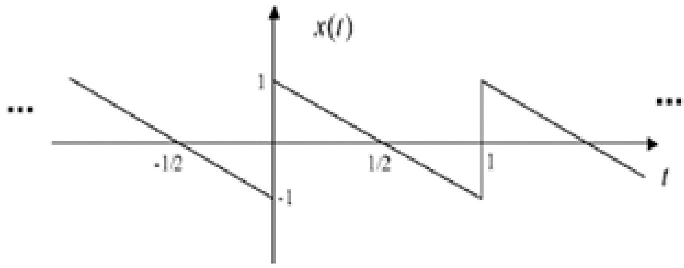
# Sheet 2

1- Find the Trigonometric FS of the following periodic sawtooth

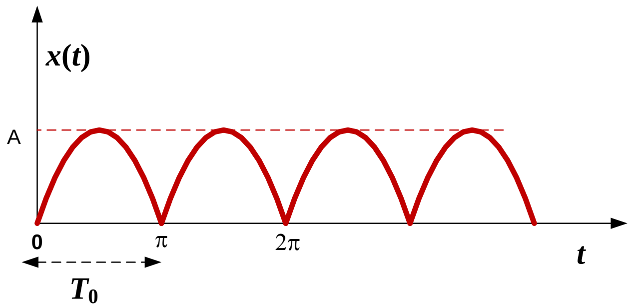



Help

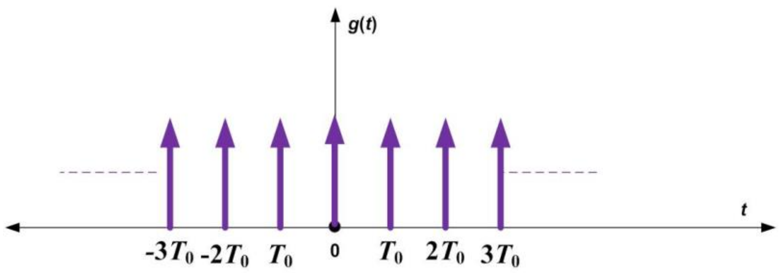
Let us find the fundamental period $T$, the fundamental frequency

$\omega_0$, and the Fourier series coefficients $a_k$ of the periodic "sawtooth" signal $x(t)$

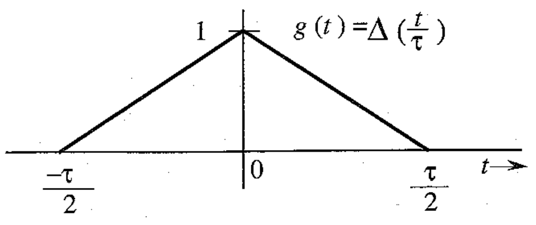
2- Find the quadrature (Trigonometric) FS for the full wave rectified sine wave shown in figure below. Then obtain the exponential FS and its spectrum




3-Obtain the Fourier series of the unit impulse response train shown below. Then, Plot the amplitude and phase spectrums of the signal.

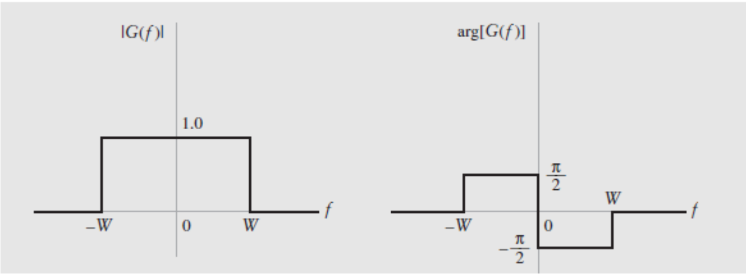



4- Using the time differentiation property, find the Fourier transform of the triangle pulse $\Delta(t/\tau)$



5- Evaluate the Fourier transform of the damped sinusoidal wave $g(t) = \exp(-t)\sin(2\pi f_c t)u(t)$, where $u(t)$ is the unit step function.

6- Determine the inverse Fourier transform of the frequency function $G(f)$ defined by the amplitude and phase spectra shown in Fig.




7- **Determine spectral density, and signal energy for the following signal**

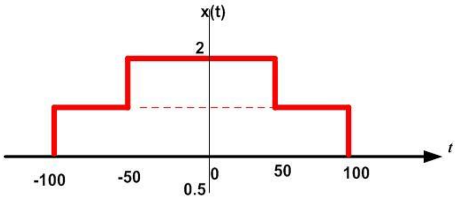
***V*(*t*)=*A* sinc[4*W*(*t*+*t*$_d$)]**

Help solution

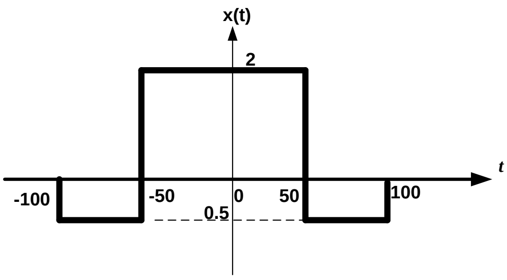
$$\frac{A^2}{16W^2}\, rect[\frac{f}{4W}]$$

8- **Find ESD for *g*(*t*)=*A rect*(*t/T*) and sketch the ESD**

9- **Find the Fourier transform of the following pulses**




10- **Find the Fourier transform of the following pulses**




# What is Hilbert transform?

**Write a short report about this transform…….**

# Extra Important Problems

**1-**Determine the FT of each of the following signals:

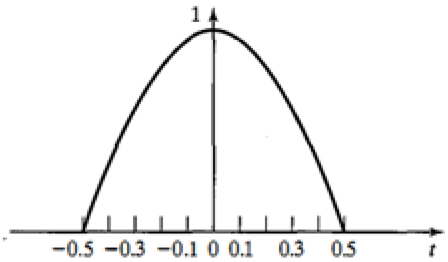
a) **$y(t)=\prod(t-3)+\prod(t+3)$**

b) **$x(t)=\prod(t/4)+\Lambda(t/2)$**

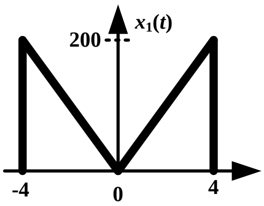
**2-**

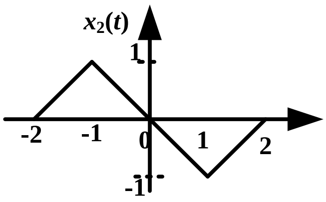
Determine the Fourier transform of the signal

$$x(t) = \begin{cases} \cos(\pi t) & |t| \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$




3- Apply Fourier transform properties to determine the Fourier transform of the following signals



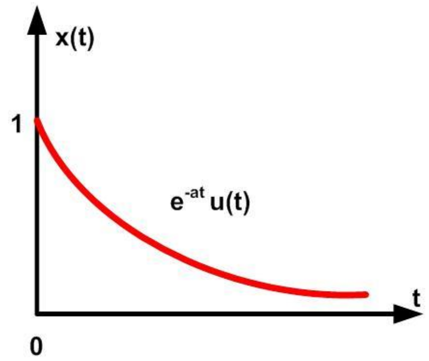




4-As shown in the figure, the decaying exponential pulse function, compute
1) The signal energy,

2) The amplitude spectrum and sketch it.

3) The percentage of total energy contained inside the frequency band $-W \le f \le W$ where $W=a/2\pi$

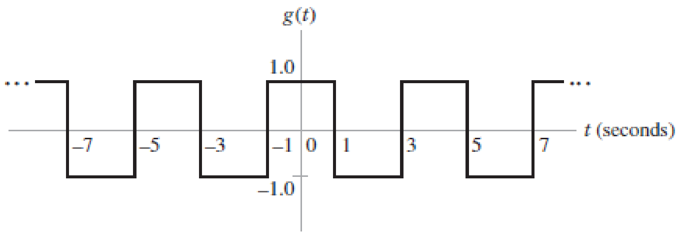
*5)* The bandwidth *W* such that 85% of the energy is contained in frequency below *B*.




6- Obtain the FT RF pulse, then find the amplitude spectrum

$$g(t) = \mathrm{rect}\left(\frac{t}{T}\right)\cos(2\pi f_c t)$$

7-Consider the square wave shown in Figure below Find the power spectral density, average power, and autocorrelation function of this square wave. Does the wave have dc power? Explain your answer.



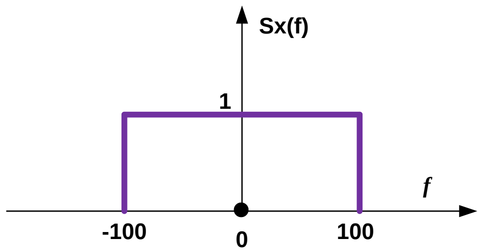

8- Consider the power signal x(t) with autocorrelation function $R_x(\tau)=200\mathrm{sinc}(200\pi t)$.

a) find and plot $S_x(f)$, the PSD of x(t)

We can obtain the $S_x(f)$ by using duality property as follows:

$$R(\tau) \xleftrightarrow{F} S(f)$$

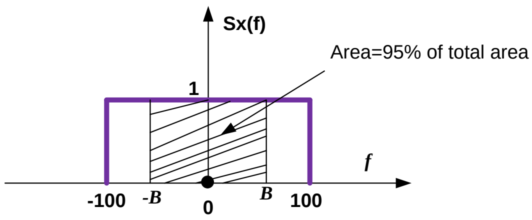
$$200\mathrm{sinc}(200\pi t) \leftrightarrow \mathrm{rect}\left(\frac{f}{200}\right)$$




b) compute the $P_x$ the total power of x(t)

$$P_x = \int_{-\infty}^{\infty} S_x(f)\,df = \int_{-100}^{100} 1\,df = 200W$$

c) compute the 95th % Bandwidth of x(t)

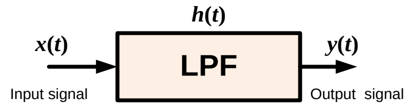



$$0.95(P_x) = \int_{-B}^{B} S_x(f)\,df$$

$$0.95 \times 200 = \int_{-B}^{B} rect(\frac{f}{100})\,df = \int_{-B}^{B} 1\,df = 2B$$

$$B = 95Hz$$

d) if x(t) is used as the input signal to ideal Low Pass Filter (LPF) with amplitude response $|H(f)|=rect(\frac{f}{100})$. Compute the output power, $P_y$



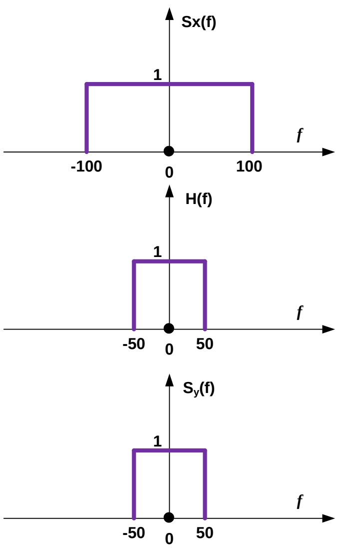


$$P_y = \int_{-\infty}^{\infty} S_y(f)\,df$$

$$S_y(f) = |H(f)|^2 S_x(f)$$

$$|H(f)|^2 = |rect(\frac{f}{100})|^2 = rect(\frac{f}{100})$$

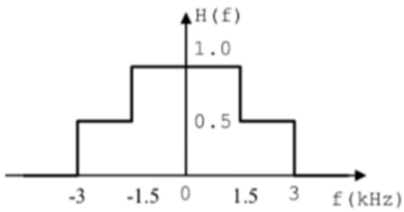
$$S_x(f) = rect(\frac{f}{200})$$



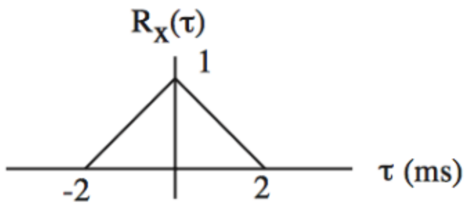

$$P_y = \int_{-\infty}^{\infty} S_y(f)\,df$$

$$P_y = \int_{-50}^{50} 1\,df = 100\ \text{W}$$

9-A zero-mean white Guassian noise signal $x(t)$ with a power spectral density of $N_0/2$=0.001 W/Hz is applied at the input of a system with frequency $H(f)$ given below. Compute and plot the power spectral density $S_y(f)$ of the output signal. Using the result in a) compute the $P_y$ the power of the output signal.




10- A signal has an autocorrelation function



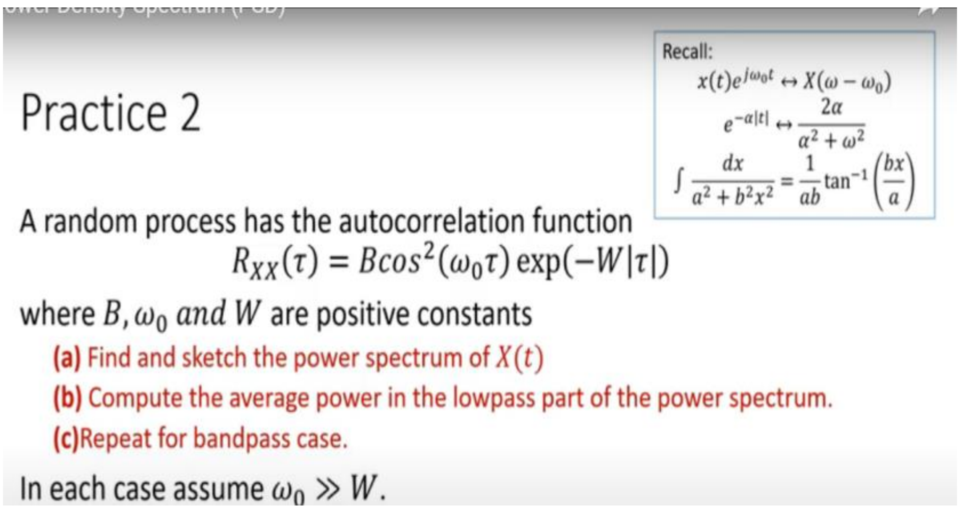

a- Find the energy spectral density function
b- What is the first null bandwidth of this signal?

$$R(\tau) \xleftrightarrow{F} \psi(f)$$

$$\Lambda\left(\frac{\tau\times10^{-3}}{2}\right) \xleftrightarrow{F} 2\times10^{-3}\operatorname{sinc}^2(2\times10^{-3}f)$$

$$\Lambda\left(\frac{\tau\times10^{-3}}{2}\right) \xleftrightarrow{F} 0.002\operatorname{sinc}^2(0.002f)$$

The first null bandwidth of the signal is given by BW= 1/0.002=500Hz




Convolution problem

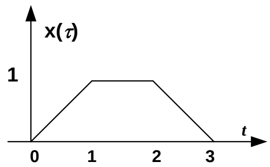
The impulse response is given by

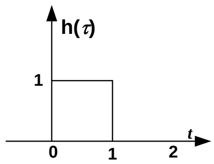
$$h(t)=\begin{cases} u(t) & 0\le t\le T \\ 0, & otherwise \end{cases}$$

The input signal $x(t)=e^{-at}\,u(t)$. Find the output of the system, $y(t)$.

<u>Convolution problem</u>

-Perform the convolution of the given signals




-Find and plot the output, y[n] of LTI system if

$x[n]=\{1,\underset{\uparrow}{2},-2\}$ and $h[n]=\{2,0,\underset{\uparrow}{1}\}$

-Find and plot the output, $y[n]=x_1[n]*x_2[n]$

$x_1[n]=\{-1,\underset{\uparrow}{2},0,1\}$ and $x_2[n]=\{3,1,\underset{\uparrow}{0},-1\}$

## Problems

- Compute convolution for the following signals:
  - A) $x(n)=[1 \quad 3 \quad \underset{\uparrow}{2} \quad 1]$ $\qquad h(n)=[1 \quad 1]$
  - B) $x(n)=[\underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \quad 5]$ $\qquad h(n)=[1 \quad -1]$
  - C) $x(n)=[2 \quad 1 \quad 3 \quad 2 \quad -1]$ $\qquad h(n)=[4 \quad 3 \quad 2 \quad 1]$

## Explain or give notes about Hilbert transform?