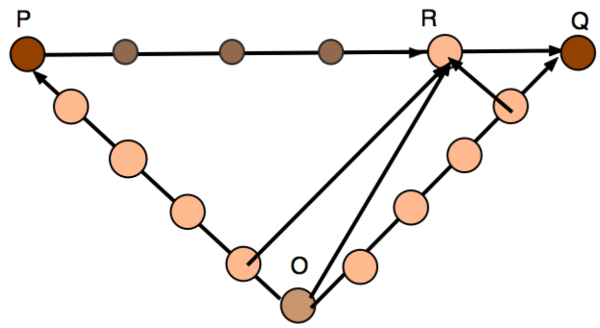
1)

Since R is the point on the line segment that is four times as far from P as it is from Q, this indicates that $\overrightarrow{PR} = \frac{4}{5}\overrightarrow{PQ}$.

Then:

$\vec{r} = \overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \overrightarrow{OP} + \frac{4}{5}\overrightarrow{PQ} = \frac{1}{5}\overrightarrow{OP} + \frac{4}{5}\left(\overrightarrow{OP} + \overrightarrow{PQ}\right) = \frac{1}{5}\overrightarrow{OP} + \frac{4}{5}\overrightarrow{OQ} = \frac{1}{5}\vec{p} + \frac{4}{5}\vec{q}$



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Let P be (a,b), Q be (c,d).

The function of the line segment of PQ is:

$$y = \frac{d-b}{c-a}(x-a) + b$$

$$\Rightarrow \ (c-a)y = (d-b)(x-a) + b(c-a)$$

$$(c-a)(y-b) = (d-b)(x-a)$$

$$\frac{y-b}{d-b} = \frac{x-a}{c-a} = t$$

$$\Rightarrow \begin{cases} y-b = t(d-b) \\ x-a = t(c-a) \end{cases} \Rightarrow \begin{cases} y = t(d-b) + b \\ x = t(c-a) + a \end{cases}$$

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The function of the line segment of PQ is:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \qquad ①$$

For $R(x,y)$:

If $x \notin [x_1, x_2]$ or $y \notin [y_1, y_2]$, then R is not on the line segment

If $x \in [x_1, x_2]$ or $y \in [y_1, y_2]$, and $R(x,y)$ satisfy equation ①, then R is on the line segment.

2)

Let $P(x_1, y_1), Q(x_2, y_2)$

$$\vec{n} * \overrightarrow{PQ} = 0$$

$\Rightarrow$ the normals are
$(y_2 - y_1, -x_2 + x_1)$ and $(y_1 - y_2, x_2 - x_1)$

Since it must be on the left wrt the direction of the line segment,
Therefore, the desired normal is

$$\frac{y_1 - y_2}{\|\overrightarrow{PQ}\|}, \frac{x_2 - x_1}{\|\overrightarrow{PQ}\|} \quad if\ x_2 - x_1 \geq 0$$

else the normal is

$$\left(\frac{y_2 - y_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}, \frac{-x_2 + x_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}\right)$$

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First, to find if the projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$ is on the line segment:
Let O' on the line of P,Q.

$$\overrightarrow{PO'} = \frac{\overrightarrow{PR} \cdot \overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} \frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \left(\frac{(x_2 - x_1)^2 (x - x_1)}{\|\overrightarrow{PQ}\|^2}, \frac{(y_2 - y_1)^2 (y - y_1)}{\|\overrightarrow{PQ}\|^2}\right)$$

From 1) part 3, we can check if O' is on the line segment, if not, the distance between an arbitrary point $R(x, y)$ and the line segment will be:

$$d = \min\big(d(P, R), d(Q, R)\big) = \min\left(\sqrt{(x_1 - x)^2 + (y_1 - y)^2}, \sqrt{(x_2 - x)^2 + (y_2 - y)^2}\right)$$

If O' is on the line segment, then the distance will be:

$$d = \frac{|(y_2 - y_1)x - (x_2 - x_1)y - y_1 x_1 + y_1 x_2|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$
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