Clump Detection and Organization through MRA

A 3D Discrete Wavelet Transform Application

Martín Villanueva[1]

[1]Departamento de Informática
Universidad Técnica Federico Santa María

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Outline

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What is a Wavelet? (2)

Wavelets are just mathematical functions $\Psi(t)$ with interesting properties:

- They have a band-pass like spectrum: $|\mathbf{\Psi}(\omega)|^2\Big|_{\omega=0} = 0$
- $\int \Psi(t)dt = 0 \rightarrow$ mean value of zero and therefore must be oscillatory (*A wave!*).
- They have concentration in both time and frequency domains (*Compact Support!*)
- The last one is achieved through the *Vanishing Moments* condition: $\int t^p \phi(t)dt = 0$

What is a Wavelet? (3)

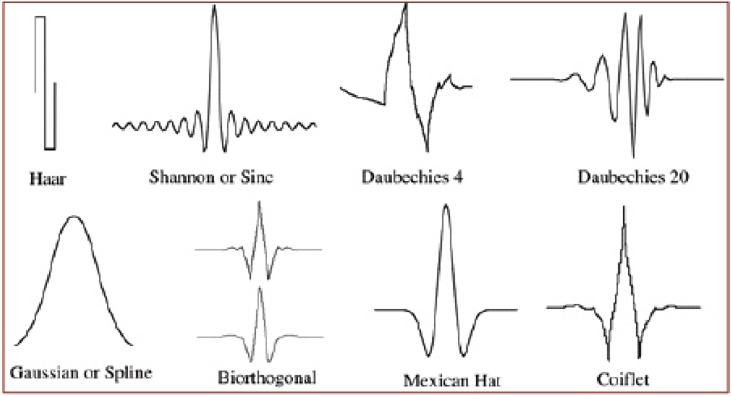

Figure 8

Examples of types of wavelets

Continuous Wavelet Transform

Idea: Decomposing a signal function $f(t)$ into a set of basis wavelet functions $\Psi_{s,\tau}(t)$:

$$\gamma(s,\tau) = \int f(t)\Psi_{s,\tau}(t)^* dt$$

where s and τ are the scale and translation. We can then reconstruct the original signal by the inverse process:

$$f(t) = \int\int \gamma(s,\tau)\Psi_{s,\tau}(t) d\tau ds$$

Here the wavelets are dilated and expanded versions of the so-called **mother wavelet** $\Psi(t)$:

$$\Psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right)$$

Discrete Wavelet Transform

Most of times, CWT can't be obtained analytically. So we have to discretize and compute it numerically. Discrete Wavelets can only be scaled and translated in discrete steps:

$$\Psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}} \Psi \left(\frac{t - k\tau_0 s_0^j}{s_0^j} \right)$$

and then an arbitrary signal can be reconstructed by:

$$f(t) = \sum_{j,k} \gamma(j,k) \Psi_{j,k}(t)$$

Problem: It still needs an infinite number of wavelets!

Discrete Wavelet Transform (2)

Solution: The scaling function!

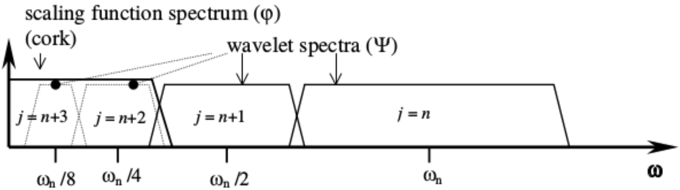

Figura: Filter Bank Decomposition

- Each wavelet has a band-pass spectrum.
- We will not cover the spectrum all way down to zero, but to use a low-pass filter to plug the hole when it is small enough.

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Subband Coding

Idea: We can see the DWT as a filter bank, that passes the signal through this filter bank, and the output of the different filter stages are the wavelet -and scaling function- coefficients.

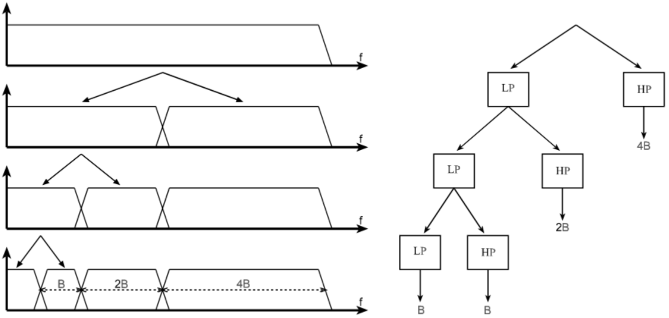

Figura: Subband Coding

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Alves et al.

The mass function of dense molecular cores and the origin of the IMF

J. Alves[1], M. Lombardi[2,⋆], and C. J. Lada[3]

Procedure: *Object identification in wavelet space*: For a given scale i, structures are isolated with **classical thresholding** at i with σ_i being the noise amplitude at scale i. A structure at scale i is connected with a structure at scale $i+1$ if its local maxima drops in structure at scale $i+1$.

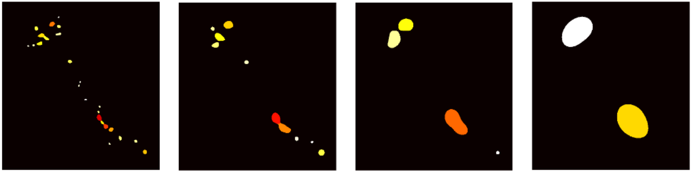
Observation: It uses a different approach of DWT, called the Stationary Wavelet Transform (SWT). Given some image, it produces a *redundant* representation on the Wavelet space, and of the *same size* of the input image.

Gregorio et al.

Automatic detection and automatic classification of structures in astronomical images

Rodrigo Gregorio[a], Mauricio Solar[a], Diego Mardones[b], Karim Pichara[c], Victor Parada[d], Ricardo Contreras[e]

Procedure: Similar to Alves, computes SWT of 2D image at different scales, but finding structures in the Wavelet space through a clump identification algorithm (like ClumpFind).



(a) Level 5 (b) Level 6 (c) Level 7 (d) Level 8

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Project

Motivation: Precisely represent the dense core formations in cold molecular clouds, and its structural relationship.

- Extend the ideas of Gregorio to 3D, so we can work with 3D Spectroscopic Data Cubes directly.
- This will be achieved by MRA with 3D Discrete Wavelet Transform.
- The isolation Steps will be carried out by algorithms like ClumpFind or FellWalker.
- The results on different levels will be summarized in a Dendrogram tree structure.

Project: Limitations

- Using 3D SWT would be terrible inefficient! As we have seen, it produces a redundant representation of the *same size* of the input. (*An sparse representation is needed!*)
- SWT for 3D wavelets isn't implemented in any software package anyway.
- We have to choose the 3D wavelet(s) (or build one) that bests extract/represents the needed features.

Project: Solution

We can use a DWT (with no redundance), which can be seen as a sparse representation of an image. Then there are two options:

- At each level, Find out a way to find structures on the Wavelet space.
- At each level, compute the IDWT and find structure on this resampled version of the image.

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Technical Details

Solution will be implemented in Matlab. Why?

- It has the best and more serious Wavelet software package.
- And it has (almost)all the other necessary stuff: Fits routines, Dendrograms software, etc.
- But, it will be necessary to implement or integrate clump detection algorithms.