```
with(VectorCalculus):
with(LinearAlgebra):
X := [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9]:
U := [u, phi]:
PSI := [Psi_1, Psi_2, Psi_3, Psi_4, Psi_5, Psi_6, Psi_7, Psi_8, Psi_9]:
γ_E := x -> norm(<x[3], x[4]>, 2, conjugate = false):
γ_M := x -> norm(<x[3] - x[1], x[4] - x[2]>, 2, conjugate = false):
f_1 := (x, u) -> x[5]:
f_2 := (x, u) -> x[6]:
f_3 := (x, u) -> x[7]:
f_4 := (x, u) -> x[8]:
f_5 := (x, u) -> -μ_E·x[1]/d^3:
f_6 := (x, u) -> -μ_E·x[2]/d^3:
f_7 := (x, u) -> -μ_E·x[3]/γ_E(x)^3 - μ_M·(x[3] - x[1])/γ_M(x)^3 + u[1]·cos(u[2])/x[9]:
f_8 := (x, u) -> -μ_E·x[4]/γ_E(x)^3 - μ_M·(x[4] - x[2])/γ_M(x)^3 + u[1]·sin(u[2])/x[9]:
f_9 := (x, u) -> C·u[1]:
k_1 := x -> 1/4·((x[3] - x[1])^2 + (x[4] - x[2])^2 - r_M^2)^2:
k_2 := x -> 1/4·((x[7] - x[5])^2 + (x[8] - x[6])^2 - V_M^2)^2:
k_3 := x -> 1/2·((x[3] - x[1])·(x[7] - x[5]) + (x[4] - x[2])·(x[8] - x[6]))^2:
k_4 := x -> piecewise(x[9] < m_r, 1/2·(m_r - x[9])^2):
f := (x, u) -> [f_1(x, u), f_2(x, u), f_3(x, u), f_4(x, u), f_5(x, u), f_6(x, u), f_7(x, u), f_8(x, u), f_9(x, u)]:
Q := (x, u) -> -x[9] + K_1·T + K_2·(k_1(x) + k_2(x) + k_3(x) + k_4(x)):
H := -Transpose(Jacobian(f(X, U), X)):
J := -Jacobian(<Q(X, U)>, X):
Model
```

$$Model \tag{1}$$

```
for i from 1 by 1 to 9 do
f(X, U)[i]
od;
```

$$x_5$$

$$x_6$$

$$x_7$$

$$x_8$$

$$-\frac{\mu_E x_1}{d^3}$$

$$-\frac{\mu_E x_2}{d^3}$$

$$-\frac{\mu_E x_3}{\left(x_3^2+x_4^2\right)^{3/2}}-\frac{\mu_M\left(x_3-x_1\right)}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{3/2}}+\frac{u\cos(\phi)}{x_9}$$

$$-\frac{\mu_E x_4}{\left(x_3^2+x_4^2\right)^{3/2}}-\frac{\mu_M\left(x_4-x_2\right)}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{3/2}}+\frac{u\sin(\phi)}{x_9}$$

$$C\,u \tag{2}$$

Rownania sprzężone

Rownania sprzężone **(3)**

for i **from** 1 **by** 1 **to** 9 **do**
for j **from** 1 **by** 1 **to** 9 **do**
$K[j] := H[i,j]\cdot PSI[j]$
od;
$K[1]+K[2]+K[3]+K[4]+K[5]+K[6]+K[7]+K[8]+K[9]$
od;

$$\frac{\mu_E \Psi_5}{d^3}+\left(-\frac{\mu_M}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{3/2}}-\frac{3}{2}\,\frac{\mu_M\left(x_3-x_1\right)\left(-2x_3+2x_1\right)}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{5/2}}\right)\Psi_7$$
$$-\frac{3}{2}\,\frac{\mu_M\left(x_4-x_2\right)\left(-2x_3+2x_1\right)\Psi_8}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{5/2}}$$

$$\frac{\mu_E \Psi_6}{d^3}-\frac{3}{2}\,\frac{\mu_M\left(x_3-x_1\right)\left(-2x_4+2x_2\right)\Psi_7}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{5/2}}+\left(-\frac{\mu_M}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{3/2}}\right.$$
$$\left.-\frac{3}{2}\,\frac{\mu_M\left(x_4-x_2\right)\left(-2x_4+2x_2\right)}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{5/2}}\right)\Psi_8$$

$$\left(\frac{\mu_E}{\left(x_3^2+x_4^2\right)^{3/2}}-\frac{3\,\mu_E x_3^2}{\left(x_3^2+x_4^2\right)^{5/2}}+\frac{\mu_M}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{3/2}}\right.$$
$$\left.-\frac{3}{2}\,\frac{\mu_M\left(x_3-x_1\right)\left(2x_3-2x_1\right)}{\left(\left(x_3-x_1\right)^2+\left(x_4-x_2\right)^2\right)^{5/2}}\right)\Psi_7+\left(-\frac{3\,\mu_E x_3 x_4}{\left(x_3^2+x_4^2\right)^{5/2}}\right.$$

$$-\frac{3}{2}\,\frac{\mu_M\,(x_4-x_2)\,(2\,x_3-2\,x_1)}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{5/2}}\Bigg)\,\Psi_8$$

$$\left(-\frac{3\,\mu_E\,x_3\,x_4}{\left(x_3^{\,2}+x_4^{\,2}\right)^{5/2}}-\frac{3}{2}\,\frac{\mu_M\,(x_3-x_1)\,(2\,x_4-2\,x_2)}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{5/2}}\right)\Psi_7+\left(\frac{\mu_E}{\left(x_3^{\,2}+x_4^{\,2}\right)^{3/2}}\right.$$

$$\left.-\frac{3\,\mu_E\,x_4^{\,2}}{\left(x_3^{\,2}+x_4^{\,2}\right)^{5/2}}+\frac{\mu_M}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{3/2}}-\frac{3}{2}\,\frac{\mu_M\,(x_4-x_2)\,(2\,x_4-2\,x_2)}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{5/2}}\right)$$

$$\Psi_8$$

$$-\Psi_1$$

$$-\Psi_2$$

$$-\Psi_3$$

$$-\Psi_4$$

$$\frac{u\cos(\phi)\,\Psi_7}{x_9^{\,2}}+\frac{u\sin(\phi)\,\Psi_8}{x_9^{\,2}} \qquad \textbf{(4)}$$

Warunki koncowe

Warunki koncowe **(5)**

for *i* **from** 1 **by** 1 **to** 9 **do**
J[1, i];
od;

$$-K_2\left(\frac{1}{2}\left((x_3-x_1)^2+(x_4-x_2)^2-r_M^{\,2}\right)(-2\,x_3+2\,x_1)+\left((x_3-x_1)\,(x_7-x_5)+(x_4-x_2)\,(x_8-x_6)\right)(-x_7+x_5)\right)$$

$$-K_2\left(\frac{1}{2}\left((x_3-x_1)^2+(x_4-x_2)^2-r_M^{\,2}\right)(-2\,x_4+2\,x_2)+\left((x_3-x_1)\,(x_7-x_5)+(x_4-x_2)\,(x_8-x_6)\right)(-x_8+x_6)\right)$$

$$-K_2\left(\frac{1}{2}\left((x_3-x_1)^2+(x_4-x_2)^2-r_M^{\,2}\right)(2\,x_3-2\,x_1)+\left((x_3-x_1)\,(x_7-x_5)+(x_4-x_2)\,(x_8-x_6)\right)(x_7-x_5)\right)$$

$$-K_2\left(\frac{1}{2}\left((x_3-x_1)^2+(x_4-x_2)^2-r_M^{\,2}\right)(2\,x_4-2\,x_2)+\left((x_3-x_1)\,(x_7-x_5)+(x_4-x_2)\,(x_8-x_6)\right)(x_8-x_6)\right)$$

$$-K_2\left(\frac{1}{2}\left((x_7-x_5)^2+(x_8-x_6)^2-V_M^{\,2}\right)(-2\,x_7+2\,x_5)+\left((x_3-x_1)\,(x_7-x_5)+(x_4\right.$$

$$-x_2)\ (x_8-x_6))\ (-x_3+x_1)\Big)$$

$$-K_2\left(\frac{1}{2}\ \left((x_7-x_5)^2+(x_8-x_6)^2-V_M{}^2\right)\ (-2\,x_8+2\,x_6)+\left((x_3-x_1)\ (x_7-x_5)+(x_4\right.\right.$$
$$\left.\left.-x_2)\ (x_8-x_6)\right)\ (-x_4+x_2)\right)$$

$$-K_2\left(\frac{1}{2}\ \left((x_7-x_5)^2+(x_8-x_6)^2-V_M{}^2\right)\ (2\,x_7-2\,x_5)+\left((x_3-x_1)\ (x_7-x_5)+(x_4\right.\right.$$
$$\left.\left.-x_2)\ (x_8-x_6)\right)\ (x_3-x_1)\right)$$

$$-K_2\left(\frac{1}{2}\ \left((x_7-x_5)^2+(x_8-x_6)^2-V_M{}^2\right)\ (2\,x_8-2\,x_6)+\left((x_3-x_1)\ (x_7-x_5)+(x_4\right.\right.$$
$$\left.\left.-x_2)\ (x_8-x_6)\right)\ (x_4-x_2)\right)$$

$$1-K_2\left(\begin{cases} x_9-m_r & x_9\le m_r \\ 0 & m_r<x_9 \end{cases}\right) \tag{6}$$

Gradient wzgldem sterowania

$M := PSI[1]\cdot f_1(X,U)+PSI[2]\cdot f_2(X,U)+PSI[3]\cdot f_3(X,U)+PSI[4]\cdot f_4(X,U)+PSI[5]\cdot f_5(X,U)$
$+PSI[6]\cdot f_6(X,U)+PSI[7]\cdot f_7(X,U)+PSI[8]\cdot f_8(X,U)+PSI[9]\cdot f_9(X,U);$

$$\Psi_1 x_5+\Psi_2 x_6+\Psi_3 x_7+\Psi_4 x_8-\frac{\Psi_5\,\mu_E\,x_1}{d^3}-\frac{\Psi_6\,\mu_E\,x_2}{d^3}+\Psi_7\left(-\frac{\mu_E\,x_3}{\left(x_3{}^2+x_4{}^2\right)^{3/2}}\right. \tag{7}$$
$$\left.-\frac{\mu_M\,(x_3-x_1)}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{3/2}}+\frac{u\cos(\phi)}{x_9}\right)+\Psi_8\left(-\frac{\mu_E\,x_4}{\left(x_3{}^2+x_4{}^2\right)^{3/2}}\right.$$
$$\left.-\frac{\mu_M\,(x_4-x_2)}{\left((x_3-x_1)^2+(x_4-x_2)^2\right)^{3/2}}+\frac{u\sin(\phi)}{x_9}\right)+\Psi_9\,C\,u$$

$diff(M,u);$
$diff(M,\phi);$

$$\frac{\Psi_7\cos(\phi)}{x_9}+\frac{\Psi_8\sin(\phi)}{x_9}+\Psi_9\,C$$
$$-\frac{\Psi_7\,u\sin(\phi)}{x_9}+\frac{\Psi_8\,u\cos(\phi)}{x_9} \tag{8}$$

Gradient z parametrów

$x_{01} := d:$
$x_{02} := 0:$
$x_{03} := r_E\cdot\cos(\theta_E):$

$x_{04} := r_E \cdot \sin(\theta_E)$:

$x_{05} := 0$:

$x_{06} := d \cdot \omega$:

$x_{07} := -(V_E + V_i) \cdot \sin(\theta_E)$:

$x_{08} := (V_E + V_i) \cdot \cos(\theta_E)$:

$x_{09} := m_i \cdot e^{C \cdot V_i}$:

$f_i := [x_{01}, x_{02}, x_{03}, x_{04}, x_{05}, x_{06}, x_{07}, x_{08}, x_{09}]$:

$param := [\theta_E, V_i]$:

$N_i := Transpose(Jacobian(f_i, param))$;

for i **from** 1 **by** 1 **to** 2 **do**

$P[i] := -N_i[i,1] \cdot PSI[1] - N_i[i,2] \cdot PSI[2] - N_i[i,3] \cdot PSI[3] - N_i[i,4] \cdot PSI[4] - N_i[i,5] \cdot PSI[5]$
$- N_i[i,6] \cdot PSI[6] - N_i[i,7] \cdot PSI[7] - N_i[i,8] \cdot PSI[8] - N_i[i,9] \cdot PSI[9]$

od;

$$\left[\left[0, 0, -r_E \sin(\theta_E), r_E \cos(\theta_E), 0, 0, -(V_E + V_i) \cos(\theta_E), -(V_E + V_i) \sin(\theta_E), 0\right],\right.$$
$$\left.\left[0, 0, 0, 0, 0, 0, -\sin(\theta_E), \cos(\theta_E), m_i e^{C V_i} C \ln(e)\right]\right]$$

$$r_E \sin(\theta_E) \Psi_3 - r_E \cos(\theta_E) \Psi_4 + (V_E + V_i) \cos(\theta_E) \Psi_7 + (V_E + V_i) \sin(\theta_E) \Psi_8$$
$$\sin(\theta_E) \Psi_7 - \cos(\theta_E) \Psi_8 - m_i e^{C V_i} C \ln(e) \Psi_9 \qquad \textbf{(9)}$$