```
with(VectorCalculus) :
with(LinearAlgebra) :
```

$X := [x_1, x_2, x_3, x_4, x_5]:$

$U := [u, \mathrm{phi}]:$

$PSI := [\mathrm{Psi}_1, \mathrm{Psi}_2, \mathrm{Psi}_3, \mathrm{Psi}_4, \mathrm{Psi}_5]:$

$\gamma_E := x \rightarrow norm(\langle x[1], x[2]\rangle, 2, conjugate = false) :$

$\gamma_M := x \rightarrow norm(\langle x[1] - x_M, x[2] - y_M\rangle, 2, conjugate = false) :$

$f_1 := (x, u) \rightarrow x[3]:$

$f_2 := (x, u) \rightarrow x[4]:$

$$f_3 := (x, u) \rightarrow -\frac{\mu_E \cdot x[1]}{\gamma_E(x)^3} - \frac{\mu_M \cdot (x[1] - x_M)}{\gamma_M(x)^3} + \frac{u[1] \cdot \cos(u[2])}{x[5]} :$$

$$f_4 := (x, u) \rightarrow -\frac{\mu_E \cdot x[2]}{\gamma_E(x)^3} - \frac{\mu_M \cdot (x[2] - y_M)}{\gamma_M(x)^3} + \frac{u[1] \cdot \sin(u[2])}{x[5]} :$$

$f_5 := (x, u) \rightarrow C \cdot u[1]:$

$$k_1 := x \rightarrow \frac{1}{4} \cdot \left(\left(x[1] - x_M\right)^2 + \left(x[2] - y_M\right)^2 - r_M^{\,2} \right)^2 :$$

$$k_2 := x \rightarrow \frac{1}{4} \cdot \left(\left(x[3] - dx_M\right)^2 + \left(x[4] - dy_M\right)^2 - V_M^{\,2} \right)^2 :$$

$$k_3 := x \rightarrow \frac{1}{2} \cdot \left(\left(x[1] - x_M\right) \cdot \left(x[3] - dx_M\right) + \left(x[2] - y_M\right) \cdot \left(x[4] - dy_M\right) \right)^2 :$$

$$k_4 := x \rightarrow piecewise\left(x[5] < m_r,\ \frac{1}{2} \cdot \left(m_r - x[5]\right)\text{^}2\right) :$$

$f := (x, u) \rightarrow [f_1(x, u), f_2(x, u), f_3(x, u), f_4(x, u), f_5(x, u)]:$

$Q := (x, u) \rightarrow -K_1 \cdot x[5] + K_2 \cdot T + \rho \cdot \left(k_1(x) + k_2(x) + k_3(x) + k_4(x)\right) :$

$H := -Transpose(Jacobian(f(X, U), X)) :$

$J := -Jacobian(\langle Q(X, U)\rangle, X) :$

Model

Model **(1)**

for i **from** 1 **by** 1 **to** 5 **do**
$f(X, U)[i]$
od;

$$x_3$$

$$x_4$$

$$-\frac{\mu_E x_1}{\left(x_1^2 + x_2^2\right)^{3/2}} - \frac{\mu_M \left(x_1 - x_M\right)}{\left(\left(x_1 - x_M\right)^2 + \left(x_2 - y_M\right)^2\right)^{3/2}} + \frac{u \cos(\phi)}{x_5}$$

$$-\frac{\mu_E x_2}{\left(x_1^2+x_2^2\right)^{3/2}}-\frac{\mu_M\left(x_2-y_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{3/2}}+\frac{u\sin(\phi)}{x_5}$$

$$C\,u \tag{2}$$

Rownania sprzężone

$$\textit{Rownania sprzężone} \tag{3}$$

for i **from** 1 **by** 1 **to** 5 **do**
 for j **from** 1 **by** 1 **to** 5 **do**
$K[j] := H[i,j]\cdot PSI[j]$
 od;
 $K[1]+K[2]+K[3]+K[4]+K[5]$
 od;

$$\left(\frac{\mu_E}{\left(x_1^2+x_2^2\right)^{3/2}}-\frac{3\,\mu_E x_1^2}{\left(x_1^2+x_2^2\right)^{5/2}}+\frac{\mu_M}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{3/2}}-\frac{3}{2}\,\frac{\mu_M\left(x_1-x_M\right)\left(2\,x_1-2\,x_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{5/2}}\right)\Psi_3+\left(-\frac{3\,\mu_E x_1 x_2}{\left(x_1^2+x_2^2\right)^{5/2}}-\frac{3}{2}\,\frac{\mu_M\left(x_2-y_M\right)\left(2\,x_1-2\,x_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{5/2}}\right)\Psi_4$$

$$\left(-\frac{3\,\mu_E x_1 x_2}{\left(x_1^2+x_2^2\right)^{5/2}}-\frac{3}{2}\,\frac{\mu_M\left(x_1-x_M\right)\left(2\,x_2-2\,y_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{5/2}}\right)\Psi_3+\left(\frac{\mu_E}{\left(x_1^2+x_2^2\right)^{3/2}}-\frac{3\,\mu_E x_2^2}{\left(x_1^2+x_2^2\right)^{5/2}}+\frac{\mu_M}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{3/2}}-\frac{3}{2}\,\frac{\mu_M\left(x_2-y_M\right)\left(2\,x_2-2\,y_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{5/2}}\right)\Psi_4$$

$$-\Psi_1$$

$$-\Psi_2$$

$$\frac{u\cos(\phi)\,\Psi_3}{x_5^2}+\frac{u\sin(\phi)\,\Psi_4}{x_5^2} \tag{4}$$

Warunki koncowe

$$\textit{Warunki koncowe} \tag{5}$$

for i **from** 1 **by** 1 **to** 5 **do**
J[1, i];
 od;

$$-\rho\left(\frac{1}{2}\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2-r_M^2\right)\left(2x_1-2x_M\right)+\left(\left(x_1-x_M\right)\left(x_3-dx_M\right)+\left(x_2-y_M\right)\left(x_4-dy_M\right)\right)\left(x_3-dx_M\right)\right)$$

$$-\rho\left(\frac{1}{2}\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2-r_M^2\right)\left(2x_2-2y_M\right)+\left(\left(x_1-x_M\right)\left(x_3-dx_M\right)+\left(x_2-y_M\right)\left(x_4-dy_M\right)\right)\left(x_4-dy_M\right)\right)$$

$$-\rho\left(\frac{1}{2}\left(\left(x_3-dx_M\right)^2+\left(x_4-dy_M\right)^2-V_M^2\right)\left(2x_3-2dx_M\right)+\left(\left(x_1-x_M\right)\left(x_3-dx_M\right)+\left(x_2-y_M\right)\left(x_4-dy_M\right)\right)\left(x_1-x_M\right)\right)$$

$$-\rho\left(\frac{1}{2}\left(\left(x_3-dx_M\right)^2+\left(x_4-dy_M\right)^2-V_M^2\right)\left(2x_4-2dy_M\right)+\left(\left(x_1-x_M\right)\left(x_3-dx_M\right)+\left(x_2-y_M\right)\left(x_4-dy_M\right)\right)\left(x_2-y_M\right)\right)$$

$$K_1-\rho\left(\begin{cases} x_5-m_r & x_5\le m_r \\ 0 & m_r<x_5\end{cases}\right) \tag{6}$$

Gradient wzgldem sterowania

$M := PSI[1]\cdot f_1(X,U)+PSI[2]\cdot f_2(X,U)+PSI[3]\cdot f_3(X,U)+PSI[4]\cdot f_4(X,U)+PSI[5]\cdot f_5(X,U);$

$$\Psi_1 x_3+\Psi_2 x_4+\Psi_3\left(-\frac{\mu_E x_1}{\left(x_1^2+x_2^2\right)^{3/2}}-\frac{\mu_M\left(x_1-x_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{3/2}}+\frac{u\cos(\phi)}{x_5}\right)+\Psi_4\left(-\frac{\mu_E x_2}{\left(x_1^2+x_2^2\right)^{3/2}}-\frac{\mu_M\left(x_2-y_M\right)}{\left(\left(x_1-x_M\right)^2+\left(x_2-y_M\right)^2\right)^{3/2}}+\frac{u\sin(\phi)}{x_5}\right)+\Psi_5 C u \tag{7}$$

$diff(M,u);$

$diff(M,\phi);$

$$\frac{\Psi_3\cos(\phi)}{x_5}+\frac{\Psi_4\sin(\phi)}{x_5}+\Psi_5 C$$

$$-\frac{\Psi_3 u\sin(\phi)}{x_5}+\frac{\Psi_4 u\cos(\phi)}{x_5} \tag{8}$$

Gradient z parametrów

$x_{01} := r_E\cdot\cos(\theta_E):$

$x_{02} := r_E\cdot\sin(\theta_E):$

$x_{03} := -(V_E+V_i)\cdot\sin(\theta_E):$

$x_{04} := (V_E+V_i)\cdot\cos(\theta_E):$

$x_{05} := m_i \cdot e^{C \cdot V_i}$:

$f_i := [x_{01}, x_{02}, x_{03}, x_{04}, x_{05}, x_{06}, x_{07}, x_{08}, x_{09}]$:

$param := [\theta_E, V_i]$:

$N_i := Transpose(Jacobian(f_i, param))$;

for i **from** 1 **by** 1 **to** 2 **do**

$P[i] := -N_i[i,1] \cdot PSI[1] - N_i[i,2] \cdot PSI[2] - N_i[i,3] \cdot PSI[3] - N_i[i,4] \cdot PSI[4] - N_i[i,5] \cdot PSI[5]$

od;

$$\left[\left[-r_E \sin(\theta_E), r_E \cos(\theta_E), -(V_E + V_i)\cos(\theta_E), -(V_E + V_i)\sin(\theta_E), 0, 0, 0, 0, 0\right], \left[0, 0, -\sin(\theta_E), \cos(\theta_E), m_i e^{C V_i} C \ln(e), 0, 0, 0, 0\right]\right]$$

$$r_E \sin(\theta_E)\Psi_1 - r_E \cos(\theta_E)\Psi_2 + (V_E + V_i)\cos(\theta_E)\Psi_3 + (V_E + V_i)\sin(\theta_E)\Psi_4$$

$$\sin(\theta_E)\Psi_3 - \cos(\theta_E)\Psi_4 - m_i e^{C V_i} C \ln(e)\Psi_5 \qquad \textbf{(9)}$$