism level of the final solution. It means that, at last, only $\lfloor\Gamma_{j,s',s}\rfloor$ unit of trains that belong to the set $Q_{j,s',s}$ are affected by disturbances, and one train (e.g., $k_{j,s',s}\in Q_{j,s',s}$) would be perturbed at most $(\Gamma_{j,s',s}-\lfloor\Gamma_{j,s',s}\rfloor)\hat{t}_{k_{j,s',s},j}$. In other words, it is assumed that only those trains that belong to set $K_{j,s',s}$, where $K_{j,s',s}\subseteq Q_{j,s',s}$ and $|K_{j,s',s}|=\lfloor\Gamma_{j,s',s}\rfloor$, in addition to train $k_{j,s',s}$, where $k_{j,s',s}\in Q_{j,s',s}$ and $k_{j,s',s}\notin K_{j,s',s}$, are disturbed in practice during passing block section j. Finally, the robust formulation of (12) is achieved by simply adding the statement (13), shown at the bottom of the page, which is known as a *protection function*, to the left side of (12). The proposed protection function is to consider maximum possible deviation regarding the amount of parameter $\Gamma_{j,s',s}$. By this alteration, the resulting robust constraint would be nonlinear. In the next step, the proposed protection function is restated as the following model:

$$\max \sum_{i\in Q_{j,s',s-1}}\left(\sum_{s''=s'}^{s-1}(d_{ijs''}\times\hat{t}_{ij})\times z_{i,j,s',s-1}\right)$$
$$\text{subject to}: \quad \sum_{i\in Q_{j,s',s-1}} z_{i,j,s',s-1}\leq\Gamma_{j,s',s-1}$$
$$0\leq z_{i,j,s',s-1}\leq 1 \quad \forall i\in Q_{j,s',s-1}. \quad (14)$$

Note that the optimal solution of model (14) consists of $\lfloor\Gamma_{j,s',s}\rfloor$ variables at 1 and one variable at $\Gamma_{j,s',s}-\lfloor\Gamma_{j,s',s}\rfloor$. Therefore, it is concluded that the proposed protection function is equal to the objective function of model (14).

The dual of model (14) is shown as the following model:

$$\min \sum_{i\in S'_{j,s-1}} p_{i,j,s',s-1}+\Gamma_{j,s',s-1}\times z'_{j,s',s-1}$$

Subject to :

$$z'_{j,s',s-1}+p_{i,j,s',s-1}\geq\sum_{s''=s'}^{s-1}(d_{ijs''}\times\hat{t}_{ij}) \quad \forall i\in Q_{j,s',s-1}$$
$$p_{i,j,s',s-1}\geq 0 \quad \forall i\in Q_{j,s',s-1}$$
$$z'_{j,s',s-1}\geq 0. \quad (15)$$

Note that $z'_{j,s',s-1}$ and $p_{i,j,s',s-1}$ are the dual variables of the first and second constraints of model (14), respectively. Finally, instead of using the nonlinear robust constraint achieved by embedding the proposed protection function (13) to the left side of (12), one can use linear constraints, i.e.,

$$\sum_{i\in S_j} dep_{ijs'}+\sum_{i\in S_j}(d_{ijs}\times t_{ij})+\sum_{i\in Q_{j,s',s-1}} p_{i,j,s',s-1}+\Gamma_{j,s',s-1}\times z'_{j,s',s-1}\leq\sum_{k\in S_j} dep_{kjs},$$
$$j\in BS, s'\in[1,|S_j|-1], s\in[s'+1,|S_j|] \quad (16)$$
$$z'_{j,s',s-1}+p_{i,j,s',s-1}\geq\sum_{s''=s'}^{s-1}(d_{ijs''}\times\hat{t}_{ij})$$
$$\forall i\in Q_{j,s',s-1}, j\in BS, s'\in[1,|S_j|-1], s\in[s'+1,|S_j|] \quad (17)$$
$$p_{i,j,s',s-1}\geq 0$$
$$\forall i\in Q_{j,s',s-1}, j\in BS, s'\in[1,|S_j|-1], s\in[s'+1,|S_j|] \quad (18)$$
$$z'_{j,s',s-1}\geq 0, \forall j\in BS, s'\in[1,|S_j|-1], s\in[s'+1,|S_j|] \quad (19)$$

$$\max_{\left\{K_{j,s',s-1}\cup\{k_{j,s',s-1}\}\,\middle|\,K_{j,s',s-1}\subseteq Q_{j,s',s-1},|K_{j,s',s-1}|=\lfloor\Gamma_{j,s',s-1}\rfloor,k_{j,s',s-1}\in Q_{j,s',s-1},k_{j,s',s-1}\notin K_{j,s',s-1}\right\}}$$
$$\left\{\sum_{i\in K_{j,s',s-1}}\sum_{s''=s'}^{s-1}(d_{ijs''}\times\hat{t}_{ij})+(\Gamma_{j,s',s-1}-\lfloor\Gamma_{j,s',s-1}\rfloor)\times\sum_{s''=s'}^{s-1}(d_{k_{j,s',s-1},js''}\times\hat{t}_{k_{j,s',s-1},j})\right\} \quad (13)$$

which are achieved through combining model (15) and (12). For more details, see [30]. The role of $\Gamma_{j,s',s-1}$ is to adjust the robustness against the level of conservatism of the solution. Considering block section j and trains with the sequence numbers $\{s', s'+1, \ldots, s-1\}$, (16)–(19) provide the condition that, if $\Gamma_{j,s',s-1}$ trains are affected by disturbances, the timetable remains robust by a probability of 100%. We have experimentally proposed index: $\Gamma_{j,s',s-1} = (s-1-s') \times \alpha \times \beta^{(s-1-s')}$, where α and β are the adjusting parameters and $0 < \alpha, \beta \leq 1$.

Parameters α and β are set according to the desired level of robustness. Simply by increasing the value of these parameters, the resulted timetable will be more robust. By adjusting $\alpha = \beta = 1$, the most conservatism timetable will be achieved.

Solving the mathematical model by common software packages such as Lingo, CPLEX, or GAMS is very time consuming, even for solving small-size instances. Thus, to have a more powerful tool to generate robust train timetables, a B&B algorithm is proposed. In the next section, we show how the required buffer times are computed to reach a known level of robustness. Then, in Section V, the proposed methods are used in the B&B algorithm.

IV. Computation of Buffer Times

To have a timetable that is benefited by a known level of robustness, two methods are presented to compute the required amount of buffer times. Generally, the amount of buffer time is directly dependent on the desired level of robustness. In the remaining part of this section, the authors, to control the robustness level (i.e., to adjust the conservatism level) of timetables, propose some methods that are simple to tune and compute.

A. First Method

The first method relies on the same assumptions considered in the presented robust model. To that end, each buffer time can be computed by the proposed protection function.

B. Second Method

In the first method, it was assumed that there is no information about the distribution function of disturbances. However, finding an adequate distribution function seems to be impossible due to a lack of categorized data to primary and secondary delays, a good estimation by experts can be invaluable. The second method is concerned with the known distributions of disturbances. In this method, to reach the desired level of robustness, the minimum required buffer time between each pair of consecutive departures/arrivals is computed in a way that a robustness measure is fulfilled.

For simplicity, throughout this paper, it is assumed that train numbers equal their sequence numbers in passing block section j, i.e., index "i" can be substituted for "s" in all the notations, and *vice versa*.

As previously stated, the solution for increasing the robustness of timetables is to augment the buffer times. The amount of buffer times must be proportional to the amount of uncertainty,

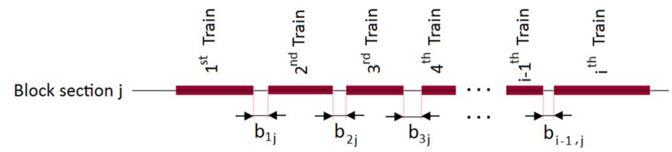

Fig. 2. Buffer times in block section j.

as well as the possibility of delay propagation among the trains. The former case is dependent on the type of trains, as well as the characteristics of the block sections. The latter one in contrast with the first case is more complicated and varied based on the possibility of delay propagation in the timetable.

Considering Fig. 2, one can find that $D_{1j} = 0$, $D_{2j} = Max\{0, (\varepsilon_{1j} - b_{1j})\}$ and $D_{3j} = Max\{0, (\varepsilon_{2j} - b_{2j}), (\varepsilon_{1j} + \varepsilon_{2j} - b_{1j} - b_{2j})\}$. By computing other next delays, D_{ij} will be

$$D_{ij} = \begin{cases} 0, & \text{if } i \leq 1 \\ Max\left\{0, \underset{l\in[2,i]}{Max}\{T_{ijl}\}\right\}, & \text{if } i > 1 \end{cases} \tag{20}$$

where $T_{ijl} = \sum_{k=1}^{l-1}(\varepsilon_{i-k,j} - b_{i-k,j})$.

Proposition 1: Consider block section j in Fig. 2. By assumption of equal probability density function for disturbance occurrence and also equal amount of buffer times, e.g., $f(\varepsilon_{ij}) = f(\varepsilon_0)$ and $b_{sj} = b_0, \forall j \in B_i, \forall i \in S_j, 1 \leq s \leq |S_j|$, we have: $E(D_{ij}) \geq E(D_{i-1,j})$.

Proof: Based on (20) and considering the aforementioned assumptions, we have

$$E(D_{i-1,j}) = E\left(\max\left\{0, \max_{l\in[2,i-1]}\{T_{i-1,j,l}\}\right\}\right)$$

$$E(D_{ij}) = E\left(\max\left\{0, \max_{l\in[2,i]}\{T_{i,j,l}\}\right\}\right)$$

$$E\left(\max\left\{0, \max_{l\in[2,i-1]}\{T_{i-1,j,l}\}, T_{i,j,l}\right\}\right) \geq E\left(\max\left\{0, \max_{l\in[2,i-1]}\{T_{i-1,j,l}\}\right\}\right).$$

Furthermore

$$E\left(\max\left\{0, \max_{l\in[2,i-1]}\{T_{i-1,j,l}\}, T_{i,j,l}\right\}\right) = E\left(\max\left\{0, \max_{l\in[2,i]}\{T_{i,j,l}\}\right\}\right)$$

Therefore, $E(D_{ij}) \geq E(D_{i-1,j})$. ■

By Proposition 1, it can be concluded that, considering the counted assumptions, to achieve a robust timetable, generally, $mb_{sj} \geq mb_{s-1,j}$.

Using the proposed method to calculate the delays, the authors propose the following inequality:

$$P(D_{sj} > \eta) < \lambda \quad \forall j \in B, \quad 1 \leq s \leq |S_j| \tag{21}$$

where η and λ, are called adjusting parameters, and $\eta \geq 0$ and $\lambda \in [0, 1)$. In other words, we aim to generate robust timetables

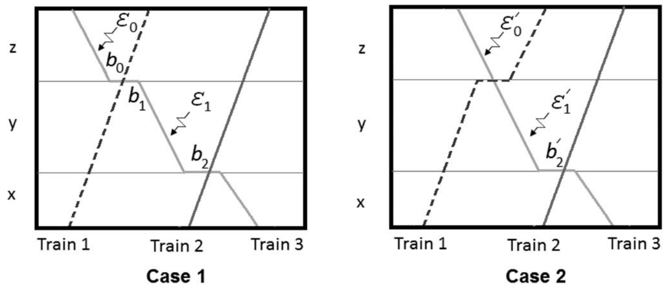

Fig. 3. Simple timetable.

in which the probability of occurring a delay bigger than η is always smaller than λ.

Proposition 2: To achieve a timetable such that its robustness covers inequality (21), $mb_{i-1,j}$ must be found based on the following model:

(Model P1) $mb_{i-1,j} = Min\, b_{i-1,j}$
subject to

$$\left(\overbrace{\iiint \ldots\ldots\ldots\ldots\ldots \int}^{l-1}_{\sum_{k=1}^{l-1}(\varepsilon_{i-k,j})<\eta+\sum_{k=1}^{l-1} b_{i-k,j}} \prod_{k=1}^{l-1} f(\varepsilon_{i-k,j}) \times \prod_{k=1}^{l-1} (d\varepsilon_{i-k,j}) \right) > 1-\lambda \forall l \in [2,i]. \quad (22)$$

Proof: Based on the proposed robustness measure, (21), $\forall l \in [2,i]$, it is intended to find the minimum required buffer times such that $P(T_{ijl} > \eta) < \lambda$, or in other words, $P(\sum_{k=1}^{l-1}(\varepsilon_{i-k,j} - b_{i-k,j}) > \eta) < \lambda$. The result immediately follows from replacing the cumulative density function of $\sum_{k=1}^{l-1} \varepsilon_{i-k,j}$, which is the sum of some independent random variables. ■

Remark 2: The distribution functions of disturbances dynamically change, depending on their position on the timetable. To illustrate, the issue considers Fig. 3.

By comparing the travel path of train 3 in Fig. 3 (case 1) with that in Fig. 3 (case 2), it can be found that the nonstop travel of train 3 in block sections z and y in Fig. 3 (case 2) leads to an increase in the potential disturbance of this train in passing block section y, in comparison with Fig. 3 (case 1). Therefore, $\varepsilon'_1 \geq \varepsilon_1$, and subsequently, we should have $b'_2 \geq b_2$. By this explanation, it is understood that finding the exact distribution functions of disturbances is not an easy task, particularly in large-scale timetables. In this paper, to have a good approximation of computing buffer times, we neglect the possible effect of remained nonabsorbed disturbances, e.g., the impact of $\varepsilon_0 - (b_0 + b_1)$ in computing the distribution function of disturbance ε_1 in Fig. 3 (case 1). It is worth noting that b_0 and b_1 are both individually computed so that each of them satisfies the desired robustness level. By this assumption, in Fig. 3 (cases 1 and 2), we have $\varepsilon_1 = \overline{\varepsilon}_1$ and $\varepsilon'_1 = \overline{\varepsilon}'_0 + \overline{\varepsilon}'_1$, respectively.

Proposition 3: By the assumption of considering equal exponential distributions for disturbances with rate parameter $1/\theta$, i.e., $f(\overline{\varepsilon}_{ij}) = (1/\theta)e^{-(1/\theta)\overline{\varepsilon}_{ij}}\ \forall i,j$, the constraint of model P1, i.e., (22), can be substituted for the following inequality (23), shown at the bottom of the page.

Proof: The result follows form, considering the fact that the sum of k independent exponentially distributed random variables, each of which has a mean of θ, is a gamma distribution with scale parameter θ and a shape parameter k. ■

V. Proposed Branch and Bound Algorithm

In this section, by using the proposed methods of computing the required buffer times, a B&B algorithm is introduced. The proposed B&B algorithm generates all the active schedules. A feasible schedule is the one that satisfies all constraints. A feasible schedule is called active if no train can be scheduled earlier by altering the train sequences on block sections and not delaying any other trains.

Each node, e.g., ν, of the branching tree corresponds with a partial schedule. Node ν is associated with a particular set called Ω_ν. If, in any steps of the B&B algorithm, set Ω_ν becomes empty, then node ν is at the bottom of the tree and represents a complete schedule that can no longer be branched. Moreover, an active node list contains the nodes that have the possibility of branching. This active node list is updated in each step of the algorithm. Whenever this list is empty, the algorithm is terminated. The proposed B&B algorithm for the robust train-timetabling problem is given as follows:

Step 1: (Initial Condition)

Consider initial node ν as the first transit doublet, which could start not later than others, e.g., (i, o_i)

$k \leftarrow 0$
ANL $\leftarrow$ Initial node ν
$\Omega_\nu \leftarrow \{(i, o_i) \quad \forall i \in T\}$
$dep^{\nu}_{io_i} \leftarrow od_i \quad \forall i \in T$
$mb^{\nu}_{io_i} \leftarrow 0 \quad \forall (i,j) \in \Omega_\nu$
$sq^{\nu}_j \leftarrow 1 \quad \forall j \in BS$
$la^{\nu}_j \leftarrow 0 \quad \forall j \in BS.$

$$\left(\sum_{m=0}^{l-2+\sum_{k=1}^{l-1} nst_{i-k,j}} \frac{\left(\frac{1}{\theta} \times \left(\eta + \sum_{k=1}^{l-1} b_{i-k,j}\right)\right)^m}{m!} e^{-\frac{1}{\theta} \times \left(\eta + \sum_{k=1}^{l-1} b_{i-k,j}\right)} \right) < \lambda \quad (23)$$

Step 2: (Block Section Selection)

If $dep^v_{io_i} < la^v_j$ then $dep^v_{io_i} \leftarrow la^v_j, \forall (i,j) \in \Omega_\nu$.

If $sq^\nu_j > 1$ then $b^\nu_{ij} \leftarrow dep^\nu_{ij} - la^v_j$, else $b^\nu_{ij} \leftarrow 0$, $\forall (i,j) \in \Omega_\nu$.

Based on the actual values of $b^\nu_{sj}, \forall s \in [1, sq^v_j]$, compute the minimum required buffer time, mb_{ij}, using model P1, $\forall (i,j) \in \Omega_\nu$.

If $mb^\nu_{ij} > b^v_{ij}$ then $dep^\nu_{ij} \leftarrow dep^\nu_{ij} + (mb^v_{ij} - b^v_{ij})$, $\forall (i,j) \in \Omega_\nu$

$$arr^\nu_{ij} \leftarrow dep^\nu_{ij} + t_{ij} \quad \forall (i,j) \in \Omega_\nu.$$

$$ma_v \leftarrow \min \left\{ arr^v_{ij} | (i,j) \in \Omega_v \right\}$$

$j^* \leftarrow$ The block section related to the ma_v

$$arr^\nu_{ij} \leftarrow 0 \quad \forall (i,j) \in \Omega_\nu.$$

Step 3: (Branching)

$$\Omega'_\nu \leftarrow \left\{ (i,j^*) \in \Omega_v | dep^v_{ij^*} < ma_v \right\}$$

For each member of Ω'_ν, generate a new node beneath node ν. For each new node, ν', let:

Transfer all data that exists in node ν to node ν'. Furthermore, consider following alterations:

$$arr^{\nu'}_{i^*j^*} \leftarrow dep^{\nu'}_{i^*j^*} + t_{i^*j^*},$$

$$la^{\nu'}_{j^*} \leftarrow arr^{\nu'}_{i^*j^*}$$

$$dep^{\nu'}_{i^*j'} \leftarrow arr^{\nu'}_{i^*j^*} + st_{i^*j^*}$$

$$dep^{\nu'}_{ij^*} \leftarrow arr^{\nu'}_{i^*j^*} \quad \forall\, (i,j^*) \in \Omega'_\nu - \{(i^*,j^*)\}$$

$$\Omega_{\nu'} \leftarrow \{\Omega_v \cup (i^*,j')\} - (i^*,j^*)$$

$$sq^{\nu'}_{j^*} \leftarrow sq^\nu_{j^*} + 1$$

where i^* is the selected train correspondence with the node ν';

where j' is the next block section that should be met by train i^* after j^*.

Step 4: (Fathoming Nodes)

Compute the lower bound (LB) based on the fast LB computing method for each new node ν'. Fathom the new node ν', if the fathoming condition is satisfied. Update the ANL.

Step 5: (Node Selection)

If $\Omega_{\nu'}$ is empty, i.e., a new schedule is achieved, update the upper bound if it is necessary. If ANL is empty, then the algorithm is terminated, Return the best-found timetable; otherwise, select one of the active nodes that belongs to the ANL based on the first node selection rule.

If $\Omega_{\nu'}$ is not empty, select one of the recently generated nodes based on the second node selection rule.

Go to step 2.

A. Fast LB Computing

For each node, an LB is considered. If the LB is greater than or equal to the UB, then the descendants of the corresponding node are eliminated. The LB in the node ν' is computed using the following:

$$\mathrm{TWT}_{v'} \leftarrow \mathrm{TWT}_v + w_{i^*} \times \left(dep^{v'}_{i^*j^*} - arr^{v'}_{i^*j''} - st_{i^*j^*} \right)$$

$$LB_{v'} \leftarrow \mathrm{TWT}_{v'} + \sum_{i \in T} \sum_{j \in BS} t_{ij}$$

where j'' is the block section that is met by train i^* before j^*. A stronger LB is proposed in Section V-E.

B. Node Selection Rules

The first rule is given as follows:

Among the new generated nodes, each two nodes are compared based on the pairwise comparison method.

- If $w_{i_1} \geq w_{i_2}$ and $arr^v_{i_1j} \leq arr^v_{i_2j}$, then the node correspondence with train i_1 dominates the one relates to i_2.
- If $w_{i_1} \geq w_{i_2}$ and $arr^v_{i_1j} > arr^v_{i_2j}$, then the node correspondence with train i_1 dominates the one relates to i_2 if the following inequality is satisfied:

$$\frac{arr^v_{i_1j} - arr^v_{i_2j}}{t_{i_1j}} < \frac{w_{i_1}}{w_{i_1} + w_{i_2}}.$$

The second rule is given as follows:

Whenever a new schedule is generated, the next node is selected based on the first-in–first-served rule.

C. Fathoming Condition

If $LB_\nu > UB$, then node ν must be fathomed.

D. Heuristic BS Algorithm

The train-timetabling problem is known to be NP-hard [34], and therefore, optimal solutions are unattainable in real large-scale samples in a reasonable amount of time.

To find a near-optimum solution for large-scale problems in a limited amount of time, the fathoming conditions are proposed here.

1) In the branching step, one can redefine $\Omega'_\nu \leftarrow \{(i,j^*) \in \Omega_v | dep^v_{ij^*} < p \times ma_v\}$, where p is called first severity parameter.
2) The number of branches in each node nb could not exceed a threshold.
3) The quantity of transit doublets that should be scheduled in a timetable equals to $\sum_{j \in BS} |S_j|$. Suppose that, in node ν, s' transit doublets are scheduled, and therefore, $\sum_{j \in BS} |S_j| - s'$ doublets are remained. In this case, if the following condition holds, node ν is fathomed:

   $LB_\nu > (1 - q \times ((\sum_{j \in BS} |S_j| - s') / \sum_{j \in BS} |S_j|)) \times UB$, where q is called second severity parameter and must be $0 < q < 1$. The value of parameter q addresses the tradeoff between finding better solutions and the required running time to achieve the final solution. Generally, in tight timetables, the authors recommend assigning a large value for parameter q.

TABLE II
COMPARISON RESULTS BETWEEN LING, THE PROPOSED B&B, AND THE PROPOSED BEAM SEARCH ALGORITHM

Item	N-n-t	N-s-t	N-b-s	Solving time by Lingo	[LB, UB] by Lingo	Solving time by B&B	Optimum solution by B&B	Best achieved LB	Solving time by BS	TWT by BS	Gap (BS-LB)
1	3	3	6	0:00:07	110	0:00:00	110	110	0:00:00	110	0
2	4	3	6	0:00:12	143	0:00:00	143	143	0:00:00	143	0
3	4	4	6	0:00:32	209	0:00:00	209	209	0:00:00	209	0
4	4	4	8	0:02:43	180	0:00:01	180	180	0:00:01	180	0
5	4	5	8	0:17:53	229	0:00:01	229	229	0:00:01	229	0
6	5	5	8	0:30:00	[175- 338]	0:00:18	308	308	0:00:10	308	0
7	5	5	10	0:30:00	[121- 521]	0:00:03	247	247	0:00:02	247	0
8	6	5	12	0:30:00	[110- 663]	0:01:18	261	261	0:00:59	261	0
9	6	5	14	0:30:00	[143- 894]	0:03:52	316	316	0:00:28	316	0
10	6	6	15	0:30:00	[93-1204]	----	----	326	0:04:48	411	85
11	6	6	18	0:30:00	[52-846]	----	----	340	0:04:22	452	112
12	6	6	24	0:30:00	[100, 2510]	----	----	338	0:05:53	578	240
13	7	7	15	0:30:00	[78, 2572]	----	----	376	0:04:45	651	275
14	7	7	18	0:30:00	[97, 3013]	----	----	398	0:05:02	665	267
15	7	7	24	0:30:00	[111, 2857]	----	----	365	0:05:52	767	402
16	8	8	15	0:30:00	[138, 1552]	----	----	498	0:06:10	859	361
17	8	8	18	0:30:00	NFS	----	----	521	0:06:23	861	340
18	8	8	24	0:30:00	NFS	----	----	532	0:07:11	942	410
19	10	10	18	0:30:00	NFS	----	----	819	0:07:31	1457	638
20	10	10	24	0:30:00	NFS	----	----	812	0:09:59	1328	516

NFS stands for no feasible solution

By employing some randomly generated samples, it is observed that the combination of $Nb = 2$ and $p = 9$ results in statistically better output than the other evaluated combinations

E. Strong LB Computing

The most frequently used LB technique in the job shop scheduling problem with the objective of total weighted tardiness is to relax the problem into a single machine one. In this paper, we have presented a stronger LB, based on the decomposition of the main problem into n distinct problems. Each derived problem is then solved by the proposed B&B. In the first problem, the origin departure time is recalculated; however, the other $n - 1$ remaining problems are solved under the assumption of relaxing origin departure time [i.e., (5)].

VI. VALIDITY OF ALGORITHM

To check the validity of the proposed B&B algorithm, the introduced robust train-timetabling model is coded in Lingo. Table II shows the comparison results among the proposed B&B algorithm coded in Visual Basic, the proposed heuristic BS, and the Lingo outputs. To compare the outcomes of the solving methods, it is assumed that $\Gamma_{j,s',s-1} = (s - 1 - s')$, $\forall j \in BS$, and the buffer time computation method in the B&B algorithm is based on the first method introduced in Section V. This table comprises the outcome of different algorithms for 20 randomly generated numerical examples. In all instances, it is assumed that all trains pass all of the block sections one after another. A time interval equal to 20 min is considered among departure times of trains from their origins. N-n-t, N-s-t, and N-b-s stand for number of northern trains, number of southern trains, and number of block sections, respectively.

The results prove that the proposed B&B is capable of solving all instances faster than the Lingo software package in all solved instances. There are cases where the software package cannot solve some problems and the B&B may fail on others. However, the BS can easily solve all problems, reaching a good feasible solution. The results demonstrate the efficiency of the proposed BS algorithm in finding near-optimal solutions in a reasonable amount of time. The reported LBs, where the optimum solution does not exist, are based on the strong LB computing method described in Section V. In this method, each problem is decomposed to three smaller problems. In other words, the railway network is separated into three parts: The first part contains the most crowded section of the problem in a size that could be solved by the B&B algorithm. As it is mentioned for the initial small problem, the origin departure times are recalculated, whereas the remaining problems are solved by the assumption of relaxing (5). The last column specifies the gap between BS and the best found LBs. The reported CPU time in Table II is achieved by a Pentium IV laptop with an Intel Core 2 Duo processor running at 2.53 GHz and 2 GB of random access memory.

To demonstrate the effectiveness of the proposed method for computing buffer times, an instance consists of a single-track railway line including ten block sections and ten trains is considered. The origin departure time for each direction is fixed as 0, 30, 60, 90, and 120. The traveling times are randomly generated from interval [10], [20]. It is assumed that all disturbances have exponential distributions with parameter $\theta = 2$ min for all trains. It is also supposed that $\eta = 3$ min; in other words, one intends to find timetables where the robustness level fulfills the condition $P(D_{sj} > 3) < \lambda$ for all sequences of all block sections. The optimum total weighted tardiness (TWT) for the following three cases are equal to 135, 171, and 242, respectively: 1) not robust, i.e., $\lambda = 1.0$; 2) $\lambda = 0.9$; and 3)

TABLE III
BUFFER TIMES FOR DIFFERENT AMOUNTS OF PARAMETER λ

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
Total buffer times(λ=1.0)	1	6	8	15	18	8	17	29	14	19
Total buffer times(λ=0.9)	1	9	11	18	21	10	25	36	18	22
Total buffer times(λ=0.1)	19	25	25	13	28	14	25	42	24	27

TABLE IV
RELATION AMONG OPTIMALITY, ROBUSTNESS, AND CAPACITY CONSUMPTION

	% Optimality alteration (TWT) when λ=1→0.5	% Capacity consumption alteration when λ=1→0.5
Ex. 1	50%	5%
Ex. 2	22%	2%
Ex. 3	34%	3%
Ex. 4	18%	2%
Ex. 5	41%	2%

$\lambda = 0.1$. For each train, the summation of the computed buffer times is reported in Table III. Moreover, the average capacity consumption of block sections for the investigated cases is equal to 262, 273, and 279 min, respectively.

To further investigate the optimality, robustness, and capacity consumption, five instances have been randomly generated. The results are illustrated in Table IV. The outcomes indicate that, by increasing the desired robustness level, the optimality reduces, whereas the capacity consumption increases.

VII. CONCLUSION

A robust train-timetabling model that has the capability of handling the disturbances that exist among traveling times is proposed in this paper. It is discussed that, in many real-world train-timetabling problems, a small change in input parameters results in having not only a nonoptimal solution but the feasibility of the final solution as well. The proposed mathematical model could guarantee that a small change in input parameters does not have any effect in feasibility and the optimal solution of the robust one. The tradeoffs among the optimality, robustness, and the capacity consumption have been discussed. The proposed method to compute the buffer times and the introduced robustness measures have been well designed to be easily tuned so that the desired robustness level of railway decision makers can be simply addressed. In addition to that, a B&B algorithm that is able to solve problems much faster than the Lingo software is proposed. To guarantee the robustness of the proposed B&B algorithm, the required buffer times among the train departure and arrival times are calculated by proposing two methods that are under the assumption of known and unknown distribution functions of disturbances, distinctively. A BS is also introduced to solve the large-scale problems. The proposed BS could achieve the optimum solutions in all the investigated instances with known optimum solution. Moreover, it has been proved that the use of B&B algorithm leads to 96% reduction in run time, whereas the BS could find an optimum solution with 99% reduction in run time in comparison with the Lingo software package. Finally, the experimental results show that the proposed algorithms are effectively reduced in the run time.

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