تمرین سری ۱

مریم کریمی جعفری - 99106617

سوال‌های عملی

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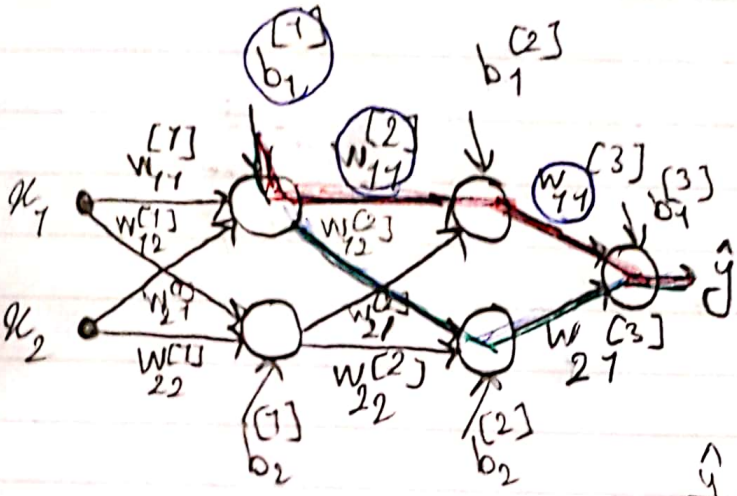

$$loss = (y - \hat{y})^2$$

$$cost\ function = \frac{1}{2n} \sum_{i=1}^{n} (y^{(i)} - \hat{y}^{(i)})^2$$

$$\hat{y} = a_1^{[3]}$$

$f_1 = f_2 \Rightarrow tanh \Rightarrow f_1' = f_2' = 1 - f_1^2$

$f_3 \Rightarrow sigmaid \Rightarrow f_3' = f_3(z)\,[1 - f_3(z)]$

$\frac{\partial loss}{\partial w_{11}^{[3]}} = ?$

$$z_1^{[3]} = w_{11}^{[3]} a_1^{[2]} + w_{21}^{[3]} a_2^{[2]} + b_1^{[3]}$$

$$\frac{\partial loss}{\partial w_{11}^{[3]}} = \frac{\partial loss}{\partial a_1^{[3]}} \frac{\partial a_1^{[3]}}{\partial z_1^{[3]}} \frac{\partial z_1^{[3]}}{\partial w_{11}^{[3]}} = \delta_1^{[3]} f_3'(z_1^{[3]}) a_1^{[2]}$$

$$\delta_1^{[3]} = -2(y - \hat{y})$$

$$f_3'(z_1^{[3]}) = \frac{1}{1+e^{-z_1^{[3]}}} \left[1 - \frac{1}{1+e^{-z_1^{[3]}}}\right]$$

$$a_1^{[2]} = z_1^{[2]}$$

$$z_1^{[2]} = w_{11}^{[2]} a_1^{[1]} + w_{21}^{[2]} a_2^{[1]} + b_1^{[2]}$$

$\frac{\partial loss}{\partial w_{11}^{[2]}} = ?$

$z_1^{[2]} = w_{11}^{[2]} a_1^{[1]} + w_{21}^{[2]} a_2^{[1]} + b_1^{[2]}$

$\frac{\partial loss}{\partial w_{11}^{[2]}} = \underbrace{\frac{\partial loss}{\partial a_1^{[3]}} \frac{\partial a_1^{[3]}}{\partial z_1^{[3]}} \frac{\partial z_1^{[3]}}{\partial a_1^{[2]}}}_{\delta_1^{[2]}} \frac{\partial a_1^{[2]}}{\partial z_1^{[2]}} \frac{\partial z_1^{[2]}}{\partial w_{11}^{[2]}}$

$\delta_1^{[2]} = \delta_1^{[3]} f_3'(z_1^{[3]}) w_{11}^{[3]}$

$\Rightarrow \frac{\partial loss}{\partial w_{11}^{[2]}} = \delta_1^{[2]} f_2'(z_1^{[2]}) a_1^{[1]} = \delta_1^{[3]} f_3'(z_1^{[3]}) w_{11}^{[3]} f_2'(z_1^{[2]}) a_1^{[1]}$

در صفحه قبل گفته شده.

$f_2'(z_1^{[2]}) = 1 - \left(\frac{e^{z_1^{[2]}} - e^{-z_1^{[2]}}}{e^{z_1^{[2]}} + e^{-z_1^{[2]}}}\right)^2$

$a_1^{[1]} = f_1(z_1^{[1]})$

$z_1^{[1]} = w_{11}^{[1]} x_1 + w_{21}^{[1]} x_2 + b_1^{[1]}$

$\frac{\partial loss}{\partial b_1^{[1]}} = ?$

۲ مسیر داریم

مسیر اول (کوتاه‌تر)

$$\frac{\partial loss}{\partial b_1^{[1]}} = \frac{\partial loss}{\partial a_1^{[3]}} \frac{\partial a_1^{[3]}}{\partial z_1^{[3]}} \frac{\partial z_1^{[3]}}{\partial a_1^{[2]}} \frac{\partial a_1^{[2]}}{\partial z_1^{[2]}} \frac{\partial z_1^{[2]}}{\partial a_1^{[1]}} \frac{\partial a_1^{[1]}}{\partial z_1^{[1]}} \frac{\partial z_1^{[1]}}{\partial b_1^{[1]}}$$

$\frac{\partial z_1^{[1]}}{\partial b_1^{[1]}} = 1$

$\delta_1^{[1]}$ مشترک

$\delta_1^{[1]} = \delta_1^{[2]} \frac{\partial a_1^{[2]}}{\partial z_1^{[2]}} \frac{\partial z_1^{[2]}}{\partial a_1^{[1]}} = \delta_1^{[2]} f_2'(z_1^{[2]}) w_{11}^{[2]}$

که قبلاً محاسبه شده

$$\Rightarrow \frac{\partial loss}{\partial b_1^{[1]}} = \delta_1^{[1]} f_1'(z_1^{[1]})$$

$$f_1'(z_1^{[1]}) = 1 - \left(\frac{e^{z_1^{[1]}} - e^{-z_1^{[1]}}}{e^{z_1^{[1]}} + e^{-z_1^{[1]}}}\right)^2$$

مسیر دوم (کمی سخت‌تر)

$$\frac{\partial loss}{\partial b_1^{[1]}} = \frac{\partial loss}{\partial a_1^{[3]}} \frac{\partial a_1^{[3]}}{\partial z_1^{[3]}} \frac{\partial z_1^{[3]}}{\partial a_2^{[2]}} \frac{\partial a_2^{[2]}}{\partial z_2^{[2]}} \frac{\partial z_2^{[2]}}{\partial a_1^{[1]}} \frac{\partial a_1^{[1]}}{\partial z_1^{[1]}} \frac{\partial z_1^{[1]}}{\partial b_1^{[1]}}$$

$\delta_1^{[1]}$ مشترک دوم

$\delta_1^{[1]} = \delta_2^{[2]} \frac{\partial a_2^{[2]}}{\partial z_2^{[2]}} \frac{\partial z_2^{[2]}}{\partial a_1^{[1]}} = \delta_2^{[2]} f_2'(z_2^{[2]}) w_{12}^{[2]}$

$$\Rightarrow \delta_1^{[1]} = \sum_{j=1}^{2} \delta_j^{[2]} f'(z_j^{[2]}) w_{1j}^{[2]}$$