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A green train scheduling model and fuzzy multi-objective optimization algorithm

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ABSTRACT

Train scheduling model is traditionally formulated to minimize the energy consumption for reducing the operation cost. As the European Union formulates the first carbon emission trading scheme in the world, it is necessary to extend the operation cost to include the expenses for buying/selling the carbon emission allowances. In this paper, we propose a multi-objective train scheduling model by minimizing the energy and carbon emission cost as well as the total passenger-time, and named it as green train scheduling model. For obtaining a non-dominated timetable which has equal satisfactory degree on both objectives, we apply a fuzzy multi-objective optimization algorithm to solve the model. Finally, we perform two numerical examples to illustrate the efficiency of the proposed model and solution methodology.

1. Introduction

The railway transport planning is a highly complex process which contains passenger demand analysis, line planning, train scheduling, rolling stock planning, crew planning and crew rostering [1]. First, demand analysis assesses the amount of passengers from certain origins to certain destinations, and line planning decides which lines will be operated and how frequently for meeting these demands. Then train scheduling determines the arrival and departure times at each station for all trains, i.e., the train timetables. Rolling stock planning assembles engines and coaches to trains and assigns them to the lines. Finally, crew planning determines the distribution of personnel in order to guarantee that each train is equipped with the necessary staff, and crew rostering constructs the rosters from the crew duties.

Train scheduling is one of the most difficult and challenging problems. An efficient train timetable should consider the tradeoff between operation efficiency and operation effectiveness which respectively represents the benefits of railway companies and passengers [2]. Efficiency is defined as the ratio of the level of a service compared to the cost for providing such a service. For a fixed level of a service, the main variable in determining the efficiency is the operation cost. Effectiveness is a measure showing how well the demand for the service is satisfied. Apart from the safety and comfort factors, convenience is the most concerning aspect for service from the viewpoint of passengers, which is related to the trip time.

The existing researches on train scheduling usually regard operation cost as energy cost (see [2–5]). However, as the formulation of carbon emission trading scheme in European Union, which is the first international trading system for carbon emissions, the carbon emission cost should be also included. Under such a scheme, each company is allocated a certain quantity of carbon emission allowances. If a company is incapable of meeting its target, it can buy allowances from others that are under their targets. Similarly, companies that prove more able to reduce their emissions are allowed to trade excess allowances to other, more polluting, enterprises.

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This paper aims to extend the traditional definition of operation cost to include the carbon emission trading expenses for the study on train scheduling problem. First, we define a carbon emission cost associated with each train timetable according to the European Union trading scheme. Furthermore, we formulate a multi-objective train scheduling model by minimizing the energy and carbon emission cost as well as the total passenger-time. For obtaining a non-dominated solution with balanced satisfaction degree on both objectives, we apply a fuzzy multi-objective optimization approach to solve the model. Finally, we present two numerical examples to illustrate the efficiency of the proposed model and solution methodology.

2. Literature review

Train scheduling aims to determine the arrival and departure times at each station for all trains on an entire line or network, i.e., the train timetables. The manual way and computer-based way are two main methods. The manual method has been used for more than 150 years, which makes a feasible timetable through a trial and error process by using a preliminary train diagram. As the ever-increasing competition among railway companies, the computer-based method draws more and more attentions because it is capable of improving the operation efficiency and effectiveness significantly. Simulation, expert system and mathematical programming are three mainly used computer-based approaches in practice. Compared with simulation [6,7] and expert system [8,9], the mathematical programming approach was not widely applied in the past because it needs a longer time to get an approximately optimal timetable. However, as the improvement of computer speed, mathematical programming gradually develops to be the most popular approach. In 1971, Amit and Goldfard [10] firstly applied the optimization techniques in train scheduling problem, which opened the door for making train timetables via mathematical programming theory. Since then, researchers proposed a large number of scheduling models and algorithms for optimizing the trip time [11], delay time [12], reliability [13], operation cost [5], deviation from a preferred timetable [14,15] and so on. A good survey on the single-objective optimization methods was presented by Cordeau et al. [16].

Train scheduling is essentially a multi-objective decision problem since an effective timetable should concern not only the benefit of railway companies but also the benefit of passengers. Generally speaking, the railway companies prefer to minimize the operation cost, which conflicts to the benefit of passengers since a lower cost will result in a longer trip time. Therefore, there are more and more researches focusing on the tradeoff study between operation cost and trip time by formulating multi-objective optimization models. For example, Higgins et al. [3] proposed a two-objective model which minimizes the delay time and the cost on fuel consumption. In 2004, Ghoseiri et al. [2] first proposed a concept of total passenger-time for measuring the satisfaction degree of passengers, and then formulated a two-objective optimization model to minimize the fuel consumption and the total passenger-time. In 2009, Yang et al. [17] proposed a fuzzy expected value model for minimizing the delay time and the total passenger-time, in which the number of passengers boarding/leaving each station is considered as a fuzzy variable. In 2011, Li et al. [4] defined a concept of speed deviation to measure the service equality, and then proposed a 3E-train scheduling model by optimizing the service equality, operation efficiency and effectiveness.

Multi-objective approaches are generally proved to be able to produce better solution than single-objective approaches because more relevant factors can be considered as optimization objectives and can be evaluated in non-commensurable units. Such an advantage is also proved in other relevant areas such as air services planning [18], bus operations planning [19], airline flight planning [20], freight train planning [21], urban school bus planning [22], transportation investment planning [23] and so on.

Train scheduling is a mixed integer programming problem, which includes not only continuous variables expressing the arrival and departure times at each station, but also binary variables controlling the routes for all trains. In order to solve such a problem, researchers designed many effective algorithms including enumeration algorithm [12], branch-and-bound algorithm [24,13,3,25], Lagrangian relaxation algorithm [26], heuristic algorithm [14,15], simulated annealing algorithm [27], and genetic algorithm [28].

3. Model formulation

Suppose that a set of trains enter a track of a rail line connecting two distinct locations. We denote inbound trains dispatching from west to east and outbound trains dispatching from east to west. Fig. 1 shows a sample of inbound and outbound trains on a rail line containing several stations.

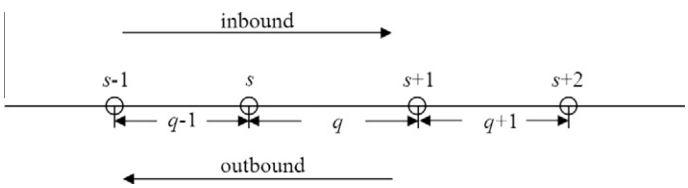

Fig. 1. Sample of a rail line showing the track segments and stations.

3.1. Notation

For good understanding about the paper, we first introduce the notation to be used throughout this paper.

Subscripts and index sets

i	denotes a train;
s	denotes a station;
q	denotes a segment between two successive stations;
s_{eq}	station at which a train enters segment q;
s_{lq}	station at which a train leaves segment q;
I	set of trains;
S	set of stations;
Q	set of segments;
Q_i	set of segments which can be used by train i;
Q_{is}^{I}	set of segments pointing into station s which can be used by train i;
Q_{is}^{O}	set of segments pointing out of station s which can be used by train i.

Parameters

v_{iq}	average velocity of train i on segment q;
u_{iq}	upper limit for the average velocity of train i on segment q;
l_{iq}	lower limit for the average velocity of train i on segment q;
R_{iq}	resistance effort on train i while it traverses segment q;
P_{iq}	required power for train i while it traverses segment q;
r_i	amount of fuel consumption per unit power output;
N_{is}	number of passengers on train i when it arrives at station s;
Y_{is}	number of passengers leaving train i at station s;
Z_{is}	number of passengers boarding train i at station s;
T_{is}^{y}	required stopping time for allowing passengers to leave train i at station s;
T_{is}^{z}	required stopping time for allowing passengers to board train i at station s;
m_i	the mass of train i;
g	the acceleration of gravity;
M	a large positive number;
w_{is}	the minimum dwell time required for train i at station s;
h_q	the minimum headway time between two trains on segment q;
d_q	length of segment q;
X_{iO_i}	the earliest departure time of train i from its origin station;
X_{iD_i}	the planned arrival time of train i at its destination station;
θ_q	gradient on segment q;
Δ	allowance for carbon emission;
λ	unit price of carbon emission allowance;
η_i	carbon emission factor of train i, i.e., the amount of carbon emission for per unit energy consumption.

Continuous decision variables

T_{is}^{a}	time at which train i arrives at station s;
T_{is}^{d}	time at which train i departs from station s;
T_{iO_i}	time at which train i departs from its origin station;
T_{iD_i}	time at which train i arrives at its destination station.

Binary decision variables

$$H_{iq} = \begin{cases} 1, & \text{if train} i \text{traverses segment} q \in Q_i \\ 0, & \text{otherwise;} \end{cases}$$

$$A_{ijq} = \begin{cases} 1, & \text{if inbound train} i \text{traverses segment} q \in Q_i \cap Q_j \text{before inbound train} j \\ 0, & \text{otherwise;} \end{cases}$$

$$B_{ijq} = \begin{cases} 1, & \text{if inbound train} i \text{traverses segment} q \in Q_i \cap Q_j \text{before outbound train} j \\ 0, & \text{otherwise;} \end{cases}$$

$$C_{ijq} = \begin{cases} 1, & \text{if outbound train} i \text{traverses segment} q \in Q_i \cap Q_j \text{before outbound train} j \\ 0, & \text{otherwise.} \end{cases}$$

3.2. Energy and carbon emission cost

For each train, the amount of fuel consumption per mass is proportional to the resistance effort and the displacement, where the resistance includes many aspects such as rolling resistance, flange resistance, axle resistance, track resistance, curve resistance, grade resistance, air resistance and so on. Davis and the American Railway Engineering Association derived a comprehensive train resistance equation, which has been incorporated into many train performance simulators and analytical models. In Davis equation, the resistance is defined as

$$R_{iq} = m_i\left(k_{1i} + k_{2i}v_{iq} + k_{3i}v_{iq}^2 + g\sin(\theta_q)\right),$$

where k_{1i}, k_{2i} and k_{3i} are resistance coefficients for train i and $g\sin(\theta_q)$ is the grade resistance per mass. Table 1 shows the values of resistance coefficients for different train types. For each segment $q \in Q_i$, the velocity is determined as

$$v_{iq} = \frac{d_q}{T^a_{is_{lq}} - T^d_{is_{eq}}}.$$

For keeping train i a constant velocity, the required tractive effort should be equal to the resistance. Then it is easy to prove that the required power is $P_{iq} = R_{iq}v_{iq}$. Since the trip time for train i traverses segment q is d_q/v_{iq}, the fuel consumption is $R_{iq}d_q r_i$. Furthermore, the fuel consumption for train i during its trip is:

$$E_i = \sum_{q\in Q_i} R_{iq}d_q r_i H_{iq}.$$

Let c denote the cost per unit fuel consumption. Then the cost on fuel consumption is

$$E = \sum_{i=1}^{I}\sum_{q\in Q_i} cR_{iq}d_q r_i H_{iq}.$$

Now, let us consider the carbon emission cost. Under the Kyoto Protocol, developed countries are permitted to use a trading system to help them meet their emission targets. European Union formulates the first international carbon emission trading scheme, which covers more than 1000 energy intensive companies across the 27 member countries. In principle, a country may allocate allowances to individual companies for the emission of a certain quantity of greenhouse gases, where one allowance represents one tonne of carbon dioxide equivalent. If a company is incapable of meeting its target, it can buy allowances from companies that are under their targets. Similarly, companies that prove more able to reduce their emissions are allowed to trade excess allowances to other, more polluting, enterprises.

If we use η_i to denote the emission factor for the engine of train i and use Δ to denote the allowance for carbon emission, then the carbon emission cost is:

$$F = \lambda\left(\sum_{i=1}^{I}\eta_i E_i - \Delta\right), \tag{1}$$

where λ is the unit price for trading the surplus emission. If the total emission is larger than Δ, then F expresses the expenses on buying the extra emission allowances. On the contrary, if the total emission is less than Δ, then F expresses the profit arising from the reduction on emission.

3.3. Total passenger-time

According to the strategic scheduling planning, each train is scheduled to stop at certain stations to allow passengers to board/leave the train. Arrival at each of these predetermined stations terminates an old sub-journey and starts a new sub-journey. Therefore, the trip of each train is divided into several sub-journeys. Fig. 2 illustrates the variation of the number of passengers on the train with respect to the time. First, train i enters segment q at time $T^a_{is_{eq}}$, the number of passengers on the

Table 1
Resistance coefficients for different train types.

	k_1(kN/t)	k_2 (kN/t (m/s))	k_3 (kN/t(m/s)^2)
British APT (advanced passenger train)	16.6	0.366	0.0260
Older British trains	15.5	0.292	0.0574
Danish IC3 (InterCity Train)-single set	19.7	0	0.0425
Danish IC3 (InterCity Train)-multiple set	19.7	0	0.0240
German IC (InterCity Train)-BR103 locomotive	16.0	0	0.0225

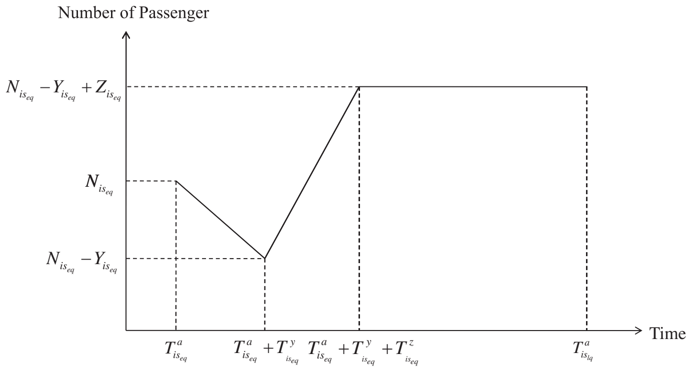

Fig. 2. Train passenger-time diagram on segment q.

train is $N_{is_{eq}}$. Furthermore, with passengers leave the train, the number of passengers on the train reduces to $N_{is_{eq}} - Y_{is_{eq}}$ at time $T^a_{is_{eq}} + T^y_{is_{eq}}$. With passengers board the train, the number of passengers on the train increases to $N_{is_{eq}} - Y_{is_{eq}} + Z_{is_{eq}}$ at time $T^a_{is_{eq}} + T^y_{is_{eq}} + T^z_{is_{eq}}$. Finally, the train departs from station s_{eq} and moves to the next station. The area under the passenger-time curve gives the total passenger-time for train i transverses the segment, that is:

$$T_{iq} = \frac{1}{2}(N_{is_{eq}} - Y_{is_{eq}} + N_{is_{eq}})T^y_{is_{eq}} + \frac{1}{2}(N_{is_{eq}} - Y_{is_{eq}} + N_{is_{eq}} - Y_{is_{eq}} + Z_{is_{eq}})T^z_{is_{eq}} + (N_{is_{eq}} - Y_{is_{eq}} + Z_{is_{eq}})\left(T^a_{is_{lq}} - T^a_{is_{eq}} - T^y_{is_{eq}} - T^z_{is_{eq}}\right)$$
$$= (N_{is_{eq}} - Y_{is_{eq}} + Z_{is_{eq}})\left(T^a_{is_{lq}} - T^a_{is_{eq}}\right) - \frac{1}{2}(N_{is_{eq}} - Y_{is_{eq}} + Z_{is_{eq}})T^y_{is_{eq}} - \frac{1}{2}Z_{is_{eq}}T^z_{is_{eq}}.$$

The total passenger-time for all trains is:

$$T = \sum_{i=1}^{I}\sum_{q\in Q_i} T_{iq}.$$

3.4. Constraints

For each train, since it cannot leave the origin station earlier than its earliest departure time, and it should arrive at the destination station before the scheduled time, we have the following constraints:

$$T_{iO_i} \geqslant X_{iO_i}, \quad \forall i \leqslant I, \tag{2}$$
$$T_{iD_i} \leqslant X_{iD_i}, \quad \forall i \leqslant I. \tag{3}$$

Assume that train i runs from station $s-1$ to station s. It is clear that the train should first choose one and only one segment to come into station s, and then choose one and only one segment pointing out of station s. Then we get the following equations:

$$\sum_{q\in Q^I_{is}} H_{iq} = \sum_{q\in Q^O_{is}} H_{iq} = 1, \quad \forall i \leqslant I, \quad s < S. \tag{4}$$

The arrival time is recorded from the corresponding incoming segment, as well as upon departure, the departure time to the corresponding outgoing segment is recorded. Since train i crosses through station s takes a fixed time $T^y_{is} + T^z_{is}$, the departure time of train i from station s and the arrival time at station s satisfy:

$$\sum_{q\in Q^I_{is}} T^d_{is} H_{iq} = \sum_{q\in Q^O_{is}} T^a_{is} H_{iq} + T^y_{is} + T^z_{is}, \quad \forall i \leqslant I, \quad s \leqslant S. \tag{5}$$

Given the upper velocity limit u_{iq} and the lower velocity limit l_{iq} for train i on segment q, the trip time limits are:

$$\frac{d_q}{u_{iq}} \leqslant T^a_{is_{lq}} - T^d_{is_{eq}} \leqslant \frac{d_q}{l_{iq}}. \tag{6}$$

For reasons of signaling, safety, etc., a headway time is required between each pair of successive trains. For any $i, j \in I, q \in Q_i \cap Q_j$, the argument breaks down into three cases. In the case of two inbound trains, we have:

$$\begin{cases} T^d_{is_{eq}} + h_q \leqslant T^d_{js_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-A_{ijq})\right], \\ T^a_{is_{lq}} + h_q \leqslant T^a_{js_{lq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-A_{ijq})\right], \\ T^d_{js_{eq}} + h_q \leqslant T^d_{is_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-A_{jiq})\right], \\ T^a_{js_{lq}} + h_q \leqslant T^a_{is_{lq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-A_{jiq})\right], \\ A_{ijq} + A_{jiq} = H_{iq}H_{jq}. \end{cases} \tag{7}$$

In the case of two outbound trains, we have:

$$\begin{cases} T^d_{is_{eq}} + h_q \leqslant T^d_{js_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-C_{ijq})\right], \\ T^a_{is_{lq}} + h_q \leqslant T^a_{js_{lq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-C_{ijq})\right], \\ T^d_{js_{eq}} + h_q \leqslant T^d_{is_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-C_{jiq})\right], \\ T^a_{js_{lq}} + h_q \leqslant T^a_{is_{lq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-C_{jiq})\right], \\ C_{ijq} + C_{jiq} = H_{iq}H_{jq}. \end{cases} \tag{8}$$

In the case of two opposite trains, the following constraints should be satisfied:

$$\begin{cases} T^a_{is_{lq}} \leqslant T^d_{js_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-B_{ijq})\right], \\ T^a_{js_{lq}} \leqslant T^d_{is_{eq}} + M\left[(1-H_{iq}) + (1-H_{jq}) + (1-B_{jiq})\right], \\ B_{ijq} + B_{jiq} = H_{iq}H_{jq}. \end{cases} \tag{9}$$

3.5. Green train scheduling model

An efficient train timetable should consider both the operation cost and the trip time, which respectively represents the benefits of railway company and passengers. In this paper, we formulate the following multi-objective optimization model which minimizes the operation cost and the total passenger-time:

$$\min f(\mathbf{x}) = \{E(\mathbf{x}) + F(\mathbf{x}),\ T(\mathbf{x})\}, \tag{10}$$

under the constraints (2)–(9), where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is an n-dimensional decision vector containing all binary and continuous variables. Since the cost function considers both the energy consumption and the carbon emission, we name it as a green train scheduling model.

Note that if all trains are powered by electricity such that there are no carbon emissions, then $\eta_i = 0$ for all i and the green train scheduling model degenerates to the multi-objective model proposed by Ghoseiri et al. [2] .

4. Solution

Fuzzy mathematical programming is an efficient approach to solve multi-objective optimization problems, which models each objective as a fuzzy set whose membership function represents the degree of satisfaction of the objective. The membership degree is usually assumed to rise linearly from zero (for the least satisfactory value) to one (for the most satisfactory value). Zimmermann [29] first used the max–min operator to aggregate the fuzzy objectives for making a compromise decision. However, it cannot guarantee a non-dominated solution and is not completely compensatory. To achieve full compensation between aggregated membership functions and to ensure a non-dominated solution, we use the extended max–min approach suggested by Lai and Hwang [30].

First, according to the single-objective optimization methods, it is easy to calculate the range for each objective. Here, we use $C_{\min}$ and $C_{\max}$ to denote the minimum and maximum operation costs, and use $T_{\min}$ and $T_{\max}$ to denote the minimum and maximum total passenger-times. Furthermore, we construct the membership function for cost objective:

$$\mu_c(x) = \begin{cases} 1, & \text{if} \quad x < C_{\min}, \\ \frac{C_{\max} - x}{C_{\max} - C_{\min}}, & \text{if} \quad C_{\min} \leqslant x \leqslant C_{\max}, \\ 0, & \text{if} \quad x > C_{\max}, \end{cases} \tag{11}$$

and the membership function for total passenger-time objective:

$$\mu_t(x) = \begin{cases} 1, & \text{if} \quad x < T_{\min}, \\ \frac{T_{\max} - x}{T_{\max} - T_{\min}}, & \text{if} \quad T_{\min} \leqslant x \leqslant T_{\max}, \\ 0, & \text{if} \quad x > T_{\max}. \end{cases} \tag{12}$$

Finally, we aggregate $\mu_c(x)$ and $\mu_t(x)$ by using the augmented max–min operator and then formulate the following single-objective optimization model:

$$\begin{cases} \max & \alpha + \varepsilon(\mu_c(\mathbf{x}) + \mu_t(\mathbf{x}))/2 \\ \text{s.t.} & \mu_c(x) \geqslant \alpha \\ & \mu_t(x) \geqslant \alpha \\ & \text{Constraints (2)–(9)} \end{cases} \tag{13}$$

where α is an auxiliary variable which represents the overall satisfactory level of compromise (to be maximized) and ε is a small positive number. Note that a non-dominated solution is always generated when α is maximized. The single-objective model (13) can be solved by using the nonlinear optimization software LINGO.

5. Numerical example

In this section, we present two examples to illustrate the efficiency of the proposed model and solution method.

Example 5.1. In this example, we consider a small rail network which includes three segments and three stations (see Fig. 3). There are two outboard trains and one inboard train, all of them leave from their origin stations to station 2 and then arrive at their destination stations. We need to select the optimal segment for outboard train to start its trip and for inboard train to complete its trip. In addition, we need to determine each train's arrival and departure times at each station. The parameter values are shown by Table 2.

In order to illustrate the efficiency of the proposed model on reducing carbon emission, we first apply the nonlinear optimization software LINGO to solve the optimal timetable without considering carbon emission cost, i.e., $\eta_1 = \eta_2 = \eta_3 = 0$. The results are shown as follows:

(a) $H_{11} = H_{13} = 1, d_{11} = 300, a_{12} = 3261.9, d_{12} = 3981.9$ and $a_{13} = 7200$;
(b) $H_{22} = H_{23} = 1, d_{21} = 0, a_{22} = 2711.7, d_{22} = 3431.7$ and $a_{23} = 6657.5$;
(c) $H_{33} = H_{32} = 1, d_{33} = 0, a_{32} = 2828.6, d_{32} = 3548.6$ and $d_{31} = 5670.4$;
(d) the energy cost is 604.5 and the total passenger-time is 691.6 h.

Take train 1 for example, it departs from its origin station at 300 s and arrives at the second station via segment 1 at 3261.9 s, after a 720 s dwell time, it leaves the second station, and arrives at its destination station at 7200 s. Based on such a timetable, it is easy to calculate that the carbon emission is 1.6526 t, and the carbon emission cost is 52.21.

Now, we consider the proposed green train scheduling model. First, it is calculated that the minimum and maximum operation costs are 567.2 and 815.8, and the minimum and maximum total passenger-times are 630 and 783.3. Furthermore, we solve the fuzzy multi-objective optimization model (13). The results are concluded as follows:

(a) $H_{12} = H_{13} = 1, d_{11} = 300, a_{12} = 2745.4, d_{12} = 3465.4$ and $a_{13} = 6774.5$;
(b) $H_{21} = H_{23} = 1, d_{21} = 0, a_{22} = 3045.4, d_{22} = 3765.4$ and $a_{23} = 7074.5$;
(c) $H_{33} = H_{32} = 1, d_{33} = 0, a_{32} = 2828.6, d_{32} = 3548.6$ and $d_{31} = 5605.7$.
(d) the energy and carbon emission cost is 660.9 and the total passenger-time is 687.8 h.

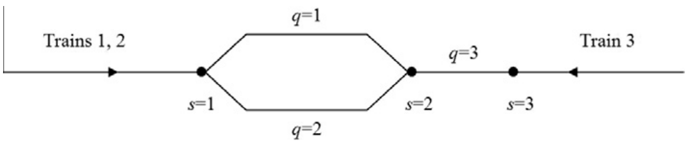

Fig. 3. A rail network containing three segments and three stations.

Table 2
The parameter values used in Example 5.1.

Parameter		Parameter		Parameter		Parameter		Parameter		Parameter		Parameter	
P_{11}	100	k_{11}	19.7	m_1	315	u_{11}	140	l_{11}	0	η_1	0.0006	X_{1D_1}	7200
P_{21}	100	k_{21}	16.6	m_2	451	u_{12}	140	l_{12}	0	η_2	0.0000	X_{2D_2}	7200
P_{33}	200	k_{31}	16.0	m_3	418	u_{13}	140	l_{13}	0	η_3	0.0008	X_{3D_3}	7200
Y_{12}	50	k_{12}	0.000	d_1	90	u_{21}	140	l_{21}	0	w_{12}	720	h_q	300
Y_{22}	50	k_{22}	0.366	d_2	80	u_{22}	140	l_{22}	0	w_{22}	720	Δ	1
Y_{32}	50	k_{32}	0.000	d_3	110	u_{23}	140	l_{23}	0	w_{32}	720	c	1
Z_{12}	50	k_{13}	0.0425	r_1	2×10^7	u_{31}	140	l_{31}	0	X_{1O_1}	0	λ	80
Z_{22}	50	k_{23}	0.0260	r_2	2×10^7	u_{32}	140	l_{32}	0	X_{2O_2}	0	θ	0
Z_{32}	50	k_{33}	0.0225	r_3	2×10^7	u_{33}	140	l_{33}	0	X_{3O_3}	0	M	100000

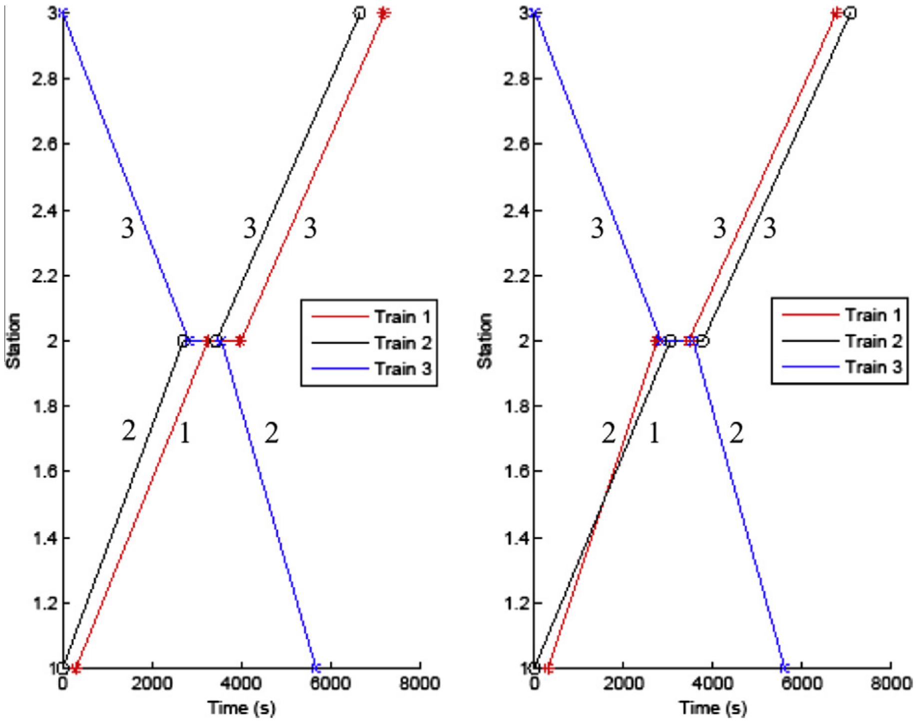

Fig. 4. The optimal timetables without and with carbon emission cost.

Take train 1 for example, it departs from its origin station at 300 s and arrives at the second station at 2738.9 s, after a dwell time, it leaves the second station, and arrives at its destination station at 6760.4 s. The quantity of carbon emission is calculated to be 1.6147 t and the carbon emission cost is calculated to be 49.17.

In Fig. 4, the left graph illustrates the timetable without carbon emission cost, and the right one illustrates the timetable with carbon emission cost. Note that the consideration of carbon emission significantly affects the choice of route and the arrival and departure times. Since train 2 has the lowest carbon emission factor, the green model allocates it a longer route. As a result, the carbon emission cost is reduced by

$$\frac{52.21 - 49.17}{52.21} \times 100\% = 5.82\%.$$

At the same time, the total passenger-time is reduced by 0.55% but the energy cost is increased by 1.19%.

Example 5.2. In this example, we consider a realistic larger-scale Wuhan–Guangzhou high-speed railway line (see Fig. 5), which includes 9 segments and 10 stations. Assume that there are three couples of trains operated between Wuhan station to Guangzhou station, and we need to determine the arrival and departure times for all trains at all stations. Table 3 shows the parameter values used in this example, and Table 4 shows the number of passengers on each train at each segment, that is, $N_{is} - Y_{is} + Z_{is}$ for each $i = 1,2,\ldots,6$ and $s = 1,2,\ldots,9$.

In what follows, we apply the nonlinear optimization software LINGO to solve the passenger-time minimization model which only concerns the total passenger-time, and the green train scheduling model which considers both the total passenger-time and the energy and carbon emission cost. In Fig. 6, the left graph shows the optimal timetable of the passenger-time minimization model, and the right graph shows the optimal timetable of the green train scheduling model.

The comparisons on the calculation results are concluded as follows:

- For reducing the energy and carbon emission cost, the green train scheduling approach enlarges the trip time for almost all trains, see Table 5. For example, since train 3 has larger values on mass, resistance coefficients and carbon emission factor, its trip time is significantly enlarged from 390.10 to 431.35.
- The passenger-time minimization model has the minimum total passenger-time 18743.14, while the value for the green train scheduling model is 20506.93, which is increased by

$$(20506.93 - 18743.14)/20506.93 = 8.60\%.$$

Fig. 5. The Wuhan–Guangzhou high-speed railway.

Table 3
The parameter values used in Example 5.2.

Parameter		Parameter		Parameter		Parameter		Parameter		Parameter		Parameter	
d_1	92	k_{11}	17	k_{14}	17	η_1	0.0005	m_4	380	u_{1q}	300	w_{4q}	720
d_2	122	k_{21}	0	k_{24}	0	η_2	0.0007	m_5	455	u_{2q}	250	w_{5q}	720
d_3	137	k_{31}	0.0215	k_{34}	0.0215	η_3	0.0008	m_6	550	u_{3q}	200	w_{6q}	720
d_4	38	k_{12}	20.7	k_{15}	20.7	η_4	0.0005	Δ	80	u_{4q}	300	r_i	2×10^7
d_5	111	k_{22}	0	k_{25}	0	η_5	0.0007	c	1	u_{5q}	250	X_{iO_i}	0
d_6	139	k_{32}	0.0565	k_{35}	0.0565	η_6	0.0008	λ	80	u_{6q}	200	X_{iD_i}	7200
d_7	115	k_{13}	28.5	k_{16}	28.5	m_1	380	θ	0	w_{1q}	720		
d_8	144	k_{23}	0.374	k_{26}	0.374	m_2	455	M	100000	w_{2q}	720		
d_9	83	k_{33}	0.0825	k_{36}	0.0825	m_3	550	h_q	300	w_{3q}	720		

Table 4
The number of passengers.

	Segment 1	Segment 2	Segment 3	Segment 4	Segment 5	Segment 6	Segment 7	Segment 8	Segment 9
Train 1	700	830	905	860	810	765	750	710	660
Train 2	650	770	970	875	780	770	765	670	610
Train 3	770	780	810	795	770	790	755	660	600
Train 4	730	800	900	845	800	755	740	725	680
Train 5	640	730	920	855	775	720	745	660	630
Train 6	620	710	905	865	780	735	755	680	620

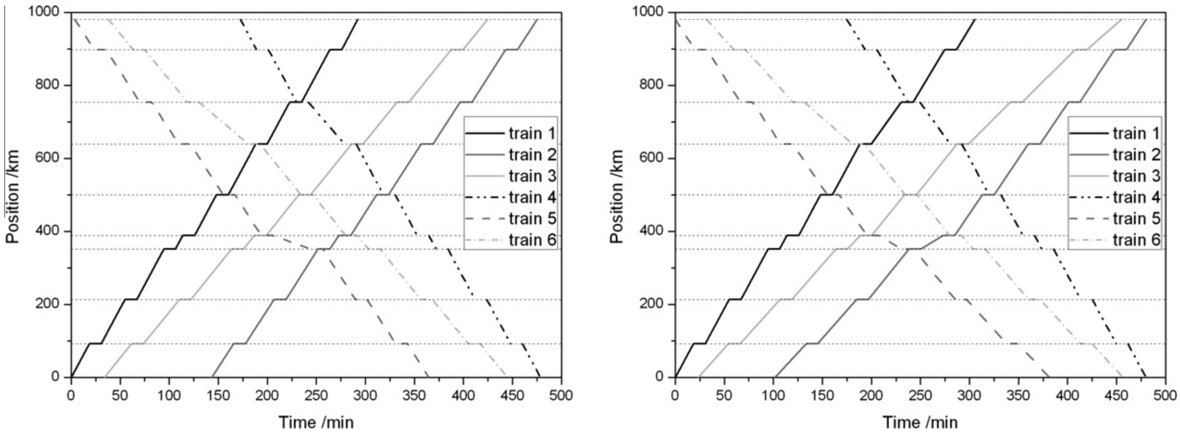

Fig. 6. The optimal timetables without and with energy and carbon emission cost.

Table 5
Comparisons on the trip times.

	Train 1	Train 2	Train 3	Train 4	Train 5	Train 6
Passenger-time minimization model	292.10	331.44	390.10	306.04	361.16	406.36
Green train scheduling model	305.49	378.63	431.35	305.32	381.58	424.76

- Based on the optimal timetable from the passenger-time minimization model, it is easy to calculate that the energy and carbon emission cost is 39414.88, while this value for the green train scheduling model is 32482.08, which implies that the green scheduling model can significantly reduce the energy and carbon emission cost by

$$(39414.88 - 32482.08)/39414.88 = 17.59\%.$$

6. Conclusion

The main contribution of this paper is to propose a green train scheduling model which minimizes the energy and carbon emission cost according to the carbon emission trading scheme of the European Union. On the consideration of the passengers' convenience, this model also minimizes the total passenger-time. The fuzzy multi-objective optimization approach was applied to solve the non-dominated timetable which has equal satisfaction degree for passenger-time and cost. Finally, two numerical examples were presented to show that the proposed model can reduce the carbon emission cost significantly compared with the existing models.

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References

[1] M.R. Bussieck, T. Winter, U.T. Zimmermann, Discrete optimization in public rail transport, Math. Program. 79 (1997) 415–444.
[2] K. Ghoseiri, F. Szidarovszky, M.J. Asgharpour, A multi-objective train scheduling model and solution, Transport. Res. B 38 (2004) 927–952.
[3] A. Higgins, E. Kozan, L. Ferreira, Optimal scheduling of trains on a single line track, Transport. Res. B 30 (2) (1996) 147–161.
[4] X. Li, L.X. Yang, K.P. Li, Z.Y. Gao, 3E-Model for temporary passenger train scheduling problem, Information – Int. Interdiscipl. J. 14 (2) (2011) 373–389.
[5] T. Lindner, Train schedule optimization in public rail transport, Ph.D. Thesis, Technische Universitat Braunschweig, 2000.
[6] O. Frank, Two-way traffic on a single line of railway, Oper. Res. 14 (1965) 801–811.
[7] E.R. Petersen, A.J. Taylor, A structured model for rail line simulation and optimization, Transport. Sci. 16 (1982) 192–206.
[8] T. Chiang, H. Hau, H. Chiang, S. Ko, C. Hsieh, Knowledge-based system for railway scheduling, Data Knowl. Eng. 27 (3) (1998) 289–312.
[9] M. Zweben, E. Davis, E. Daun, M.J. Deale, Scheduling and rescheduling with iterative repair, IEEE Trans. Syst. Man Cybern. 23 (6) (1993) 1588–1596.
[10] I. Amit, D. Goldfard, The timetable problem for railways, Development in Operations Research, vol. 2, Gordon and Breach, New York, 1971, pp. 379–387.
[11] B. Szpigel, Optimal train scheduling on a single track railway, Operations Research, vol. 72, North- Holland, Amsterdam, Netherlands, 1973, pp. 343–352.

[12] R.L. Sauder, W.M. Westerman, Computer aided train dispatching: decision support through optimization, Interfaces 13 (6) (1983) 24–37.
[13] D. Jovanovic, P.T. Harker, Tactical scheduling of rail operations: the SCAN I system, Transport. Sci. 25 (1) (1991) 46–64.
[14] M. Carey, A model strategy for train pathing with choice of lines platforms and routes, Transport. Res. B 28 (1994) 333–353.
[15] M. Carey, Extending a train pathing model from one-way to two-way track, Transport. Res. B 28 (1994) 395–400.
[16] J.F. Cordeau, P. Toth, D. Vigo, A survey of optimization models for train routing and scheduling, Transport. Sci. 32 (1998) 380–404.
[17] L. Yang, K.P. Li, Z.Y. Gao, Train timetable problem on a single-line railway with fuzzy passenger demand, IEEE Trans. Fuzzy Syst. 17 (3) (2009) 617–629.
[18] J. Flynn, S. Ratick, A multiobjective hierarchical covering model for the essential air services program, Transport. Sci. 22 (1988) 139–147.
[19] G.H. Tzeng, T.A. Shiau, Multiple objectives programming for bus operation: a case study for Taipei city, Transport. Res. B 22 (1988) 195–206.
[20] D. Teodorovic, E. Krcmar-Nozic, Multicriteria model to determine flight frequencies on an airline network under competitive conditions, Transport. Sci. 23 (1989) 14–25.
[21] Z. Fu, M. Wright, Train plan model for British rail freight services through the Channel Tunnel, J. Oper. Res. Soc. 45 (1994) 384–391.
[22] R. Bowerman, B. Hall, P. Calamai, A multi-objective optimization approach to urban school bus routing: formulation and solution method, Transport. Res. A 29 (2) (1995) 107–123.
[23] J.Y. Teng, G.H. Tzeng, A multiobjective programming approach for selecting non-independent transportation investment alternatives, Transport. Res. B 30 (1996) 291–307.
[24] E.R. Kraft, A branch and bound procedure for optimal train dispatching, Transport. Res. Forum 28 (1987) 263–276.
[25] L. Yang, L. Liu, Fuzzy fixed charge solid transportation problem and algorithm, Appl. Soft Comput. 7 (3) (2007) 879–889.
[26] U. Brannlund, P.O. Lindberg, A. Nou, J.E. Nilsson, Railway timetabling using lagrangian relaxation, Transport. Sci. 32 (1998) 358–369.
[27] M.Y. Isaai, An intelligent search technique for solving train scheduling problems, simulated annealing and constraint satisfaction, Sci. Iran. 14 (5) (2007) 442–449.
[28] M.B. Khan, D. Zhang, M. Jun, Z. Li, An intelligent technique to train scheduling problem based on genetic algorithm, in: Proceedings of the Second International Conference on Engineering Technologies, 2006, pp. 593–598.
[29] H.J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Set Syst. 1 (1978) 45–55.
[30] Y.J. Lai, C.L. Hwang, Fuzzy multiple objective decision making: methods and applications, Lecture Notes in Economics and Mathematical Systems, vol. 404, Springer-Verlag, New York, 1994.